

## Supporting Information

# Sodalite-like carbon based superconductors with $T_c$ about 77 K at ambient pressure

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# 1 Computational details

## A. Details on Eliashberg equation

Because the superconducting pairing is only related to the electron bands near the Fermi energy, the energy shift  $\chi(\vec{k}, i\omega_n)$  becomes 0, the  $T_c$  merely involves the determination of the renormalization function  $Z(i\omega_n)$  and the order parameter  $\phi(i\omega_n)$ , which are calculated by the following Migdal-Eliashberg equations:<sup>1,2</sup>

$$Z(i\omega_n) = 1 + \frac{\pi T}{\omega_n N_F} + \sum_m \frac{\omega_m}{\sqrt{\omega_m^2 + \Delta^2(i\omega_m)}} \times \lambda(\omega_n - \omega_m) \delta(\varepsilon) \quad (\text{S1})$$

$$\phi(i\omega_n) = Z(i\omega_n) \Delta(i\omega_n) + \frac{\pi T}{N_F} \sum_m \frac{\Delta(i\omega_m)}{\sqrt{\omega_m^2 + \Delta^2(i\omega_m)}} \times [\lambda(\omega_n - \omega_m) - N_F \mu^*] \delta(\varepsilon) \quad (\text{S2})$$

In Eqs. S(1) and S(2), the  $N_F$  is the density of electronic states at the Fermi energy,  $\Delta(i\omega_n)$  is superconducting gap,  $\varepsilon$  is electron and phonon energies,  $\mu^*$  is the Coulomb pseudopotential,  $T$  is the absolute temperature,  $\omega$  is the frequency,  $i\omega_n$  is the  $n$ th fermion Matsubara frequencies with  $n=0, \pm 1, \pm 2, \dots$ , and  $i\omega_n$  is given by

$$i\omega_n = i(2n + 1)\pi T \quad (\text{S3})$$

The function  $\lambda(\omega_n - \omega_m)$  describes the effective electron-electron attraction via exchange of phonons, which is connected to Eliashberg EPC spectral function  $\alpha^2 F(\omega)$  through the relation:<sup>3-5</sup>

$$\lambda(\omega_n - \omega_m) = 2 \int_0^\infty \frac{\alpha^2 F(\omega)}{(\omega_n - \omega_m)^2 + \omega^2} \omega d\omega \quad (\text{S4})$$

where  $\alpha^2 F(\omega)$  is defined as<sup>3,5</sup>

$$\alpha^2 F(\omega) = \frac{1}{2\pi N_F} \sum_{qv} \frac{\gamma_{qv}}{\omega_{qv}} \delta(\omega - \omega_{qv}) \quad (\text{S5})$$

where  $q$  is the wave vector,  $\omega_{qv}$  is the screened phonon frequency and  $\gamma_{qv}$  is the phonon linewidth, which can be determined by<sup>3,5</sup>

$$\gamma_{qv} = 2\pi \omega_{qv} \sum_{mn} \sum_k \omega_k |g_{mn}^v(\mathbf{k}, \mathbf{q})|^2 \delta(\varepsilon_{m, \mathbf{k}+\mathbf{q}} - \varepsilon_F) \times \delta(\varepsilon_{n, \mathbf{k}} - \varepsilon_F) \quad (\text{S6})$$

The electron-phonon matrix elements  $g_{mn}^v(\mathbf{k}, \mathbf{q})$  are defined as<sup>3,5</sup>

$$g_{mn}^v(\mathbf{k}, \mathbf{q}) = \left(\frac{\hbar}{2M\omega_{qv}}\right)^{\frac{1}{2}} \langle m, \mathbf{k} + \mathbf{q} | \delta_{qv} V_{SCF} | n, \mathbf{k} \rangle \quad (\text{S7})$$

In Eq. (S7),  $V_{SCF}$  is the self-consistent potential. The bare electronic Bloch state and ionic mass are labeled  $|n, \mathbf{k}\rangle$  and  $M$ . Eqs. S(1) and S(2) established a coupled nonlinear system, from which we identify the renormalization function  $Z(i\omega_n)$  and superconducting gap  $\Delta(i\omega_n)$  by self-consistently calculations in Matsubara space with Coulomb pseudopotential  $\mu^* = 0.1$ . Then, the solutions of the Eliashberg equations on the real energy axis are performed by analytic continuation based on pade functions. The  $T_c$  is determined as the temperature at which the gap is equals to zero.

### B. Details on Allen-Dynes modified McMillan equation

Based on a reanalysis of Eliashberg theory and newly available computational checks in superconductors, Allen and Dynes<sup>3,5</sup> developed and improved the McMillan equation. For  $\lambda \leq 1.5$  they point out that the  $T_c$  can be obtained by solving the following equation:

$$T_c = \frac{\omega_{log}}{1.2} \exp \left[ -\frac{1.04(1 + \lambda)}{\lambda - \mu^*(1 + 0.62\lambda)} \right] \quad (\text{S8})$$

where  $\omega_{log}$  is the logarithmic averaged frequency, can be calculated as

$$\omega_{log} = \exp \left[ \frac{2}{\lambda} \int_0^\infty \frac{d\omega}{\omega} \alpha^2 F(\omega) \ln(\omega) \right] \quad (\text{S9})$$

For  $\lambda > 1.5$ , the correction factors  $f_1$  and  $f_2$  were introduced and the Eq. (S8) was modified as:

$$T_c = \frac{f_1 f_2 \omega_{log}}{1.2} \exp \left[ -\frac{1.04(1 + \lambda)}{\lambda - \mu^*(1 + 0.62\lambda)} \right] \quad (\text{S10})$$

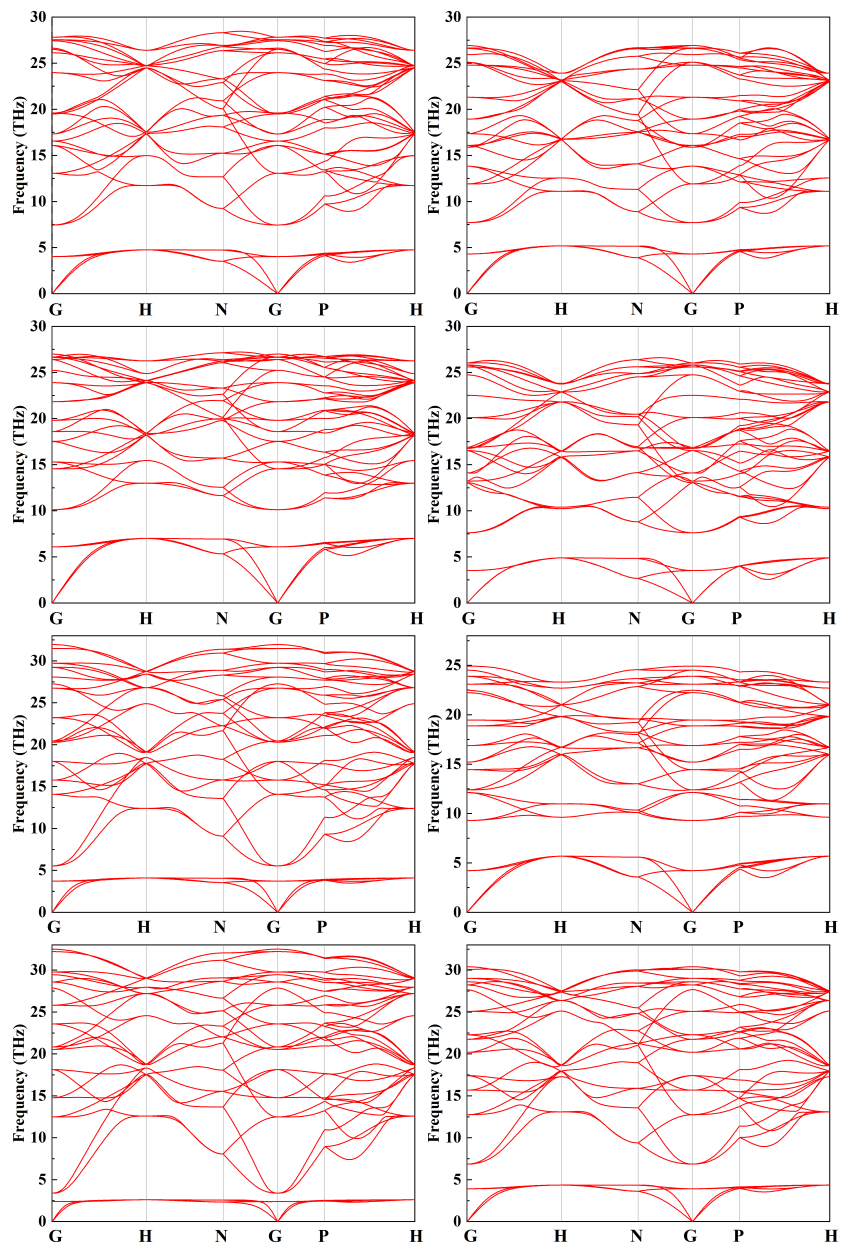
where the correction factors  $f_1$  and  $f_2$  are given by:

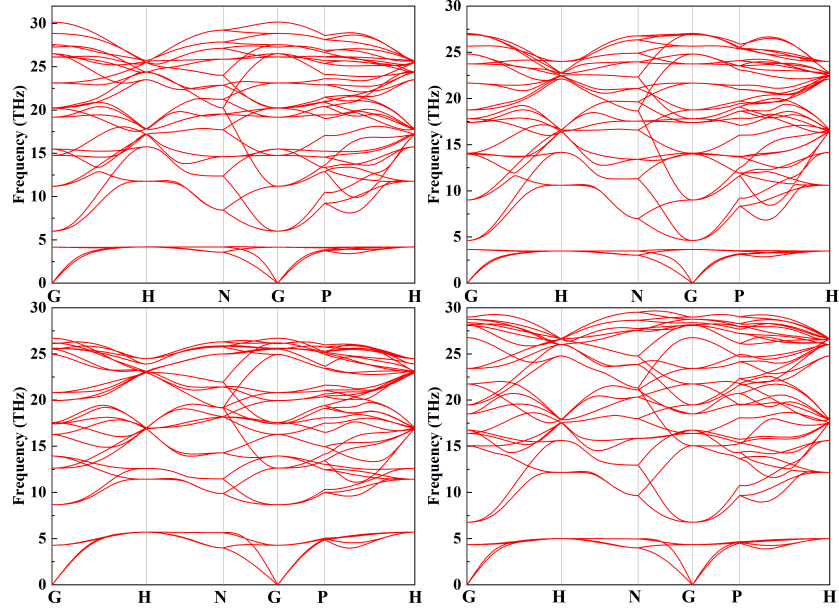
$$f_1 = \sqrt[3]{\left[ 1 + \left( \frac{\lambda}{2.46(1 + 3.8\mu^*)} \right) \right]^{3/2}}, \quad f_2 = 1 + \frac{(\frac{\bar{\omega}^2}{\omega_{log}} - 1)\lambda^2}{\lambda^2 + \left[ 1.82(1 + 6.3\mu^*) \frac{\bar{\omega}^2}{\omega_{log}} \right]^2} \quad (\text{S11})$$

In Eq. (S11), the mean square frequency is defined as:

$$\bar{\omega} = \sqrt{\frac{2}{\lambda} \int \alpha^2 F(\omega) \omega d\omega} \quad (\text{S12})$$

## 2 Supplemental Figures and Tables

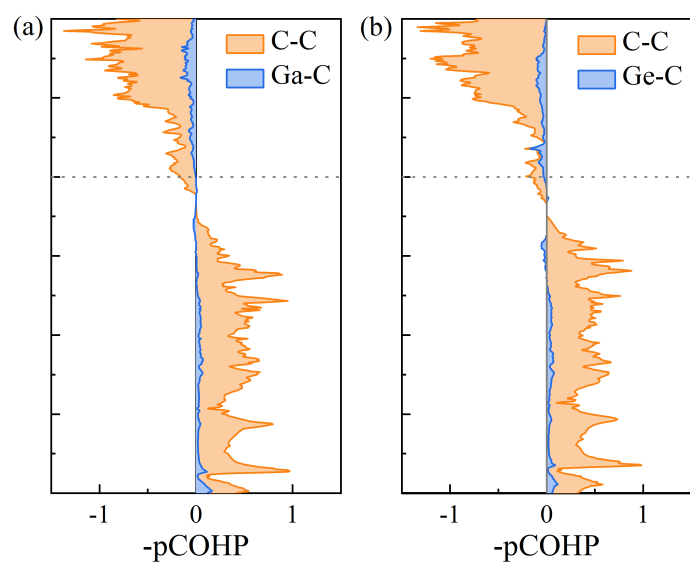




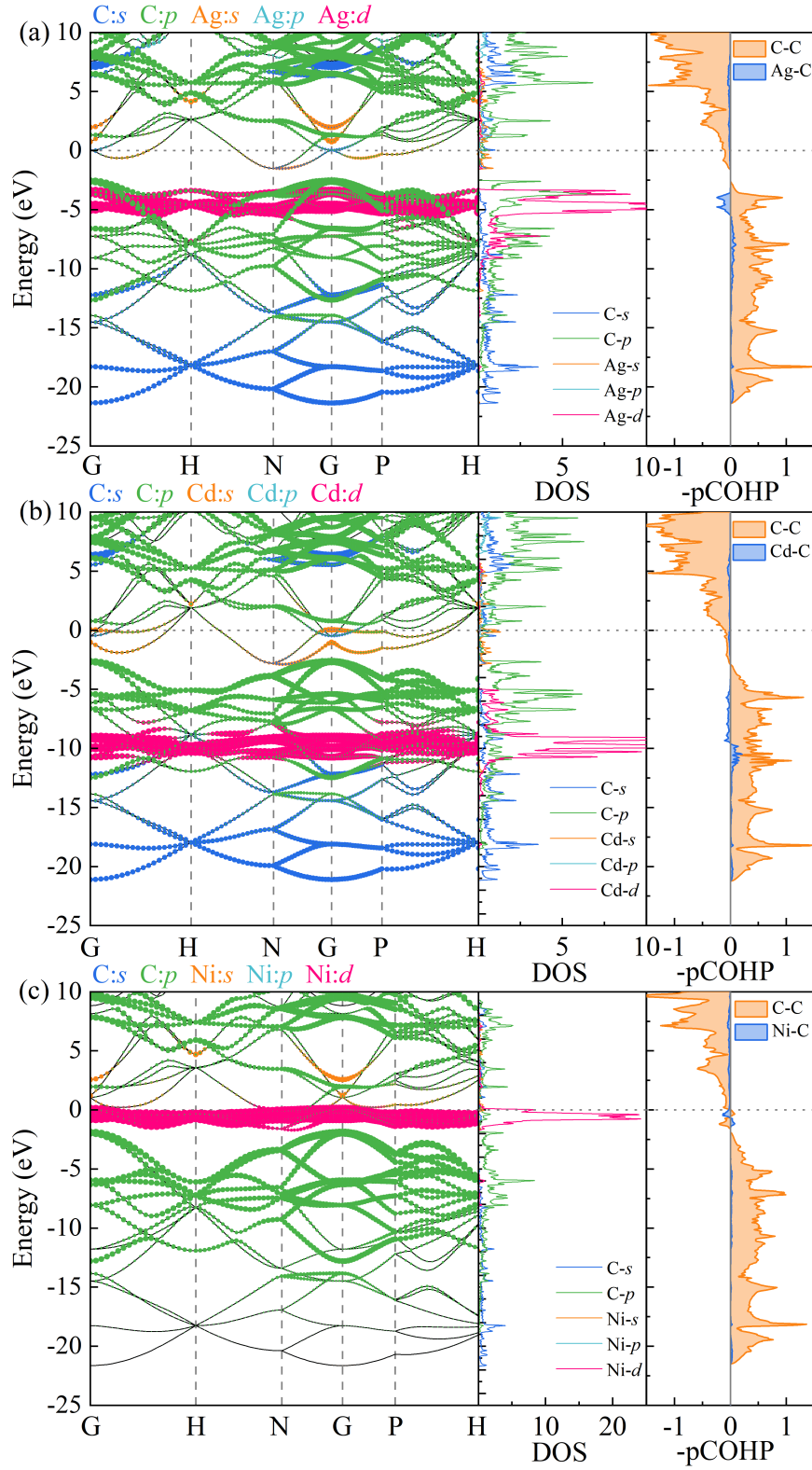
**Fig. S1** Calculated phonon dispersion curves of XC<sub>6</sub> (X = Ag, As, Br, Cd, Cu, I, Ni, Pd, Rh, Ru, Se and Zn).

Table S1: The bond length of C-C and X-C (X = Ga and Ge), Bader charge analysis for C and X, negative integrated COHP (-ICOHP) of C-C and X-C, and electronic density of states at the fermi level  $N(E_f)$  of  $\text{GaC}_6$  and  $\text{GeC}_6$ .

Parameter		$\text{GaC}_6$	$\text{GeC}_6$
Distance ( $\text{\AA}$ )	C-C	1.64	1.65
	X-C	2.59	2.60
Bader charge	C	4.12	4.13
	X	2.26	3.20
-ICOHP (eV/atom pair)	C-C	7.44	7.85
	X-C	0.61	0.56
$N(E_f)$		2.75	4.55
Lattice constants		$a=4.63896\text{\AA}$	$a=4.65532\text{\AA}$

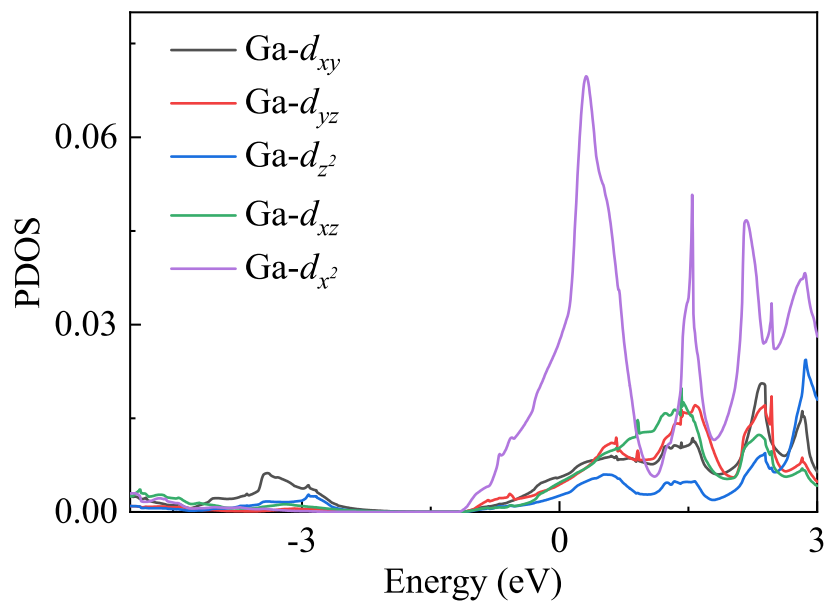


**Fig. S2** Negative projected crystal orbital Hamiltonian population (-pCOHP) of (a) GaC<sub>6</sub> and (b) GeC<sub>6</sub>.

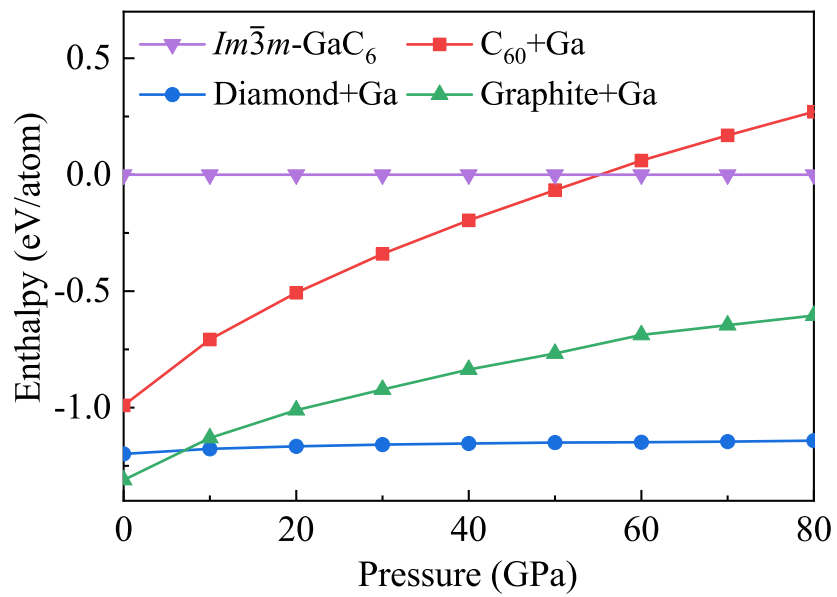


**Fig. S3** Calculated band structures, electronic density of states, COHP analysis of (a)  $\text{AgC}_6$ , (b)  $\text{CdC}_6$  and (c)  $\text{NiC}_6$ .

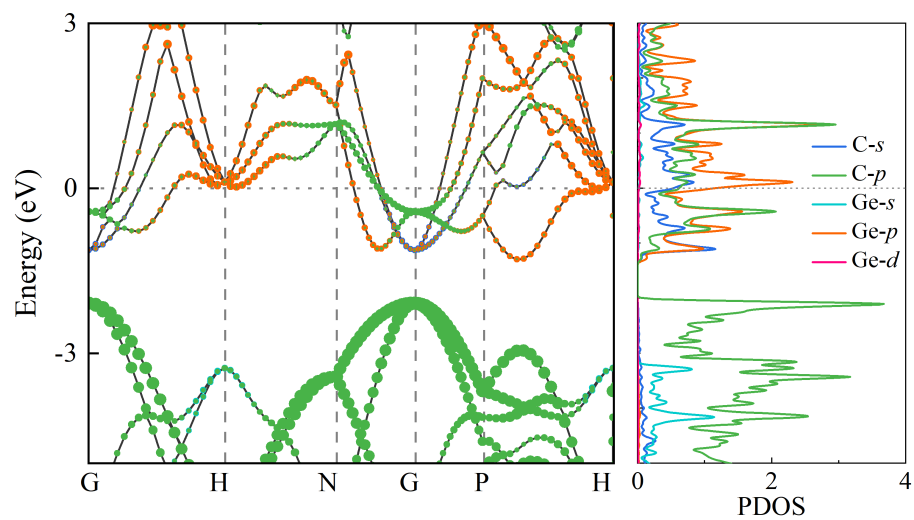




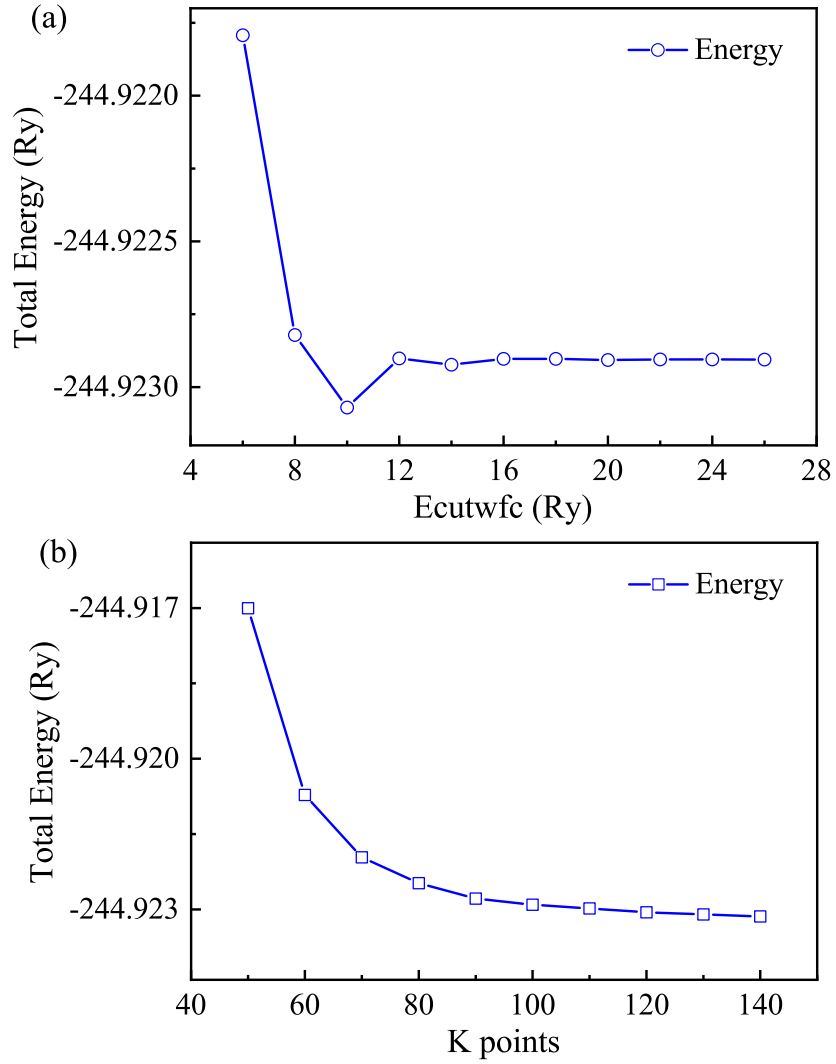
**Fig. S4** The projected DOS (PDOS) for Ga  $d$ -orbitals.



**Fig. S5** The enthalpies per atom and decomposition enthalpies as a function of pressure for  $Im\bar{3}m$ -GaC<sub>6</sub>.



**Fig. S6** Calculated band structures and electronic density of states of  $\text{GeC}_6$ .



**Fig. S7** The convergence tests for the energy of  $Im\bar{3}m$ -GaC<sub>6</sub> at ambient pressure with (a) the cutoff energy and (b) the k-points sampling, respectively.

Table S2: The bond length, Bader charge, electronic density of states at the fermi level  $N(E_f)$ , electron-phonon coupling (EPC) parameters  $\lambda$ , logarithmic average phonon frequency  $\omega_{log}$ , superconducting critical temperatures  $T_c$ , negative integrated COHP (-ICOHP) and lattice constants of  $XC_6$  ( $X = Ag, Se, Br, I, Ag$  and  $Zn$ ). The Mc-A-D, G-K and IE correspond to the  $T_c$  obtained by the Allen-Dynes modified McMillan equation, the Gor'kov and Kresin equation and the isotropic Eliashberg equation, respectively.

		AsC <sub>6</sub>	SeC <sub>6</sub>	BrC <sub>6</sub>	IC <sub>6</sub>	AgC <sub>6</sub>	ZnC <sub>6</sub>
Distance (Å)	C-C	1.65	1.66	1.66	1.71	1.64	1.63
	X-C	2.61	2.62	2.63	2.71	2.60	2.57
Bader charge	C	4.09	4.04	4.01	4.08	4.08	4.09
	X	4.43	5.73	6.92	6.55	10.5	11.46
$N(E_f)$		3.55	3.94	11.84	7.12	1.95	1.98
EPC	$\omega_{log}$ , K	420.87	516.35	701.03	524.11	646.34	755.70
	$\lambda$	1.02	0.61	0.45	0.60	0.74	0.68
	$T_c$ (Mc-A-D), K	30	12	5	12	26	24
	$T_c$ (G-K), K	46	14	4	10	38	37
	$T_c$ (IE), K	35	13	6	14	37	35
-ICOHP (eV/atom pair)	C-C	7.94	7.58	8.00	7.22	7.79	7.72
	X-C	0.46	0.33	0.25	0.34	0.19	0.10
Lattice constants (Å)		a=4.66200	a=4.68112	a=4.70170	a=4.84309	a=4.64989	a=4.59934

Table S3: The bond length, Bader charge, electronic density of states at the fermi level  $N(E_f)$ , electron-phonon coupling (EPC) parameters  $\lambda$ , logarithmic average phonon frequency  $\omega_{log}$ , superconducting critical temperatures  $T_c$ , negative integrated COHP (-ICOHP) and lattice constants of  $XC_6$  ( $X = Cd, Ni, Cu, Ru, Rh$  and  $Pd$ ). The Mc-A-D, G-K and IE correspond to the  $T_c$  obtained by the Allen-Dynes modified McMillan equation, the Gor'kov and Kresin equation and the isotropic Eliashberg equation, respectively.

		CdC <sub>6</sub>	NiC <sub>6</sub>	CuC <sub>6</sub>	RuC <sub>6</sub>	RhC <sub>6</sub>	PdC <sub>6</sub>
Distance (Å)	C-C	1.67	1.59	1.61	1.62	1.62	1.63
	X-C	2.64	2.52	2.54	2.57	2.57	2.57
Bader charge	C	4.11	4.09	4.09	4.1	4.08	4.07
	X	11.33	9.45	10.49	7.37	8.52	9.60
$N(E_f)$		2.82	10.87	0.80	9.10	3.65	
EPC	$\omega_{log}$ , K	670.64	241.21	529.10	240.62	449.31	
	$\lambda$	0.58	0.86	0.60	0.86	0.63	
	$T_c$ (Mc-A-D), K	14	13	12	13	12	
	$T_c$ (G-K), K	14	15	11	11	13	
	$T_c$ (IE), K	16	14	10	12	15	
-ICOHP (eV/atom pair)	C-C	7.29	8.04	8.48	7.73	7.75	7.39
	X-C	0.14	0.21	0.17	0.44	0.34	0.11
Lattice constants (Å)		a=4.71477	a=4.51025	a=4.54181	a=4.59235	a=4.59384	a=4.60279

## References

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