## Supplementary Information:

# Molecular mechanisms and energetics of lipid droplet formation and directional budding 

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## Supplementary Text

## S1. Assembling initial slabs

We started by assembling initial slabs, wherein a given volume $V$ of oil is covered by two flat lipid monolayers from below and above with their hydrophobic chains facing the oil, see Fig. 1 l and supplementary Fig. S1b. Initial monolayers were assembled by placing different number of lipids $N_{l i p}$, equally divided between two monolayers of fixed width $L$ in a cubic simulation box with periodic boundary conditions. Each slab with a given number of lipids $N_{l i p}$ has then a fixed projected area-per-lipid $A_{l p}=2 A_{s} / N_{l i p}$, defined as the average area-per-lipid projected into the horizontal plane.

## S2. Shape parameters of LDs

We first computed the thickness of the ER-bilayer $l_{b}=4.08 \mathrm{~nm}$ as the distance between the peaks of the density of lipid head groups of a flat bilayer, see supplementary Fig. S1a. The monolayer thickness was then taken to be $l_{m}=0.5 l_{b}=2.04 \mathrm{~nm}$. We assumed that LD-monolayers of all LDs, obtained from DPD simulations, have the shape of spherical caps to minimize membrane bending cost for a given membrane surface area. To find the shape of symmetrical LD lenses and spherical LDs, we fitted spherical caps to the simulation results and computed their shape parameters according to Figure 2a. Complete spheres of size $R=0.5 D$ were fitted to the asymmetric budding LDs. The contact point of the ER-bilayer with these spheres were then used to find the LD angles as shown in the right panel of Fig. 5b. The spherical caps of symmetrical LDs were parametrized by the radius of the sphere and the Cartesian coordinates $x_{s}, y_{s}$, and $z_{s}$ of its center. The sum $S$ of the mean square distances of all beads $i$ of LD lipids from the surface of the cap

$$
\begin{equation*}
S=\sum_{i}\left(\sqrt{\left(x_{i}-x_{s}\right)^{2}+\left(y_{i}-y_{s}\right)^{2}+\left(z_{i}-z_{s}\right)^{2}}-R\right)^{2} \tag{S1}
\end{equation*}
$$

was then minimized by setting the derivatives

$$
\begin{equation*}
\partial S / \partial x_{s}=\partial S / \partial y_{s}=\partial S / \partial z_{s}=\partial S / \partial R=0 \tag{S2}
\end{equation*}
$$

leading to a system of four equations from which the radius $R$ of the cap and the Cartesian coordinates of the cap center were obtained. The contact angle was then computed from the fitted cap. In this way, we fitted spherical caps to LD-monolayers and obtained the radii and lens angles of the luminal and cytosolic LD-monolayers for different LD shapes.

## S3. Membrane tension and bending stiffness

To find membrane properties such as membrane thickness and bending stiffness of bilayers and monolayers, we simulated small flat bilayers and small slabs. The slabs were simulated in a cubic box of fixed width 25.6 nm and initial height 25.6 nm with $N_{\text {oil }}=10,363$ oil molecules surrounded by 35,852 water beads forming slabs of size $D=24.2 \mathrm{~nm}$, see Fig. 17. The number of lipids $N_{l i p}$, equally partitioned between the two slab monolayers, was varied to generate slabs with different $A_{l p}$, see supplementary Table S 2 . Flat bilayers were also simulated in cubic boxes of the same size with the same $N_{l i p}$ and thus the same $A_{l p}$ as the slabs, see the snapshot in supplementary Fig. S2c and the data in supplementary Table S2.

Both slabs and flat bilayers were first equilibrated for $2 \mu s$ in an NPT ensemble using a Berendsen barostat in the vertical direction $z$, see supplementary Fig. S2a,c, adjusting the pressure at $23.7 k_{B} T / d^{3}=190.4 \times 10^{9} \mathrm{mN} / \mathrm{m}^{2}$. During NPT simulations the height of the simulation boxes changed to adjust the pressure while their widths remained constant at 25.6 nm . We note that the density of the simulation box, including both water and oil, slightly varies during NPT simulations but the water density remains at $5.86 / \mathrm{nm}^{3}$ as shown in the density plots in the bottom panels of Fig. S1. Membrane tensions were then computed in following $22 \mu s$-long runs in an NVT ensemble with fixed simulation boxes at
room temperature. We used the integral

$$
\begin{equation*}
\Sigma=\int_{-h}^{h} s(z) d z=\int_{-h}^{h}\left[P_{N}(z)-P_{T}(z)\right] d z \tag{S3}
\end{equation*}
$$

of the stress profile $s(z)=P_{N}(z)-P_{T}(z)$ to find both monolayer and bilayer tensions, $\Sigma_{m}$ and $\Sigma_{b}$, respectively. Here, $P_{N}$ and $P_{T}$ denote normal and tangential pressure components, computed across an almost planar patch of a bilayer or a monolayer membrane in the $z$ direction perpendicular to the membrane surface. ${ }^{[12]}$ We first calculated membrane tensions inside small slabs of size $D=24.2 \mathrm{~nm}$ to demonstrate that monolayer tensions increase linearly with $A_{l p}$. The Cartesian coordinate system was placed in the center of the slabs and the stress profile was computed inside a box of width 13 nm and height $2 h=20 \mathrm{~nm}$, see supplementary Fig. S2a. The monolayer tensions were then calculated as

$$
\begin{align*}
\Sigma_{l m} & =\int_{0}^{h} s(z) d z=\int_{0}^{h}\left[P_{N}(z)-P_{T}(z)\right] d z  \tag{S4}\\
\Sigma_{c m} & =\int_{0}^{-h} s(z) d z=\int_{0}^{-h}\left[P_{N}(z)-P_{T}(z)\right] d z
\end{align*}
$$

The monolayer tension $\Sigma_{m}=0.5\left(\Sigma_{l m}+\Sigma_{c m}\right)$ was taken to be the average of almost equal tensions in the outer (cytosolic) and inner (luminal) monolayers. Supplementary Figure S2b shows the stress profiles of some of these small slabs with corresponding $\Sigma_{m}$ 's and $A_{l p}$ 's whose values are listed in supplementary Table S2.

We computed the monolayer tension $\Sigma_{m}$ in the initial flat monolayers of three small slabs of size $D=24.2 \mathrm{~nm}$ and area $A_{s}=655.4 \mathrm{~nm}^{2}$, with different $A_{l p}$ 's, see Fig. 110. The slab area was chosen to be smaller than $A_{s m}=1115.3 \mathrm{~nm}^{2}$ for $L=25.6 \mathrm{~nm}$ to prevent the slab-to-lens transition. The monolayer tension increased from $0.4 \mathrm{mN} / \mathrm{m}$ for $A_{l p}=0.72 \mathrm{~nm}^{2}$ to $10.5 \mathrm{mN} / \mathrm{m}$ for $A_{l p}=0.82 \mathrm{~nm}^{2}$ corresponding to monolayers with $N_{l i p}=1800$ and $N_{\text {lip }}=1600$ lipids, respectively, see supplementary Fig. S2b and Table S2. We thus verified that lipid membranes with larger $A_{l p}$ 's experience larger membrane tensions.

We then used equation (S3) to obtain stress profiles of flat bilayers in a box of width

13 nm and initial height $h=5 \mathrm{~nm}$, see supplementary Fig. S2c. Supplementary Figure S2d shows the resulting stress profiles across two flat bilayers. For both bilayers and monolayers (slabs), membrane tensions and areas-per-lipid were averaged over 400 simulation frames. These simulations were performed for $20 \mu s$ in an NVT ensemble during which 400 equally distanced frames were picked to compute the average $\Sigma$ 's and $A_{l p}$ 's as listed in supplementary Table S2. These tensions, plotted against the corresponding $A_{l p}$ values by blue and red dots in supplementary Fig. S2e, were then used to compute the bending rigidities of the ER-bilayer and LD-monolayers.

We fitted a line to the data of $\Sigma_{m}$ versus $A_{l p}$, and found $A_{l 0}^{m}=0.715 \mathrm{~nm}^{2}$ corresponding to zero tension $\Sigma_{m}=0$. The area extension elastic modulus was then obtained as the product $K_{A}^{m}=A_{l 0}^{m}\left(d \Sigma_{m} / d A_{l}^{m}\right)=68.3 \mathrm{pN} / n m$, where $d \Sigma_{m} / d A_{l}^{m}=95.6 \mathrm{pN} / \mathrm{nm}^{3}$. ${ }^{3}$ The bending rigidity of the monolayer was then found as $\kappa_{m}=(1 / 12) K_{A}^{m} l_{m}^{2}=23.7 p N \cdot n m$, where $l_{m}=2.04 \mathrm{~nm}$ is the monolayer thickness i.e. half the bilayer thickness $l_{m}=0.5 l_{b}$. A similar procedure was used to find the bending rigidity of the ER-bilayer as:

$$
\begin{align*}
d \Sigma_{b} / d A_{l}^{b} & =215.3 \mathrm{pN} / \mathrm{nm}^{3} \\
A_{l 0}^{b} & =0.778 \mathrm{~nm}^{2}  \tag{S5}\\
K_{A}^{b} & =A_{l 0}^{b}\left(d \Sigma_{b} / d A_{l}^{b}\right)=167.5 \mathrm{pN} / \mathrm{nm} \\
\kappa_{b} & =(1 / 48) K_{A}^{b} l_{b}^{2}=58.1 \mathrm{pN} \cdot \mathrm{~nm}
\end{align*}
$$

In this way, we found the bending rigidities, $\kappa_{b}=58.1 p N \cdot n m$ and $\kappa_{m}=23.7 p N \cdot n m$, of the ER-bilayer and LD-monolayers, respectively.

## S4. LD parameters

Spontaneous formation of LDs from initial oil slabs were simulated in a fixed simulation box of size $(48 \times 48 \times 72) n m^{3}$ composed of $N_{\text {lip }}$ lipids placed in two monolayers exposed to water from above and below with a fixed total of 972000 beads. The number of lipids was then
varied, as listed in supplementary Table $S 3$, to change the projected area-per-lipid $A_{l p}$ for different LD sizes $D=29.5,31$, and 33.2 nm as seen in the morphological diagram in Fig. 11. Spontaneous formation of different LD structures occurred in $5 \mu s$ in an NPT ensemble at constant pressure $190.4 \times 10^{9} \mathrm{mN} / \mathrm{m}^{2}$, using a Berendsen barostat in the vertical direction $z$ perpendicular to the slab monolayers. The initial slabs transformed to a variety of LD structures as shown in Fig. 11.

## S5. The transition of LD lenses to spherical LDs

We performed the transition simulations starting from an almost spherical LD of size $D=$ 29.5 nm with the largest lens angle $\theta=86^{\circ}$, spontaneously formed from an initial slab with $N_{\text {oil }}=21,000$ and $N_{\text {lip }}=6960$. We then increased the width $L$ of the simulation box in an NPT ensemble with a barostat in the vertical direction $z$ perpendicular to the ER-bilayer for a constant number of lipids and water beads.

We increased the width of the simulation box at a rate $0.4 \mathrm{pm} / \mathrm{ns}$ resulting in a quasiequilibrium transition with intermediate equilibrium LD lenses. ${ }^{[4]}$ The procedure was then repeated successively to find different lenses shown in Fig. 20. After reaching the target box width for each lens angle, we continued the simulation for $8 \mu s$, this time in the NVT ensemble, the last $6 \mu s$ of which was used to compute membrane tensions by averaging over 120 frames, and $A_{l}$ values, and LD shape parameters by averaging over 12 frames. LD-tensions were computed, using the equations (S4), in a box of width 8 nm and height 48 nm , which contained almost flat monolayer patches as shown in supplementary Fig. S4. ER-tensions were computed using equation (S3) inside a box of size $(8 \times 48 \times 48) \mathrm{nm}^{3}$, see supplementary Fig. S4. The same boxes were used to compute $A_{l}$ values for both the ER-bilayers and LD-monolayers. Supplementary Table S4 lists the LD parameters for the resulting seven LDs shown in Fig. 2b. To compare different LD sizes, we also performed the transition simulations for another LD of size $D=27.7 \mathrm{~nm}$ with $N_{\text {oil }}=16800$ and $N_{\text {lip }}=6800$, the parameters of which are presented in supplementary Table S5. See the
supplementary text for details on line tension calculation during LD transition.

## S6. Line tensions in symmetrical LDs

The free energy of a symmetrical LD with the surrounding ER-bilayer is given by

$$
\begin{equation*}
F_{s}=2 E_{b s}+A_{b s} \Sigma_{b}+2 A_{m s} \Sigma_{m}+l \lambda \tag{S6}
\end{equation*}
$$

Here, $E_{b s}=2 \kappa_{m} A_{m s} / R^{2}$ is the bending energy of the two symmetrical LD-monolayers, $\Sigma_{b}$ and $\Sigma_{m}$ are the ER and LD-tensions, and $\lambda$ is the line tension acting along the perimeter $l=2 \pi R_{c o}$ of the contact circle with radius $R_{c o} . A_{b s}=L^{2}-\pi R^{2} \sin ^{2} \theta$ is the area of the surrounding ER-bilayer of width $L, A_{m s}=2 \pi R^{2}(1-\cos \theta)$ is the surface area of the spherical monolayers with identical radii $R$.

Minimizing the Lagrangian $\mathcal{L}_{s}=F_{s}-\gamma V_{s}$ with respect to $R$ and $\theta$ using a Lagrange multiplier $\gamma$ to fulfil a constraint on fixed LD size $V_{s}=\pi D^{3} / 6$, leads to a system of three equations

$$
\begin{align*}
V_{s}= & (2 \pi / 3) R^{3}(2+\cos \theta)(1-\cos \theta)^{2} \\
\frac{\partial \mathcal{L}_{s}}{\partial R}= & 2 \pi R^{2} \gamma(2+\cos \theta)(1-\cos \theta)^{2}-2 \pi R \sin ^{2} \theta \Sigma_{b}+8 \pi R(1-\cos \theta) \Sigma_{m}+2 \pi \sin \theta \lambda=0 \\
\frac{\partial \mathcal{L}_{s}}{\partial \theta}= & \left(2 \pi \gamma R^{3} / 3\right)\left[2 \sin \theta(2+\cos \theta)(1-\cos \theta)-\sin \theta(1-\cos \theta)^{2}\right]+8 \pi \kappa_{m} \sin \theta \\
& +4 \pi R^{2} \sin \theta \Sigma_{m}-\pi R^{2} \sin 2 \theta \Sigma_{b}+(2 \pi R \cos \theta) \lambda=0 \tag{S7}
\end{align*}
$$

with four variables $R, \theta, \lambda, \gamma$. To solve this underdetermined system, we substituted the values of $R, \Sigma_{b}$, and $\Sigma_{m}$, obtained from the DPD simulations, into the system and solved equations (S7) for the remaining three unknowns $\lambda, \theta$, and $\gamma$. The results for two LD sizes are listed in Tables S4 and S5.

## S7. Membrane elasticity model for axisymmetric LDs

To determine the minimum energy shapes of symmetric lenses, we adapted a computational approach used in ${ }^{[5]}$ based on the elasticity theory of fluid membranes. Here, the bottom monolayer of the lens is described as a smooth surface in terms of two parameters, the angle $\gamma$ of rotation around the axis of symmetry, and the arc length $s$ along longitudes. This surface then has Cartesian coordinates $X=R(s) \cos \gamma, Y=R(s) \sin \gamma$ and $Z=Z(s)$, where $0 \leq \gamma \leq 2 \pi$ and $0 \leq s \leq s_{1}$ and $R$ is the distance from the $Z$-axis, which is the axis of symmetry. We introduce a tangent angle $\Psi$ such that $d R / d s=\cos \Psi(s)$ and $d Z / d s=\sin \Psi(s)$. In this parametrization, the volume of the lens is

$$
\begin{equation*}
V=2 \pi \int_{0}^{s_{1}} R^{2}(s) \frac{\mathrm{d} Z}{\mathrm{~d} s} \mathrm{~d} s \tag{S8}
\end{equation*}
$$

and the total area of the two monolayers enclosing the lens is

$$
\begin{equation*}
A=4 \pi \int_{0}^{s_{1}} R(s) \mathrm{d} s \tag{S9}
\end{equation*}
$$

The lower limit of integration, $s=0$, corresponds to the "south pole" of the lens at which $\Psi(s=0)=0, R(s=0)=0$ and $Z(s=0)=0$. With these boundary conditions we have $R(s)=\int_{0}^{s} \cos \Psi\left(s^{\prime}\right) \mathrm{d} s^{\prime}$ and $Z(s)=\int_{0}^{s} \sin \Psi\left(s^{\prime}\right) \mathrm{d} s^{\prime}$. The upper limit of integration, $s=s_{1}$, corresponds to the lens rim, where $\Psi\left(s_{1}\right)=0$ and $R\left(s_{1}\right)=R_{1}$ to connect the lens surface smoothly to a flat bilayer at a distance $R_{1}$ from the axis of symmetry. The circumference of this rim is

$$
\begin{equation*}
\ell=2 \pi R_{1}=2 \pi \int_{0}^{s_{1}} \cos \Psi(s) \mathrm{d} s \tag{S10}
\end{equation*}
$$

The energy of the system $E=E_{\mathrm{b}, \mathrm{m}}+E_{\mathrm{b}, \mathrm{b}}+E_{\mathrm{s}, \mathrm{m}}+E_{\mathrm{s}, \mathrm{b}}+E_{1}$ comprises five terms: the bending energy $E_{\mathrm{b}, \mathrm{m}}$ of the two monolayers, the bending energy of the flat bilayer, $E_{\mathrm{b}, \mathrm{b}}=0$, the surface energy $E_{\mathrm{s}, \mathrm{m}}$ of the monolayers, the surface energy $E_{\mathrm{s}, \mathrm{b}}$ of the bilayer, and the line energy $E_{1}$ of the lens rim. Since the principal curvatures of the lens surface are given by
$C_{1}=d \Psi / d s$ and $C_{2}=\sin \Psi / R$ in the arc-length parametrization, ${ }^{\sqrt{67]}}$ the bending energy of the two monolayers is

$$
\begin{equation*}
E_{\mathrm{b}, \mathrm{~m}}=2 \pi \kappa_{\mathrm{m}} \int_{0}^{s_{1}} R(s)\left(\frac{\mathrm{d} \Psi}{\mathrm{~d} s}+\frac{\sin (\Psi(s))}{R(s)}-C_{0}\right)^{2} \mathrm{~d} s \tag{S11}
\end{equation*}
$$

where $\kappa_{\mathrm{m}}$ and $C_{0}$ are the bending rigidity modulus and the spontaneous curvature of the monolayers, respectively. The surface energy of the monolayers $E_{\mathrm{s}, \mathrm{m}}=A \Sigma_{\mathrm{s}}$, where $\Sigma_{\mathrm{s}}$ denotes the surface tension of the monolayers, and the lens surface area $A$ is given by equation S9. The surface energy of the bilayer $E_{\mathrm{s}, \mathrm{b}}=\left(L^{2}-\pi R_{1}^{2}\right) \Sigma_{\mathrm{b}}$, where $\Sigma_{\mathrm{b}}$ denotes the surface tension of the bilayer, and $L$ is the lateral size of the system. The energy term $L^{2} \Sigma_{\mathrm{b}}$ does not depend on the shape of the lens and can be omitted in numerical calculations. Finally, the line energy $E_{1}=\lambda \ell=2 \pi R_{1} \lambda$, where $\lambda$ denotes the line tension. It is convenient to use dimensionless variables in numerical calculations. Here we introduce $\tau=s / s_{1}$ with $\tau \in[0,1], \psi(\tau)=\Psi(s), r(\tau)=R(s) / s_{1}$ and $z(\tau)=Z(s) / s_{1}$. Then, using equation (S8), $s_{1}$ can be directly related to the volume $V$ of the lens

$$
\begin{equation*}
V=2 \pi s_{1}^{3} \int_{0}^{1} r^{2}(\tau) \sin (\psi(\tau)) \mathrm{d} \tau \tag{S12}
\end{equation*}
$$

In addition, with this choice of dimensionless variables, the total energy of the system can be written as $E=E_{\mathrm{b}}+E_{\mathrm{m}}+E_{1}$ with

$$
\begin{align*}
& E_{\mathrm{b}}=E_{\mathrm{b}, \mathrm{~m}}+E_{\mathrm{b}, \mathrm{~b}}=2 \pi \kappa_{\mathrm{m}} \int_{0}^{1} r(\tau)\left(\frac{\mathrm{d} \psi}{\mathrm{~d} \tau}+\frac{\sin (\psi(\tau))}{r(\tau)}-C_{0} s_{1}\right)^{2} \mathrm{~d} \tau  \tag{S13}\\
& E_{\mathrm{s}}=E_{\mathrm{s}, \mathrm{~m}}+E_{\mathrm{s}, \mathrm{~b}}=2 \pi s_{1}^{2}\left(2 \Sigma_{\mathrm{m}} \int_{0}^{1} r(\tau) \mathrm{d} \tau-\frac{1}{2} \Sigma_{\mathrm{b}} \int_{0}^{1} \cos (\psi(\tau)) \mathrm{d} \tau\right) \tag{S14}
\end{align*}
$$

and

$$
\begin{equation*}
E_{1}=2 \pi s_{1} \lambda \int_{0}^{1} \cos (\psi(\tau)) \mathrm{d} \tau \tag{S15}
\end{equation*}
$$

In equations S13), (S14) and S15), parameters $\kappa_{\mathrm{m}}, C_{0}, \Sigma_{\mathrm{m}}, \Sigma_{\mathrm{b}}$, and $\lambda$ characterize the mechanical properties of the system whereas $s_{1}$ is determined by the volume $V$ of the lens via equation S12. Since $r(\tau)$ and $z(\tau)$ are given by $\psi(\tau)$ via

$$
\begin{equation*}
r(\tau)=\int_{0}^{\tau} \cos \psi\left(\tau^{\prime}\right) \mathrm{d} \tau^{\prime} \tag{S16}
\end{equation*}
$$

and

$$
\begin{equation*}
z(\tau)=\int_{0}^{\tau} \sin \psi\left(\tau^{\prime}\right) \mathrm{d} \tau^{\prime} \tag{S17}
\end{equation*}
$$

the shape of the lens is entirely determined by the function $\psi(\tau)$, and the total energy $E=E_{\mathrm{b}}+E_{\mathrm{m}}+E_{1}$ is a functional of $\psi(\tau)$.

To minimize the total energy $E$ with respect to the lens shape, as given by the function $\psi(\tau)$, it is convenient to approximate $\psi(\tau)$ by a Fourier series ${ }^{[8] 9]}$

$$
\begin{equation*}
\psi(\tau)=\sum_{n=1}^{N} a_{n} \sin (n \pi \tau) \tag{S18}
\end{equation*}
$$

that fulfils the boundary conditions $\psi(\tau=0)=0$ and $\psi(\tau=1)=0$. Here, $N$ is the number of Fourier amplitudes $a_{n}$. The total energy $E=E_{\mathrm{b}}+E_{\mathrm{m}}+E_{\mathrm{l}}$, as given by equations (S13), (S14) and (S15), becomes now a function of $N$ variables $\left\{a_{n}\right\}$.

We minimized $E$ with respect to the set of Fourier amplitudes $\left\{a_{n}\right\}$ using a simulated annealing method. We performed the numerical calculations with $N=100$ Fourier modes as in. ${ }^{[8[9]}$ We assumed $C_{0}=0$ whereas the values of parameters $\kappa_{\mathrm{m}}, \Sigma_{\mathrm{m}}, \Sigma_{\mathrm{b}}, \lambda$ and $V$ were taken from the optimal fits of the spherical cup model to the lens shapes obtained in the DPD simulations. The resulting membrane profiles $\psi=\psi(s)$ are shown in Figure 3 by yellow curves juxtaposed on DPD simulation snapshots for different LD lenses.

## S8. LD budding by lipid exchange

Starting from an almost spherical LD of size $D=29.5 \mathrm{~nm}$, we created an asymmetric lipid number at constant $N_{\text {lip }}$ (see Table S6) by exchanging lipids from the luminal to the cytosolic LD-monolayer. The resulting LDs with a given $\Delta$ were then equilibrated for $2 \mu s$ in the NPT ensemble at constant pressure $190.4 \times 10^{9} \mathrm{mN} / \mathrm{m}^{2}$ using a Berendsen barostat in the vertical direction. The equilibration was followed by a $6 \mu s$-long NVT simulation during the last $4 \mu s$ of which membrane properties were averaged over 80 simulation frames. Supplementary Table S6 lists the LD parameters for the resulting LDs shown in Fig. 4 a with different cytosolic angles $\theta_{c}$.

## S9. Size-dependent LD-tension

Figure 2r shows ER and LD-tensions as a function of $\theta$ for an LD of size $D=29.5 \mathrm{~nm}$. To estimate finite size effect on the LD-tensions, we repeated the same calculations for a smaller LDs of size $D=20.3 \mathrm{~nm}$ and 27.7 nm . Although LD-tensions inside spherical LDs seem to be smaller for the larger LD, see supplementary Fig. S5, their values are within the errorbars of one another. Therefore, we simulated more LDs with different sizes, four of which were used to explore size-dependent behavior of LD-tensions as shown in Fig. 6a. Supplementary Table S7 lists LD parameters for these four LDs which spontaneously formed from initial slabs.

## Supplementary Figures



Figure S1: Coarse-grained membrane and neutral lipids with their densities: (a) A lipid bilayer with the corresponding density profiles of lipid heads, lipid tails, and water beads. The distance between the peaks of the lipid heads density defines the bilayer thickness $l_{b}$. Also shown is the structure of a lipid with its head $H$ and tail $T$ beads. (b) The density profiles of lipid tails and heads, and water for a slab of size $D=24.2 \mathrm{~nm}$. As expected water has a density of $3 / d^{3}=5.86 / \mathrm{nm}^{3}$. Also shown is an oil molecule with its oil beads $O$.


Figure S2: Finding membrane stiffness and thickness: (a) Small slabs of size $D=24.2 \mathrm{~nm}$ used to compute bending stiffness and membrane thickness of LD-monolayers with boxes of height $2 h=20 \mathrm{~nm}$ and width 13 nm for calculating stress profiles as shown in (b) for three slabs. (c) Small flat bilayers with boxes of height $2 h=10 \mathrm{~nm}$ and width 13 nm for calculating stress profiles in bilayers as shown in (d) for two bilayers. (e) Monolayer and bilayer tensions, $\Sigma_{m}$ and $\Sigma_{b}$, plotted against $A_{l p}$ with red and blue dots. The dashed lines, fitted to the data, were used to calculate membrane stiffness.

$$
D=26.3 \mathrm{~nm}
$$

$$
A_{l p}=0.68 \mathrm{~nm}^{2}
$$



$$
D=26.3 \mathrm{~nm}
$$

$A_{l p}=0.64 \mathrm{~nm}^{2}$


$$
\begin{aligned}
& D=28 \mathrm{~nm} \\
& A_{l p}=0.64 \mathrm{~nm}^{2}
\end{aligned}
$$



Figure S3: Multi-spherical LDs: In addition to single spherical LDs, multi-spherical LDs of smaller sizes form for sufficiently small area-per-lipids of initial slabs. Green and orange dots represent oil and lipid molecules, respectively. The bottom snapshots of the middle and right panels show the lateral cross sections of double LD structures.


Figure S4: Stress calculation boxes: Boxes of width 5 and 8 nm and height 48 nm used to compute LD and ER-tensions, $\Sigma_{m}$ and $\Sigma_{b}$, and their areas-per-lipid.


Figure S5: Size-dependent behaviour of ER and LD-tensions: ER and LD-tensions versus lens angle $\theta$ for two LDs of sizes $D=29.5$ and $D=27.7 \mathrm{~nm}$. LD-tensions of almost spherical LDs seem to be almost invariable for the two LD sizes.

## Supplementary Tables

Table S1: DPD Interaction force parameters $f_{i j}\left[k_{B} T / d\right]$

|  | $j=W$ | $j=H$ | $j=T$ | $j=O$ |
| :---: | :---: | :---: | :---: | :---: |
| $i=W$ | 25 | 30 | 75 | 75 |
| $i=H$ | 30 | 30 | 50 | 50 |
| $i=T$ | 75 | 50 | 10 | 11 |
| $i=O$ | 75 | 50 | 11 | 10 |

The interaction force parameter $f_{i j}$ between oil $O$, water $W$, lipid head $H$, and lipid tail $T$ beads.

Table S2: Properties of small slabs and flat bilayers

| $N_{l i p}$ | $A_{l}^{m}, A_{l}^{b}\left[\mathrm{~nm}^{2}\right]$ | $\Sigma_{m}[m N / m]$ | $\Sigma_{b}[m N / m]$ |
| :---: | :---: | :---: | :---: |
| 1800 | 0.72 | $0.4 \pm 0.6$ | - |
| 1700 | 0.77 | $4.6 \pm 0.3$ | $-1.8 \pm 0.5$ |
| 1600 | 0.82 | $10.5 \pm 0.2$ | $8.7 \pm 0.2$ |
| 1500 | 0.87 | $15.5 \pm 0.2$ | $20.3 \pm 0.3$ |
| 1400 | 0.93 | $19.9 \pm 0.2$ | $32.4 \pm 0.3$ |

Areas-per-lipid and tensions of flat bilayers and small slabs of size $D=24.2 \mathrm{~nm}$ ( $\Sigma_{b}$ and $\Sigma_{m}$ respectively) used to find the bending stiffness and membrane thickness of the ER-bilayer and LD-monolayers. All slabs contain the oil volume $V=4264.6 \mathrm{~nm}^{3}$ with slab area $A_{s}=655.4 \mathrm{~nm}^{2}$ and minimum slab area $A_{s m}=1115.3 \mathrm{~nm}^{2}$. The monolayer thickness is $l_{m}=2.04 \mathrm{~nm}$.

Table S3: Properties of initial slabs used for identifying relevant LD parameters

| $N_{\text {oil }}$ | $D[n m]$ | $V\left[n m^{3}\right]$ | $N_{\text {lip }}$ | $A_{l p}\left[n m^{2}\right]$ | $A_{s m}\left[n m^{2}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 21000 | 29.5 | 8601.6 | 6800 | 0.68 | 2987.5 |
|  |  |  | 6400 | 0.72 |  |
|  |  |  | 6000 | 0.77 |  |
|  |  |  | 5600 | 0.822 |  |
| 24881 | 31 | 10192 | 7200 | 0.64 | 3058.8 |
|  |  |  | 6428 | 0.716 |  |
|  |  |  | 5625 | 0.819 |  |
| 31500 | 33.2 | 10902.4 | 6800 | 0.68 | 3169.7 |
|  |  |  | 6400 | 0.72 |  |
|  |  |  | 6000 | 0.77 |  |

Properties of slabs with different sizes used for spontaneous formation of LDs and identifying LD parameters. The data corresponds to slabs the resulting LDs of which are shown in Figure 1c. The slab area is $A_{s}=2304 \mathrm{~nm}^{2}$ and the monolayer thickness is $l_{m}=2.04 \mathrm{~nm}$ for all slabs.

Table S4: Properties of symmetrical LD lenses with size $D=29.5 \mathrm{~nm}$

| $L[n m]$ | $R[\mathrm{~nm}]$ | $\theta\left[{ }^{\circ}\right]$ | $\Sigma_{b}[\mathrm{mN} / \mathrm{m}]$ | $\Sigma_{m}[\mathrm{mN} / \mathrm{m}]$ | $\lambda[\mathrm{pN}]$ | $\gamma\left[10^{9} \mathrm{mN} / \mathrm{m}^{2}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 47.2 | $14.2 \pm 0.3$ | $86.3 \pm 0.1$ | $0.3 \pm 0.3$ | $4.1 \pm 0.5$ | $10 \pm 4.4$ | -0.615 |
| 48 | $15.9 \pm 0.3$ | $75.7 \pm 0.07$ | $1.3 \pm 0.4$ | $4.8 \pm 0.5$ | $-3.4 \pm 7.2$ | -0.641 |
| 48.8 | $18 \pm 0.4$ | $67 \pm 0.04$ | $3.2 \pm 0.5$ | $5.6 \pm 0.2$ | $-8.4 \pm 8.7$ | -0.659 |
| 49.6 | $21.9 \pm 0.3$ | $55.6 \pm 0.03$ | $5.5 \pm 0.5$ | $7.1 \pm 0.3$ | $-36.1 \pm 10.9$ | -0.68 |
| 50.4 | $23.3 \pm 0.3$ | $52.7 \pm 0.03$ | $8.5 \pm 0.2$ | $9.1 \pm 0.5$ | $-38.2 \pm 11.8$ | -0.811 |
| 51.2 | $25.2 \pm 0.3$ | $49.2 \pm 0.03$ | $11.3 \pm 0.5$ | $10.8 \pm 0.4$ | $-45.7 \pm 13.8$ | -0.885 |
| 52 | $27 \pm 0.5$ | $46.4 \pm 0.03$ | $15.5 \pm 0.1$ | $11.7 \pm 0.4$ | $-31.1 \pm 21$ | -0.892 |

Properties of LD lenses obtained by increasing the width $L$ of the simulation box of an almost spherical LD of size $D=29.5 \mathrm{~nm}$ with smallest box width $L=47.2 \mathrm{~nm}$.

Table S5: Properties of symmetrical LD lenses with size $D=27.7 \mathrm{~nm}$

| $L[n m]$ | $R[\mathrm{~nm}]$ | $\theta\left[{ }^{\circ}\right]$ | $\Sigma_{b}[\mathrm{mN} / \mathrm{m}]$ | $\Sigma_{m}[\mathrm{mN} / \mathrm{m}]$ | $\lambda[p N]$ | $\gamma\left[10^{9} \mathrm{mN} / \mathrm{m}^{2}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 48 | $14.1 \pm 0.4$ | $79.7 \pm 0.1$ | $0.27 \pm 0.15$ | $4.2 \pm 0.5$ | $-2.8 \pm 3.3$ | -0.901 |
| 48.8 | $15.9 \pm 0.3$ | $70.1 \pm 0.06$ | $1.5 \pm 0.2$ | $5.2 \pm 0.3$ | $-17.2 \pm 4.3$ | -0.702 |
| 49.6 | $18.8 \pm 0.6$ | $59.9 \pm 0.04$ | $3.8 \pm 0.4$ | $6.3 \pm 0.5$ | $-30.3 \pm 10.6$ | -0.713 |
| 50.4 | $21.8 \pm 0.5$ | $52.4 \pm 0.04$ | $6.8 \pm 0.3$ | $7.7 \pm 0.4$ | $-35.9 \pm 10$ | -0.744 |
| 51.2 | $26.5 \pm 0.7$ | $44.4 \pm 0.03$ | $10.1 \pm 0.3$ | $8.7 \pm 0.3$ | $-36.6 \pm 9.8$ | -0.687 |
| 52 | $27.5 \pm 0.4$ | $43 \pm 0.03$ | $13.8 \pm 0.2$ | $11.6 \pm 0.4$ | $-52.9 \pm 11.6$ | -0.873 |

Properties of LD lenses obtained by increasing the width $L$ of the simulation box of an almost spherical LD of size $D=27.7 \mathrm{~nm}$ with the smallest box width $L=48 \mathrm{~nm}$.

Table S6: Properties of asymmetric LDs budding to the cytosol

| $N_{c m}$ <br> $N_{l m}$ | $\theta_{c}\left[{ }^{\circ}\right]$ | $\Sigma_{b}[m N / m]$ | $\Sigma_{c m}[m N / m]$ <br> $\Sigma_{l m}[m N / m]$ | $\Delta$ |
| :---: | :---: | :---: | :---: | :---: |
| 7000 | $91.6 \pm 1$ | $0.073 \pm 0.16$ | $3.85 \pm 0.39$ <br> $4.8 \pm 0.7$ | 0 |
| 7000 |  | $-0.19 \pm 0.23$ | $4.7 \pm 1.1$ <br> $4 \pm 1.1$ | 0.021 |
| 7150 | $101.3 \pm 3.3$ | -2.7 |  |  |
| 6850 |  | $0.05 \pm 0.34$ | $4.7 \pm 0.4$ <br> $4.9 \pm 0.7$ | 0.043 |
| 6700 | $108.9 \pm 2.4$ |  |  |  |
| 7450 | $115.8 \pm 1.3$ | $-0.21 \pm 0.24$ | $3.9 \pm 0.6$ <br> $4.6 \pm 0.6$ | 0.064 |
| 7550 |  |  |  |  |
| 7600 | $126.4 \pm 3.3$ | $-0.075 \pm 0.26$ | $4.3 \pm 1$ <br> $4.2 \pm 1.2$ | 0.086 |
| 7750 | $133.3 \pm 2.6$ | $-0.033 \pm 0.25$ | $4.3 \pm 0.9$ <br> 6250 | $0.1 \pm 1$ |
| 7900 | $142.4 \pm 2.3$ | $-0.018 \pm 0.27$ | $4.7 \pm 0.7$ <br> 6100 | $0.9 \pm 0.6$ |

Properties of spherical LDs obtained by exchanging lipids from the luminal to the cytosolic monolayer of an initial spherical LD of size $D=29.5 \mathrm{~nm}$ in a fixed simulation box of width $L=72 \mathrm{~nm}$. The initial spherical LD emerges into a completely budded LD attached to the ER upon exchanging lipids at a fixed total number of lipids $N_{l i p}=14000 . N_{c m}$ and $N_{l m}$ denote number of lipids in the cytosolic and luminal leaflets of the LD-monolayers and the ER bilayer. LD angles are calculated based on the LD budding model (Fig. 5b) with $\theta_{l}=180-\theta_{c}$.

Table S7: Properties of almost spherical LDs with different sizes

| $N_{\text {oil }}$ | $D[n m]$ | $N_{\text {lip }}$ | $A_{l p}\left[n m^{2}\right]$ |
| :---: | :---: | :---: | :---: |
| 5500 | 20.3 | 6400 | 0.76 |
| 16800 | 27.7 | 6800 | 0.68 |
| 21000 | 29.5 | 6960 | 0.66 |
| 31500 | 33.2 | 7200 | 0.64 |

Projected areas-per-lipid $A_{l p}$ for three nearly-spherical LDs of different sizes.

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