Supplementary Information

Pure elongation flow of an electrorheological fluid: insights on wall slip from electrorheology

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1. Slip-layer Model:

Assumptions: In the slip layer, flow occurs only in radial direction. Flow is laminar and fully developed. Flow is very slow such that all inertial terms are negligible, which leads to the following reduced form of momentum balance in radial direction:

$$0 = -\frac{\partial P}{\partial r} - \frac{\partial \tau_{rz}}{\partial z} \tag{a}$$

Assuming linear variation of τ_{rz} in the slip layer,

$$\frac{\partial \tau_{rz}}{\partial z} = \frac{\tau_{rz}}{z} \tag{b}$$

From eqn. (a) & (b), $-\frac{\partial P}{\partial r} = \frac{\tau_{rz}}{z}$

Assuming Newtonian flow behaviour within Slip layer, $\tau_{rz} = \mu \frac{\partial v_r}{\partial z}$ and we obtain

$$-\frac{\partial P}{\partial r} = \mu \frac{\partial v_r}{\partial z}$$
(c)

$$\left(\frac{1}{\mu}\right)\left(-\frac{\partial P}{\partial r}\right)\int_0^{\delta} z\partial z = \int_0^{V_r} \partial v_r$$

$$\left(\frac{1}{\mu}\right)\left(-\frac{\partial P}{\partial r}\right)\frac{\delta^2}{2} = V_r \tag{d}$$

Volume conservation in the bulk material (assuming slip layer volume is insignificant as compared to bulk volume):

$\pi r^2 h =$ constant

$$\Rightarrow r^2\frac{dh}{dt} + h\,(2r)\frac{dr}{dt} = 0$$

$$\Rightarrow r^2 U + 2rhV_r = 0 \quad \Rightarrow \quad V_r = \frac{-\,rU}{2h} \tag{e}$$

From eqn. (d) and (e):

$$\left(-\frac{\partial P}{\partial r}\right)\frac{\delta^2}{2\mu} = \frac{-\,rU}{2h} \quad \Rightarrow \quad \frac{\partial P}{\partial r} = \frac{r\mu U}{h\delta^2}$$

(f)

Integrating eqn. (f) over r to R

$$\int_{P}^{P_R} \partial P = \frac{\mu U}{h\delta^2}\int_{r}^{R} r\,\partial r \tag{g}$$

(P_R is the pressure at lateral surface of sample, and P is the pressure at radial location r)

$$P_R - P = \frac{\mu U}{h\delta^2}\frac{(R^2 - r^2)}{2}$$

$$P = P_R + \frac{\mu U}{2h\delta^2}(r^2 - R^2) \tag{h}$$

Normal force on solid plate is given by,

$$F_N = -\int_0^R (P + \tau_{zz})2\pi r dr$$

However, slip-layer model is based on lubrication approximation, which renders, $\tau_{zz} = 0$ on a solid surface for incompressible Newtonian fluid, (R. B. Bird, R. C. Armstrong and O. Hassager,

Dynamics of polymeric liquids. Volume 1, 2nd Ed. Fluid mechanics., John Wiley & Sons, Inc., United States, 1987)

Hence,

$$F_N = -\int_0^R (P) 2\pi r dr$$

$$F_N = -\int_0^R \left(P_R + \frac{\mu U}{2h\delta^2}(r^2 - R^2)\right) 2\pi r dr F_N$$

$$F_N = -2\pi P_R \int_0^R r dr + \frac{\mu U 2\pi}{2h\delta^2}\left[R^2 \int_0^R r dr - \int_0^R r^3 dr\right]$$

$$F_N = -\pi P_R R^2 + \frac{\pi \mu U}{h\delta^2}\left(\frac{R^4}{2} - \frac{R^4}{4}\right)$$

$$F_N = -P_R \frac{\pi R^2 h}{h} + \frac{\pi \mu U}{h\delta^2}\left(\frac{R^4}{4}\right) \quad \Rightarrow \quad F_N = -P_R \frac{\Omega}{h} + \frac{\mu U}{4h\delta^2}\left(\frac{\pi^2 R^4 h^2}{\pi h^2}\right)$$

$$F_N = \frac{-P_R \Omega}{h} + \frac{\mu U}{4\pi\delta^2}\frac{\Omega^2}{h^3}$$

2. Fitting to higher gap regime, and zoomed view of superposition at lower value of normal force

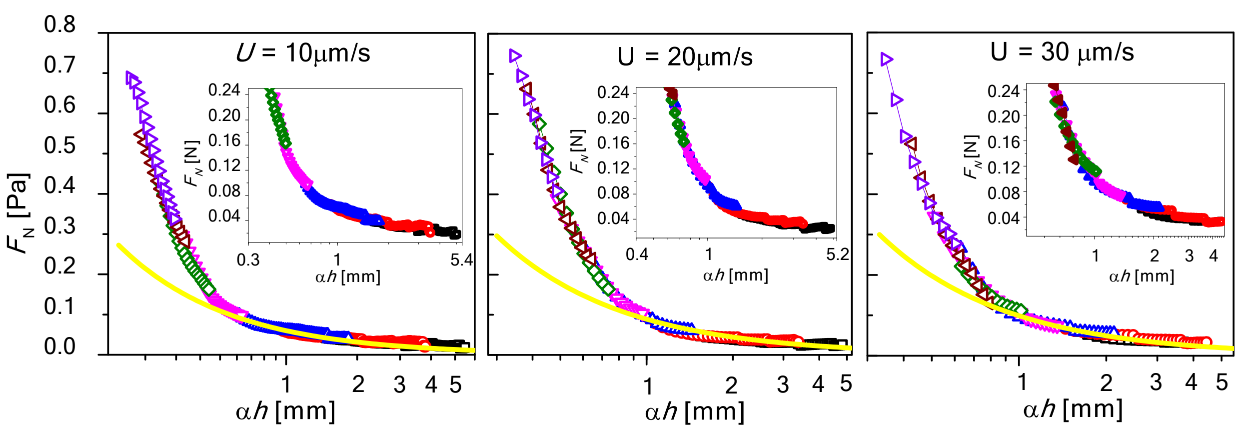

Figure SI-1: Fitting of function, $F_N = a/h$ to higher gap regime, shown by yellow solid lines. Inset shows enlarged view of superposition at low values of normal force.

3. Fitting parameters "a" and "b" of equation: $F_N = \frac{a}{h} + \frac{b}{h^3}$, ($a = -P_R \cdot \Omega$, $b = \frac{\mu U \Omega^2}{4\pi\delta^2}$)

Table SI-1.a : Values of "a", using which magnitude of P_R has been calculated.

E ***(kV/mm)***	U = **10*µm/s***		U = **20*µm/s***		U = **30*µm/s***	
	a **(*N·mm*)**	$\lvert P_R \rvert$ **(*Pa*)**	a **(*N·mm*)**	$\lvert P_R \rvert$ **(*Pa*)**	a **(*N·mm*)**	$\lvert P_R \rvert$ **(*Pa*)**
0.25	0.0600	122.2930	0.0800	163.0573	0.0998	203.4140
0.35	0.0750	152.8662	0.1000	203.8217	0.1247	254.2675
0.5	0.1052	214.5491	0.1454	296.4679	0.1720	350.7138
0.75	0.2000	407.6433	0.2000	407.6433	0.2558	521.5744
1	0.2307	470.3577	0.2352	479.5804	0.3118	635.6688
1.25	0.2727	555.8772	0.2580	525.9914	0.3441	701.4276
1.5	0.3157	643.6473	0.2962	603.9160	0.3838	782.3616

Table SI-1.b : Values of "b", using which slip layer thickness (δ) has been calculated, (shown in fig. 7).

E ***(kV/mm)***	U = **10*µm/s***	U = **20*µm/s***	U = **30*µm/s***
	b **(*N·mm³*)**	b **(*N·mm³*)**	b **(*N·mm³*)**
0.25	0.0072	0.0329	0.0279
0.35	0.0140	0.0642	0.0544
0.5	0.0388	0.1977	0.1429
0.75	0.2666	0.5140	0.4703
1	0.4096	0.8370	0.8514
1.25	0.6761	1.1043	1.1439

1.5	0.9417	1.7679	1.5873

4. Pure elongation flow results for stainless-steel surface (both top and bottom plate):

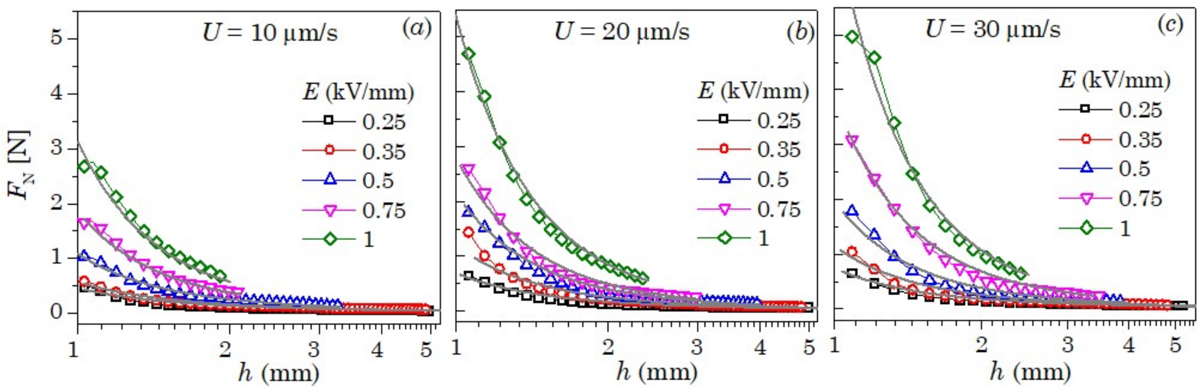

Figure SI-2: Normal force, F_N as a function of inter-plate gap, h (starting from initial gap of 1mm) under different electric-field strengths, for pulling velocity, U =10μm/s in (a), 20μm/s in (b), and 30μm/s in (c). The ordinate range is identical for all plots. The flow behavior in this region has been predicted using slip-layer model (eqn. 4), shown by grey solid line.

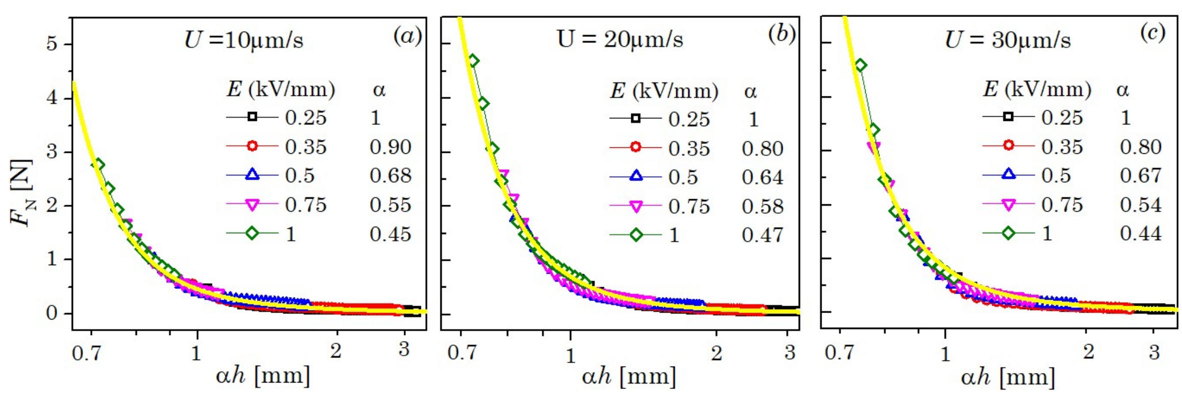

Figure SI-3: Electric field-gap master curves obtained by horizontal shifting of normal force-gap curves for pulling velocity, U =10μm/s in (a), 20μm/s in (b), and 30μm/s in (c). The y-axis range is identical for all plots. The yellow solid lines are the fits to master curve generated using slip-layer model (eqn.4).

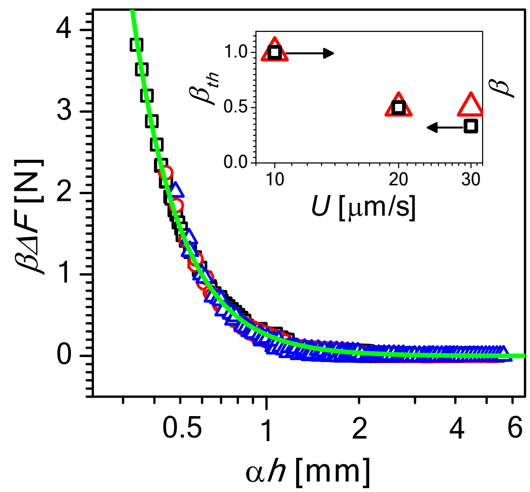

Figure SI-4: Electric field-gap-velocity superposition for force-gap data. Solid green line is the power law fit: $\beta\Delta F = c(\alpha h)^{-3}$, to master curve. Inset: experimental (β) and theoretical (β_{th}) shift factors for three pulling velocities.

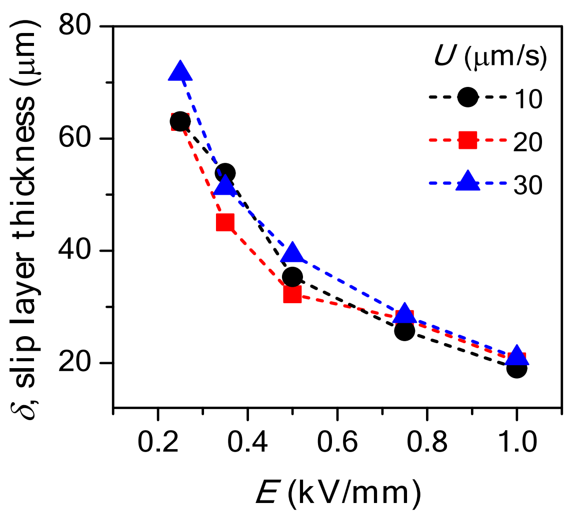

Figure SI-5: Slip layer thickness as a function of electric field strength for elongation flow data using stainless steel surface.