Supplementary Information

Effect of airflow rate and drainage on the properties of 2D smectic liquid crystal foams

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1. Fabrication of 2D liquid crystal foams

Our research involved creating 2D liquid crystal foams. We conducted experiments to determine how the thickness of the container affects the foam's structure, as shown in Figure S1. We used the same airflow rate (1 mL/min) and containers of varying thicknesses (2 mm, 4 mm, 6 mm, and 8 mm) to create the foam. Our findings revealed that a container thickness of 2 mm is the threshold required to produce 2D foams (a one-layer of bubbles). Any container with a greater thickness will result in multiple layers and the formation of 3D LC foams, which is not the focus of this study.

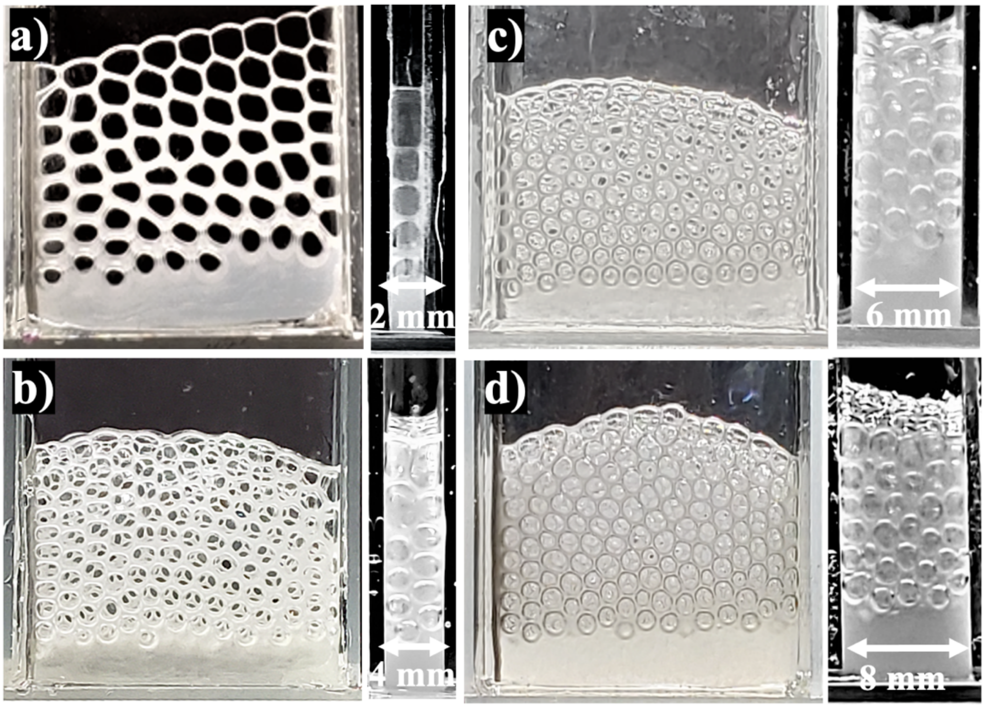

Figure S1. Effect of the container thickness on the structure of LC foam. Front view (left) and side view (right) of LC foams fabricated under the airflow rate of 1 mL/min in rectangular containers of thickness: **(a)** 2 mm, **(b)** 4 mm, **(c)** 6 mm, and **(d)** 8mm.

2. Data analysis

Each sample is analyzed vertically from bottom to top. The foam structure is divided into 4 different sections (Bins), as shown in Figure S2-a:

- Bin 1 in the black box, located approximately 18 mm above the base of the container.
- Bin 2 in the red box, located approximately 27 mm above the base of the container.
- Bin 3 in the blue box, located approximately 37 mm above the base of the container.
- Bin 4 in the pink box, located approximately 47 mm above the base of the container.

To analyze our data, we utilize *ImageJ*, an image processing software. We start by adjusting the pictures into 8-bit black and white images. Next, we use the Particle Analysis tool to identify and extract specific cell areas: the black regions inside the cells in Figure S2-a. It's important to note here that our measurements exclude the cells near the boundary to prevent any inaccuracies in the data analysis.

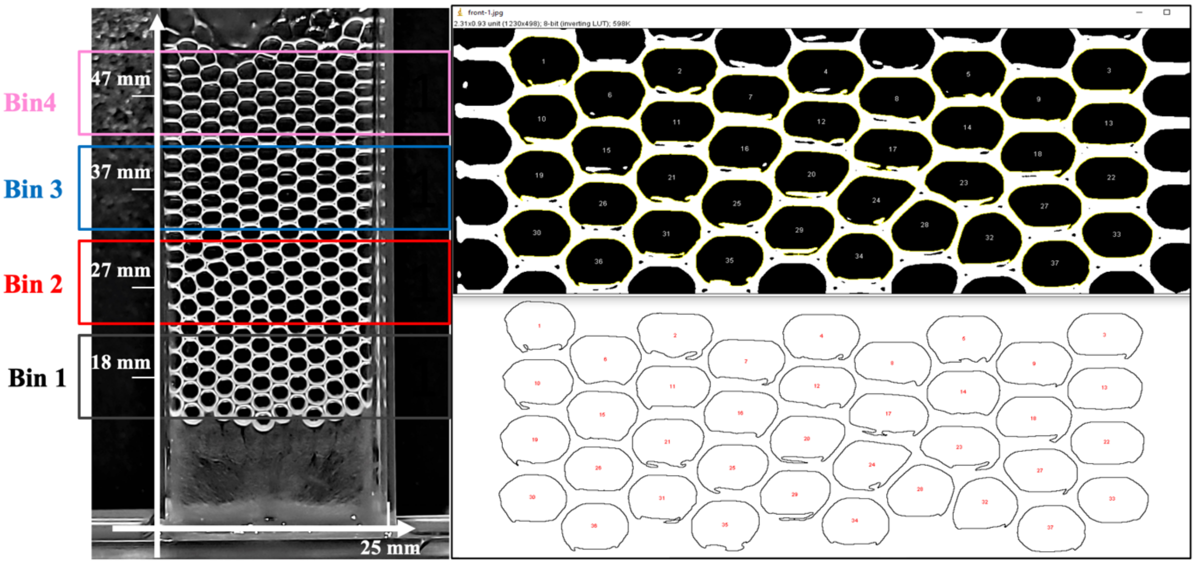

Figure S2: Image analysis of the foam using *ImageJ* software. **(a)** Each sample is divided vertically into 4 Bins from bottom to top. **(b)** An example showing the measurement of the cell numbers and areas using *ImageJ*.

3. Density profile of foam under gravity

In this section, we derive the expression of the density profile of foams under gravity using the case of simple fluids, adopted from the reference[1]. For a foam cell, the pressure difference across a single surface of a Plateau border is given by Laplace law:

$$\Delta P = p_g - p_l = \frac{\gamma}{r}$$

where p_g is the pressure of the gas, which considered constant, p_l is pressure of the liquid, γ is the surface tension between the fluid and gas, and *r* is the radius of the Plateau border.

Under equilibrium, the pressure inside the liquid must follow the conventional hydrostatic law, where the pressure changes as a function of the vertical position x as:

$$p_l = p_0 + \rho g(x - x_0),$$

where p_0 is the liquid pressure at the top layer of the foam ($x = x_0$), ρ is the density of the fluid, and g is the gravitational acceleration. Applying the Laplace law, we can determine the radius $r(x)$ of the Plateau border as a function of the vertical position as:

$$r(x) = \frac{\gamma}{p_g - p_0 + \rho g(x_0 - x)}.$$

Here, we assume the bubble as a circle with an outer radius *R* and inner radius r. Since the liquid fraction describes the concentration of the liquid content with the bubble, the liquid fraction of 2D foam can be estimated as[1]:

$$\varphi \sim \tilde{c}\frac{r^2}{R^2},$$

where, $\tilde{c}$ is a geometric parameter, which depends on the structure of the 2D foam. Now, we can present the Plateau border radius r in terms of the liquid fraction φ and the bubble volume (with radius *R*). This leads to the expression for the liquid fraction as a function of height as:

$$\varphi(x) = \frac{\tilde{c}\,(\gamma/R)^2}{[p_g - p_0 - \rho g(x - x_0)]^2}.$$

As mentioned earlier, the constant p_0 is the liquid pressure at the top layer of the foam. To determine the value of p_0, we need to consider the bottom layer of the foam in contact with the reservoir at x_b, where the liquid fraction φ reaches the wet foam limit φ^{cri} :

$$\varphi(x_b) = \varphi^{cri}.$$

We apply this condition at $x = x_b$. We obtain the expression for the liquid pressure at the top layer as:

$$p_0 = p_g - \rho g(x_b - x_0) - \left(\frac{\tilde{c}}{\varphi^{cri}}\right)^{\frac{1}{2}}\left(\frac{\gamma}{R}\right).$$

From this equation we get:

$$p_g - p_0 = \rho g(x_b - x_0) + \left(\frac{\tilde{c}}{\varphi^{cri}}\right)^{\frac{1}{2}}\left(\frac{\gamma}{R}\right).$$

We substitute this expression into the liquid fraction function $\varphi(x)$, and eliminate the pressure terms p_g and p_0. A new liquid fraction function that depends on the vertical position x can be determined:

$$\varphi(x) = \frac{\tilde{c}\,({}^{\gamma}/_{R})^2}{[\,\rho g(x_b - x_0) + (\frac{\tilde{c}}{\varphi^{cri}})^{\frac{1}{2}}(\frac{\gamma}{R}) - \rho g(x - x_0)]^2}\,,$$

We simplify this equation and obtain:

$$\varphi(x) = \left\{\frac{\tilde{c}^{\,\frac{1}{2}}\,({}^{\gamma}/_{R})}{\rho g(x_b - x) + \left(\frac{\tilde{c}}{\varphi^{cri}}\right)^{\frac{1}{2}}\left(\frac{\gamma}{R}\right)}\right\}^2,$$

or

$$\varphi(\Delta x) = \left\{\frac{\tilde{c}^{\,\frac{1}{2}}\,({}^{\gamma}/_{R})}{\rho g(\Delta x) + \left(\frac{\tilde{c}}{\varphi^{cri}}\right)^{\frac{1}{2}}\left(\frac{\gamma}{R}\right)}\right\}^2,$$

where $\Delta x = x_b - x$. This expression of liquid fraction is used to determine if the density profile of smectic foams acts similarly to that of simple fluid foams, as described in the main manuscript.

References

[1] Weaire. D and Hutzler. S, The physics of foams. United Kingdom, Clarendon Press, 1999.