Bowtie

Report date	Feb 19, 2024, 3:01:28 PM

Contents

1 Global Definitions

Date	Feb 12, 2024, 7:04:07 AM

GLOBAL SETTINGS

Name	Bowtie.mph
Path	C:\Users\hl5630\Downloads\bowtie.mph
Version	COMSOL Multiphysics 6.2 (Build: 290)
Unit system	SI

USED PRODUCTS

Microfluidics Module
COMSOL Multiphysics

COMPUTER INFORMATION

CPU	Intel64 Family 6 Model 85 Stepping 7, 4 cores, 8 GB RAM
Operating system	Windows 10

1.1 PARAMETERS

PARAMETERS 1

Name	Expression	Value	Description
Loc	65[um]	6.5E−5 m	

2 Component 1

Date	Feb 12, 2024, 6:59:13 AM

SETTINGS

Description	Value
Unit system	Same as global system (SI)
Geometry shape function	Automatic
Avoid inverted elements by curving interior domain elements	Off

SPATIAL FRAME COORDINATES

First	Second	Third
r	phi	z

MATERIAL FRAME COORDINATES

First	Second	Third
R	PHI	Z

GEOMETRY FRAME COORDINATES

First	Second	Third
Rg	PHIg	Zg

MESH FRAME COORDINATES

First	Second	Third
Rm	PHIm	Zm

2.1 DEFINITIONS

2.1.1 Coordinate Systems

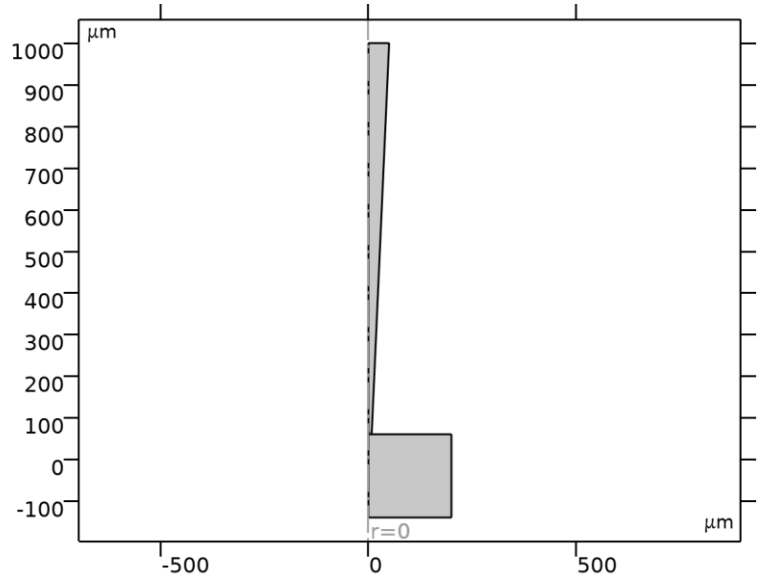
Boundary System 1

Coordinate system type	Boundary system
Tag	sys1

COORDINATE NAMES

First	Second	Third
t1	to	n

2.2 GEOMETRY 1



Geometry 1

UNITS

Length unit	µm
Angular unit	deg

GEOMETRY STATISTICS

Description	Value
Space dimension	2
Number of domains	2
Number of boundaries	12
Number of vertices	11

2.2.1 Rectangle 1 (r1)

POSITION

Description	Value
Position	{0, -140}

SIZE

Description	Value
Width	200
Height	200

INFORMATION

Description	Value
Last build time	< 1 second
Built with	COMSOL 6.2.0.290 (win64), Feb 19, 2024, 2:52:52 PM

2.2.2 Polygon 1 (pol1)

OBJECT TYPE

Description	Value
Type	Solid

COORDINATES

Description	Value
Data source	Vectors
r	{0, 8, 50, 0}
z	{60, 60, 1000, 1000}

INFORMATION

Description	Value
Last build time	< 1 second
Built with	COMSOL 6.2.0.290 (win64), Feb 19, 2024, 2:52:52 PM

2.2.3 Rectangle 2 (r2)

POSITION

Description	Value
Position	{0, Loc}
Base	Center

SIZE

Description	Value
Width	2.5
Height	4.42

INFORMATION

Description	Value
Last build time	< 1 second
Built with	COMSOL 6.2.0.290 (win64), Feb 19, 2024, 2:52:52 PM

2.2.4 Difference 1 (dif1)

INFORMATION

Description	Value
Last build time	< 1 second
Built with	COMSOL 6.2.0.290 (win64), Feb 19, 2024, 2:52:52 PM

2.2.5 Form Union (fin)

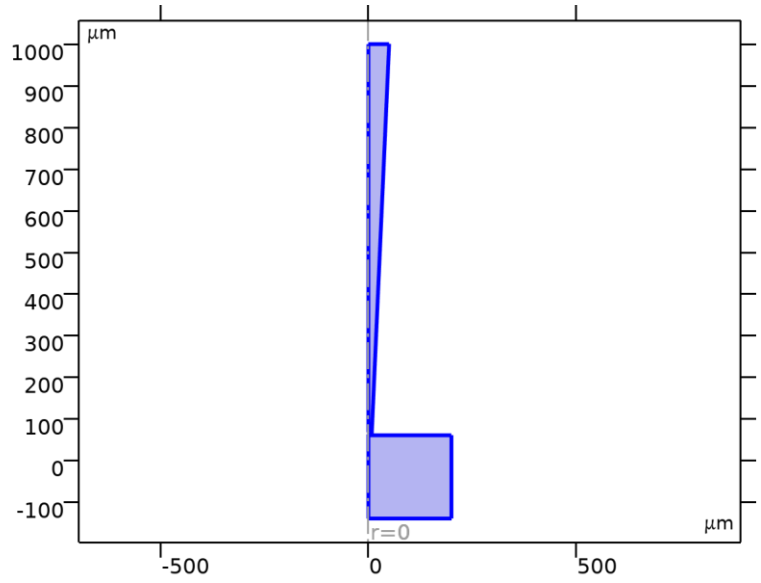
INFORMATION

Description	Value
Details	{Formed union of 1 solid object., Union has 2 domains, 12 boundaries, and 11 vertices.}
Last build time	< 1 second
Built with	COMSOL 6.2.0.290 (win64), Feb 19, 2024, 2:52:52 PM

2.3 TRANSPORT OF DILUTED SPECIES

USED PRODUCTS

Microfluidics Module
COMSOL Multiphysics



Transport of Diluted Species

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

EQUATIONS

$$\nabla \cdot \mathbf{J}_i = R_i$$

$$\mathbf{J}_i = -D_i \nabla c_i - z_i u_{m,i} F c_i \nabla V$$

2.3.1 Interface Settings

Discretization

SETTINGS

Description	Value
Concentration	Linear

SETTINGS

Description	Value
Equation form	Study controlled

Species Activity

SETTINGS

Description	Value
Species activity	Ideal

Transport Mechanisms

SETTINGS

Description	Value
Convection	Off
Migration in electric field	On
Mass transfer in porous media	Off

2.3.2 Variables

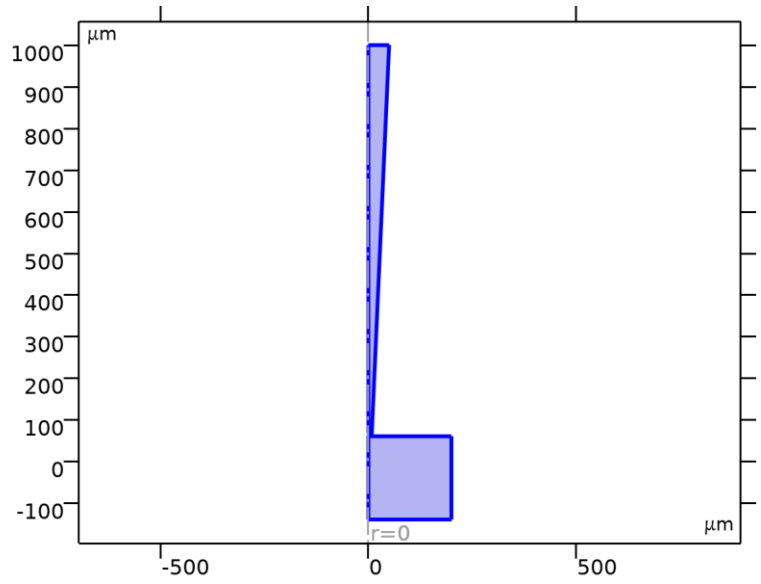
Name	Expression	Unit	Description	Selection	Details
tds.d	1	1	Out-of-plane geometry extension	Global	
tds.f_cNa	1	1	Activity coefficient	Domains 1–2	
tds.f_cCl	1	1	Activity coefficient	Domains 1–2	
tds.nr	nr	1	Normal vector, r-component	Boundary 4	
tds.nphi	0	1	Normal vector, phi-component	Boundary 4	
tds.nz	nz	1	Normal vector, z-component	Boundary 4	
tds.nr	dnr	1	Normal vector, r-component	Boundaries 1–3, 5–12	

Name	Expression	Unit	Description	Selection	Details
tds.nphi	0	1	Normal vector, phi-component	Boundaries 1–3, 5–12	
tds.nz	dnz	1	Normal vector, z-component	Boundaries 1–3, 5–12	
tds.nR	nR	1	Normal vector, R-component	Boundary 4	
tds.nPHI	0	1	Normal vector, PHI-component	Boundary 4	
tds.nZ	nZ	1	Normal vector, Z-component	Boundary 4	
tds.nR	dnR	1	Normal vector, R-component	Boundaries 1–3, 5–12	
tds.nPHI	0	1	Normal vector, PHI-component	Boundaries 1–3, 5–12	
tds.nZ	dnZ	1	Normal vector, Z-component	Boundaries 1–3, 5–12	
tds.nRg	nRg	1	Normal vector, Rg-component	Boundary 4	
tds.nPHIg	0	1	Normal vector, PHIg-component	Boundary 4	
tds.nZg	nZg	1	Normal vector, Zg-component	Boundary 4	
tds.nRg	dnRg	1	Normal vector, Rg-component	Boundaries 1–3, 5–12	
tds.nPHIg	0	1	Normal vector, PHIg-component	Boundaries 1–3, 5–12	
tds.nZg	dnZg	1	Normal vector, Zg-component	Boundaries 1–3, 5–12	
tds.nrmesh	nrmesh	1	Normal vector (mesh), r-component	Boundary 4	
tds.nphimesh	0	1	Normal vector (mesh), phi-component	Boundary 4	
tds.nzmesh	nzmesh	1	Normal vector (mesh), z-component	Boundary 4	
tds.nrmesh	dnrmesh	1	Normal vector (mesh), r-component	Boundaries 1–3, 5–12	
tds.nphimesh	0	1	Normal vector	Boundaries 1–	

Name	Expression	Unit	Description	Selection	Details
			(mesh), phi-component	3, 5–12	
tds.nzmesh	dnzmesh	1	Normal vector (mesh), z-component	Boundaries 1–3, 5–12	
tds.nrc	nrc/tds.ncLen	1	Normal vector, r-component	Boundaries 1–12	
tds.nphic	0	1	Normal vector, phi-component	Boundaries 1–12	
tds.nzc	nzc/tds.ncLen	1	Normal vector, z-component	Boundaries 1–12	
tds.ncLen	sqrt(nrc^2+nzc^2+eps)	1	Help variable	Boundaries 1–12	
tds.R_cNa	0	mol/(m³·s)	Total rate expression	Domains 1–2	+ operation
tds.cP_cNa	0	mol/kg	Concentration species adsorbed to the solid	Domains 1–2	+ operation
tds.cP_cNa	0	mol/kg	Concentration species adsorbed to the solid	Boundaries 1–12	+ operation
tds.KP_cNa	0	m³/kg	Adsorption isotherm, first concentration derivative	Domains 1–2	+ operation
tds.KP_cNa	0	m³/kg	Adsorption isotherm, first concentration derivative	Boundaries 1–12	+ operation
tds.Rads_cNa	0	mol/(m³·s)	Total adsorption rate	Domains 1–2	+ operation
tds.DiT_cNa	0	m²/s	Turbulent diffusivity	Domains 1–2	
tds.cVar_cNa	cNa	mol/m³	Species	Boundaries 1–12	
tds.cVar_cNa	cNa	mol/m³	Species	Points 1–11	
tds.R_cCl	0	mol/(m³·s)	Total rate expression	Domains 1–2	+ operation
tds.cP_cCl	0	mol/kg	Concentration species adsorbed to the solid	Domains 1–2	+ operation
tds.cP_cCl	0	mol/kg	Concentration	Boundaries 1–	+ operation

Name	Expression	Unit	Description	Selection	Details
			species adsorbed to the solid	12	
tds.KP_cCl	0	m^3/kg	Adsorption isotherm, first concentration derivative	Domains 1–2	+ operation
tds.KP_cCl	0	m^3/kg	Adsorption isotherm, first concentration derivative	Boundaries 1–12	+ operation
tds.Rads_cCl	0	$mol/(m^3 \cdot s)$	Total adsorption rate	Domains 1–2	+ operation
tds.DiT_cCl	0	m^2/s	Turbulent diffusivity	Domains 1–2	
tds.cVar_cCl	cCl	mol/m^3	Species	Boundaries 1–12	
tds.cVar_cCl	cCl	mol/m^3	Species	Points 1–11	
tds.poro	1	1	Porosity	Domains 1–2	
tds.theta_g	0	1	Gas volume fraction	Domains 1–2	
tds.theta_l	1	1	Liquid volume fraction	Domains 1–2	
tds.theta	tds.poro	1	Mobile fluid volume fraction	Domains 1–2	

2.3.3 Species Charges



Species Charges

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

Charge

SETTINGS

Description	Value
Charge number	{1, -1}

Model Input

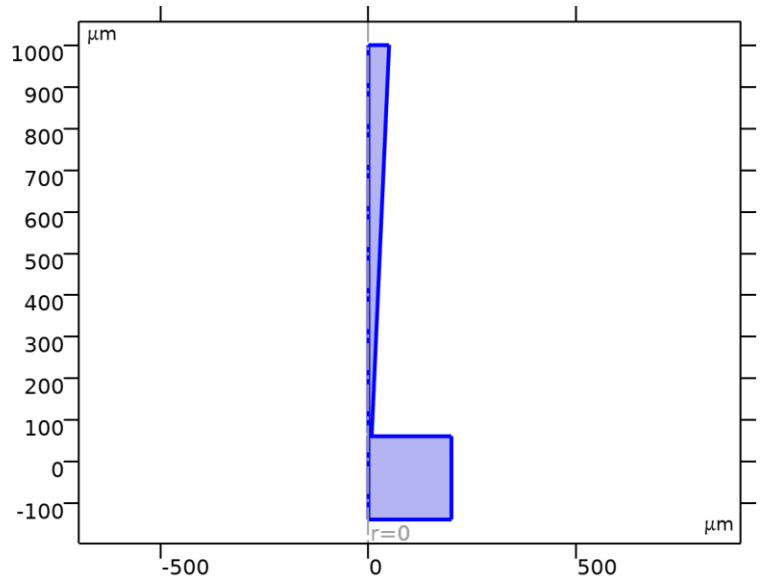
SETTINGS

Description	Value
Temperature	Common model input

Variables

Name	Expression	Unit	Description	Selection
tds.z_cNa	1	1	Charge number	Domains 1–2
tds.z_cCl	-1	1	Charge number	Domains 1–2

2.3.4 Transport Properties 1



Transport Properties 1

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

EQUATIONS

$$\nabla \cdot \mathbf{J}_i = R_i$$
$$\mathbf{J}_i = -D_i \nabla c_i - z_i u_{m,i} F c_i \nabla V$$

Diffusion

SETTINGS

Description	Value	Unit
Source	Material	
Material	None	
Diffusion coefficient	User defined	
Diffusion coefficient	1.33E-9	m^2/s
Diffusion coefficient	User defined	
Diffusion coefficient	2.03E-9	m^2/s

Migration in Electric Field

SETTINGS

Description	Value
Electric potential	Electric potential (es)

Description	Value
Mobility	Nernst - Einstein relation

Coordinate System Selection

SETTINGS

Description	Value
Coordinate system	Global coordinate system

Model Input

SETTINGS

Description	Value	Unit
Temperature	User defined	
Temperature	293.15	K

Variables

Name	Expression	Unit	Description	Selection	Details
domflux.cNar	2*(tds.dflux_cNar+tds.mflux_cNar)*pi*r*tds.d	mol/(m·s)	Domain flux, r-component	Domains 1–2	
domflux.cNaz	2*(tds.dflux_cNaz+tds.mflux_cNaz)*pi*r*tds.d	mol/(m·s)	Domain flux, z-component	Domains 1–2	
domflux.cClr	2*(tds.dflux_cClr+tds.mflux_cClr)*pi*r*tds.d	mol/(m·s)	Domain flux, r-component	Domains 1–2	
domflux.cClz	2*(tds.dflux_cClz+tds.mflux_cClz)*pi*r*tds.d	mol/(m·s)	Domain flux, z-component	Domains 1–2	
tds.ndflux_cNa	tds.dflux_cNar*tds.nrc+tds.dflux_cNaphi*tds.nphic+tds.dflux_cNaz*tds.nzc	mol/(m^2·s)	Normal diffusive flux	Boundaries 2, 4–5, 7–12	
tds.nmflux_cNa	tds.mflux_cNar*tds.nrc+tds.mflux_cNaphi*tds.nphic+tds.mflux_cNaz*tds.nzc	mol/(m^2·s)	Normal electrophoretic flux	Boundaries 2, 4–5, 7–12	
tds.ntflux_cNa	tds.bndFlux_cNa	mol/(m^2·s)	Normal total flux	Boundaries 2, 4–5, 7–12	
tds.ndflux_cCl	tds.dflux_cClr*tds.nrc+tds.dflux_cClphi*tds.nphic+tds.dflux_cClz*tds.nzc	mol/(m^2·s)	Normal diffusive flux	Boundaries 2, 4–5, 7–12	

Name	Expression	Unit	Description	Selection	Details
tds.nmflux_cCl	tds.mflux_cClr*tds.nrc+tds.mflux_cClphi*tds.nphic+tds.mflux_cClz*tds.nzc	mol/(m²·s)	Normal electrophoretic flux	Boundaries 2, 4–5, 7–12	
tds.ntflux_cCl	tds.bndFlux_cCl	mol/(m²·s)	Normal total flux	Boundaries 2, 4–5, 7–12	
tds.bndFlux_cNa	0.25*(uflux_spatial(cNa)-dflux_spatial(cNa))/(pi*r*tds.d)	mol/(m²·s)	Boundary flux	Boundary 4	Meta
tds.bndFlux_cNa	-dflux_spatial(cNa)/tds.d	mol/(m²·s)	Boundary flux	Boundaries 1, 3, 6	
tds.bndFlux_cNa	-0.5*dflux_spatial(cNa)/(pi*r*tds.d)	mol/(m²·s)	Boundary flux	Boundaries 2, 5, 7–12	Meta
tds.bndFlux_cCl	0.25*(uflux_spatial(cCl)-dflux_spatial(cCl))/(pi*r*tds.d)	mol/(m²·s)	Boundary flux	Boundary 4	Meta
tds.bndFlux_cCl	-dflux_spatial(cCl)/tds.d	mol/(m²·s)	Boundary flux	Boundaries 1, 3, 6	
tds.bndFlux_cCl	-0.5*dflux_spatial(cCl)/(pi*r*tds.d)	mol/(m²·s)	Boundary flux	Boundaries 2, 5, 7–12	Meta
tds.DF_cNarr	1.33E-9[m^2/s]	m²/s	Fluid diffusion coefficient, rr-component	Domains 1–2	
tds.DF_cNaphir	0	m²/s	Fluid diffusion coefficient, phir-component	Domains 1–2	
tds.DF_cNazr	0	m²/s	Fluid diffusion coefficient, zr-component	Domains 1–2	
tds.DF_cNarphi	0	m²/s	Fluid diffusion coefficient, rphi-component	Domains 1–2	
tds.DF_cNaphiphi	1.33E-9[m^2/s]	m²/s	Fluid diffusion coefficient, phiphi-component	Domains 1–2	
tds.DF_cNazphi	0	m²/s	Fluid diffusion	Domains 1–2	

Name	Expression	Unit	Description	Selection	Details
			coefficient, zphi-component		
tds.DF_cNarz	0	m^2/s	Fluid diffusion coefficient, rz-component	Domains 1–2	
tds.DF_cNaphiz	0	m^2/s	Fluid diffusion coefficient, phiz-component	Domains 1–2	
tds.DF_cNazz	1.33E-9[m^2/s]	m^2/s	Fluid diffusion coefficient, zz-component	Domains 1–2	
tds.D_cNarr	tds.DF_cNarr+tds.DiT_cNa	m^2/s	Diffusion coefficient, rr-component	Domains 1–2	+ operation
tds.D_cNaphir	tds.DF_cNaphir	m^2/s	Diffusion coefficient, phir-component	Domains 1–2	+ operation
tds.D_cNazr	tds.DF_cNazr	m^2/s	Diffusion coefficient, zr-component	Domains 1–2	+ operation
tds.D_cNarphi	tds.DF_cNarphi	m^2/s	Diffusion coefficient, rphi-component	Domains 1–2	+ operation
tds.D_cNaphiphi	tds.DF_cNaphiphi+tds.DiT_cNa	m^2/s	Diffusion coefficient, phiphi-component	Domains 1–2	+ operation
tds.D_cNazphi	tds.DF_cNazphi	m^2/s	Diffusion coefficient, zphi-component	Domains 1–2	+ operation
tds.D_cNarz	tds.DF_cNarz	m^2/s	Diffusion coefficient, rz-component	Domains 1–2	+ operation
tds.D_cNaphiz	tds.DF_cNaphiz	m^2/s	Diffusion coefficient, phiz-component	Domains 1–2	+ operation
tds.D_cNazz	tds.DF_cNazz+tds.DiT_cNa	m^2/s	Diffusion coefficient, zz-component	Domains 1–2	+ operation
tds.DF_cClrr	2.03E-9[m^2/s]	m^2/s	Fluid diffusion coefficient, rr-component	Domains 1–2	
tds.DF_cClphir	0	m^2/s	Fluid diffusion	Domains 1–2	

Name	Expression	Unit	Description	Selection	Details
			coefficient, phir-component		
tds.DF_cClzr	0	m^2/s	Fluid diffusion coefficient, zr-component	Domains 1–2	
tds.DF_cClrphi	0	m^2/s	Fluid diffusion coefficient, rphi-component	Domains 1–2	
tds.DF_cClphiphi	2.03E-9[m^2/s]	m^2/s	Fluid diffusion coefficient, phiphi-component	Domains 1–2	
tds.DF_cClzphi	0	m^2/s	Fluid diffusion coefficient, zphi-component	Domains 1–2	
tds.DF_cClrz	0	m^2/s	Fluid diffusion coefficient, rz-component	Domains 1–2	
tds.DF_cClphiz	0	m^2/s	Fluid diffusion coefficient, phiz-component	Domains 1–2	
tds.DF_cClzz	2.03E-9[m^2/s]	m^2/s	Fluid diffusion coefficient, zz-component	Domains 1–2	
tds.D_cClrr	tds.DF_cClrr+tds.DiT_cCl	m^2/s	Diffusion coefficient, rr-component	Domains 1–2	+ operation
tds.D_cClphir	tds.DF_cClphir	m^2/s	Diffusion coefficient, phir-component	Domains 1–2	+ operation
tds.D_cClzr	tds.DF_cClzr	m^2/s	Diffusion coefficient, zr-component	Domains 1–2	+ operation
tds.D_cClrphi	tds.DF_cClrphi	m^2/s	Diffusion coefficient, rphi-component	Domains 1–2	+ operation
tds.D_cClphiphi	tds.DF_cClphiphi+tds.DiT_cCl	m^2/s	Diffusion coefficient, phiphi-component	Domains 1–2	+ operation
tds.D_cClzphi	tds.DF_cClzphi	m^2/s	Diffusion coefficient, zphi-component	Domains 1–2	+ operation

Name	Expression	Unit	Description	Selection	Details
tds.D_cClrz	tds.DF_cClrz	m^2/s	Diffusion coefficient, rz-component	Domains 1–2	+ operation
tds.D_cClphiz	tds.DF_cClphiz	m^2/s	Diffusion coefficient, phiz-component	Domains 1–2	+ operation
tds.D_cClzz	tds.DF_cClzz+tds.DiT_cCl	m^2/s	Diffusion coefficient, zz-component	Domains 1–2	+ operation
tds.Dav_cNa	0.5*(tds.D_cNarr+tds.D_cNazz)	m^2/s	Average diffusion coefficient	Domains 1–2	
tds.Dav_cCl	0.5*(tds.D_cClrr+tds.D_cClzz)	m^2/s	Average diffusion coefficient	Domains 1–2	
tds.tflux_cNar	tds.dflux_cNar+tds.mflux_cNar	$mol/(m^2·s)$	Total flux, r-component	Domains 1–2	+ operation
tds.tflux_cNaphi	tds.dflux_cNaphi+tds.mflux_cNaphi	$mol/(m^2·s)$	Total flux, phi-component	Domains 1–2	+ operation
tds.tflux_cNaz	tds.dflux_cNaz+tds.mflux_cNaz	$mol/(m^2·s)$	Total flux, z-component	Domains 1–2	+ operation
tds.dfluxMag_cNa	sqrt(tds.dflux_cNar^2+tds.dflux_cNaphi^2+tds.dflux_cNaz^2)	$mol/(m^2·s)$	Diffusive flux magnitude	Domains 1–2	
tds.tfluxMag_cNa	sqrt(tds.tflux_cNar^2+tds.tflux_cNaphi^2+tds.tflux_cNaz^2)	$mol/(m^2·s)$	Total flux magnitude	Domains 1–2	
tds.dpflux_cNar	0	$mol/(m^2·s)$	Dispersive flux, r-component	Domains 1–2	
tds.dpflux_cNaphi	0	$mol/(m^2·s)$	Dispersive flux, phi-component	Domains 1–2	
tds.dpflux_cNaz	0	$mol/(m^2·s)$	Dispersive flux, z-component	Domains 1–2	
tds.mflux_cNar	tds.z_cNa*F_const*cNa*(-tds.um_cNarr*d(tds.V,r)-tds.um_cNarz*d(tds.V,z))	$mol/(m^2·s)$	Electrophoretic flux, r-component	Domains 1–2	
tds.mflux_cNaphi	tds.z_cNa*F_const*cNa*(-tds.um_cNaphir*d(tds.V,r)-	$mol/(m^2·s)$	Electrophoretic flux, phi-component	Domains 1–2	

Name	Expression	Unit	Description	Selection	Details
	tds.um_cNaphiz*d(tds.V,z))				
tds.mflux_cNaz	tds.z_cNa*F_const*cNa*(-tds.um_cNazr*d(tds.V,r)-tds.um_cNazz*d(tds.V,z))	mol/(m²·s)	Electrophoretic flux, z-component	Domains 1–2	
tds.mfluxMag_cNa	sqrt(tds.mflux_cNar^2+tds.mflux_cNaphi^2+tds.mflux_cNaz^2)	mol/(m²·s)	Electrophoretic flux magnitude	Domains 1–2	
tds.tflux_cClr	tds.dflux_cClr+tds.mflux_cClr	mol/(m²·s)	Total flux, r-component	Domains 1–2	+ operation
tds.tflux_cClphi	tds.dflux_cClphi+tds.mflux_cClphi	mol/(m²·s)	Total flux, phi-component	Domains 1–2	+ operation
tds.tflux_cClz	tds.dflux_cClz+tds.mflux_cClz	mol/(m²·s)	Total flux, z-component	Domains 1–2	+ operation
tds.dfluxMag_cCl	sqrt(tds.dflux_cClr^2+tds.dflux_cClphi^2+tds.dflux_cClz^2)	mol/(m²·s)	Diffusive flux magnitude	Domains 1–2	
tds.tfluxMag_cCl	sqrt(tds.tflux_cClr^2+tds.tflux_cClphi^2+tds.tflux_cClz^2)	mol/(m²·s)	Total flux magnitude	Domains 1–2	
tds.dpflux_cClr	0	mol/(m²·s)	Dispersive flux, r-component	Domains 1–2	
tds.dpflux_cClphi	0	mol/(m²·s)	Dispersive flux, phi-component	Domains 1–2	
tds.dpflux_cClz	0	mol/(m²·s)	Dispersive flux, z-component	Domains 1–2	
tds.mflux_cClr	tds.z_cCl*F_const*cCl*(-tds.um_cClrr*d(tds.V,r)-tds.um_cClrz*d(tds.V,z))	mol/(m²·s)	Electrophoretic flux, r-component	Domains 1–2	
tds.mflux_cClphi	tds.z_cCl*F_const*cCl*(-tds.um_cClphir*d(tds.V,r)-tds.um_cClphiz*d(tds.V,z))	mol/(m²·s)	Electrophoretic flux, phi-component	Domains 1–2	
tds.mflux_cClz	tds.z_cCl*F_const*c	mol/(m²·s)	Electrophoretic	Domains 1–2	

Name	Expression	Unit	Description	Selection	Details
	Cl*(-tds.um_cClzr*d(tds.V,r)-tds.um_cClzz*d(tds.V,z))		flux, z-component		
tds.mfluxMag_cCl	sqrt(tds.mflux_cClr^2+tds.mflux_cClphi^2+tds.mflux_cClz^2)	mol/(m²·s)	Electrophoretic flux magnitude	Domains 1–2	
tds.cNa_material	cNa*spatial.detF	mol/m³	Concentration	Domains 1–2	
tds.dflux_cNar	-tds.D_cNarr*cNar-tds.D_cNarz*cNaz	mol/(m²·s)	Diffusive flux, r-component	Domains 1–2	+ operation
tds.dflux_cNaphi	-tds.D_cNaphir*cNar-tds.D_cNaphiz*cNaz	mol/(m²·s)	Diffusive flux, phi-component	Domains 1–2	+ operation
tds.dflux_cNaz	-tds.D_cNazr*cNar-tds.D_cNazz*cNaz	mol/(m²·s)	Diffusive flux, z-component	Domains 1–2	+ operation
tds.grad_cNar	cNar	mol/m⁴	Concentration gradient, r-component	Domains 1–2	
tds.grad_cNaphi	0	mol/m⁴	Concentration gradient, phi-component	Domains 1–2	
tds.grad_cNaz	cNaz	mol/m⁴	Concentration gradient, z-component	Domains 1–2	
tds.cCl_material	cCl*spatial.detF	mol/m³	Concentration	Domains 1–2	
tds.dflux_cClr	-tds.D_cClrr*cClr-tds.D_cClrz*cClz	mol/(m²·s)	Diffusive flux, r-component	Domains 1–2	+ operation
tds.dflux_cClphi	-tds.D_cClphir*cClr-tds.D_cClphiz*cClz	mol/(m²·s)	Diffusive flux, phi-component	Domains 1–2	+ operation
tds.dflux_cClz	-tds.D_cClzr*cClr-tds.D_cClzz*cClz	mol/(m²·s)	Diffusive flux, z-component	Domains 1–2	+ operation
tds.grad_cClr	cClr	mol/m⁴	Concentration gradient, r-component	Domains 1–2	
tds.grad_cClphi	0	mol/m⁴	Concentration gradient, phi-component	Domains 1–2	
tds.grad_cClz	cClz	mol/m⁴	Concentration	Domains 1–2	

Name	Expression	Unit	Description	Selection	Details
			gradient, z-component		
tds.um_cNarr	tds.D_cNarr/(R_const*tds.T)	s·mol/kg	Mobility, rr-component	Domains 1–2	
tds.um_cNaphir	tds.D_cNaphir/(R_const*tds.T)	s·mol/kg	Mobility, phir-component	Domains 1–2	
tds.um_cNazr	tds.D_cNazr/(R_const*tds.T)	s·mol/kg	Mobility, zr-component	Domains 1–2	
tds.um_cNarphi	tds.D_cNarphi/(R_const*tds.T)	s·mol/kg	Mobility, rphi-component	Domains 1–2	
tds.um_cNaphiphi	tds.D_cNaphiphi/(R_const*tds.T)	s·mol/kg	Mobility, phiphi-component	Domains 1–2	
tds.um_cNazphi	tds.D_cNazphi/(R_const*tds.T)	s·mol/kg	Mobility, zphi-component	Domains 1–2	
tds.um_cNarz	tds.D_cNarz/(R_const*tds.T)	s·mol/kg	Mobility, rz-component	Domains 1–2	
tds.um_cNaphiz	tds.D_cNaphiz/(R_const*tds.T)	s·mol/kg	Mobility, phiz-component	Domains 1–2	
tds.um_cNazz	tds.D_cNazz/(R_const*tds.T)	s·mol/kg	Mobility, zz-component	Domains 1–2	
tds.um_cClrr	tds.D_cClrr/(R_const*tds.T)	s·mol/kg	Mobility, rr-component	Domains 1–2	
tds.um_cClphir	tds.D_cClphir/(R_const*tds.T)	s·mol/kg	Mobility, phir-component	Domains 1–2	
tds.um_cClzr	tds.D_cClzr/(R_const*tds.T)	s·mol/kg	Mobility, zr-component	Domains 1–2	
tds.um_cClrphi	tds.D_cClrphi/(R_const*tds.T)	s·mol/kg	Mobility, rphi-component	Domains 1–2	
tds.um_cClphiphi	tds.D_cClphiphi/(R_const*tds.T)	s·mol/kg	Mobility, phiphi-component	Domains 1–2	
tds.um_cClzphi	tds.D_cClzphi/(R_const*tds.T)	s·mol/kg	Mobility, zphi-component	Domains 1–2	
tds.um_cClrz	tds.D_cClrz/(R_const*tds.T)	s·mol/kg	Mobility, rz-component	Domains 1–2	
tds.um_cClphiz	tds.D_cClphiz/(R_const*tds.T)	s·mol/kg	Mobility, phiz-component	Domains 1–2	
tds.um_cClzz	tds.D_cClzz/(R_const*tds.T)	s·mol/kg	Mobility, zz-component	Domains 1–2	
tds.V	model.input.V	V	Electric potential	Domains 1–2	Meta
tds.T	tds.cdm1.minput_temperature	K	Temperature	Domains 1–2	

Name	Expression	Unit	Description	Selection	Details
tds.Rlin_cNa	0	1/s	Linear source term coefficient	Domains 1–2	+ operation
tds.Res_cNa	d(cNa*tds.z_cNa*F_const*(-tds.um_cNarr*d(tds.V,r)-tds.um_cNarz*d(tds.V,z)),r)+if(abs(r)<0.001*h_spatial,d(cNa*tds.z_cNa*F_const*(-tds.um_cNarr*d(tds.V,r)-tds.um_cNarz*d(tds.V,z)),r),cNa*tds.z_cNa*F_const*(-tds.um_cNarr*d(tds.V,r)-tds.um_cNarz*d(tds.V,z))/r)+d(cNa*tds.z_cNa*F_const*(-tds.um_cNazr*d(tds.V,r)-tds.um_cNazz*d(tds.V,z)),z)-cNa*tds.Rlin_cNa-tds.R_cNa	mol/(m³·s)	Equation residual	Domains 1–2	
tds.Rlin_cCl	0	1/s	Linear source term coefficient	Domains 1–2	+ operation
tds.Res_cCl	d(cCl*tds.z_cCl*F_const*(-tds.um_cClrr*d(tds.V,r)-tds.um_cClrz*d(tds.V,z)),r)+if(abs(r)<0.001*h_spatial,d(cCl*tds.z_cCl*F_const*(-tds.um_cClrr*d(tds.V,r)-tds.um_cClrz*d(tds.V,z)),r),cCl*tds.z_cCl*F_const*(-tds.um_cClrr*d(tds.V,r)-tds.um_cClrz*d(tds.V,z))/r)+d(cCl*tds.z_	mol/(m³·s)	Equation residual	Domains 1–2	

Name	Expression	Unit	Description	Selection	Details
	cCl*F_const*(-tds.um_cClzr*d(tds.V,r)-tds.um_cClzz*d(tds.V,z)),z)-cCl*tds.Rlin_cCl-tds.R_cCl				

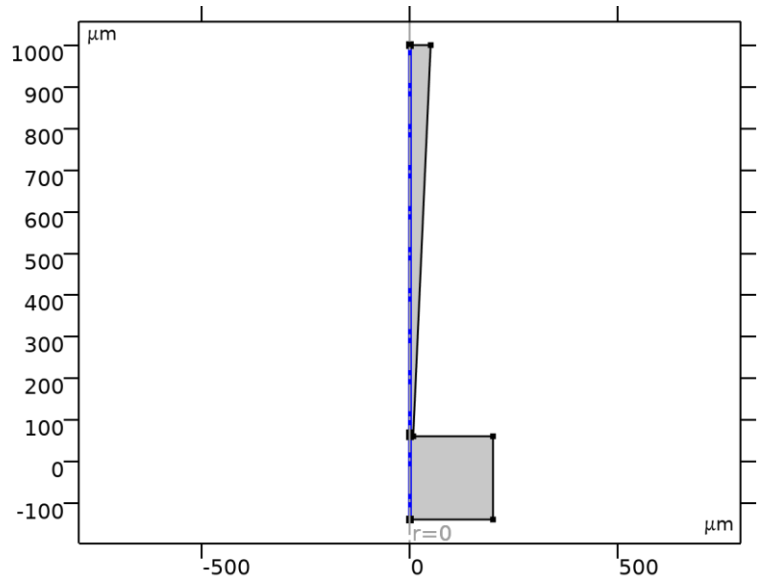
Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection
cNa	Lagrange (Linear)	mol/m³	Concentration	Spatial	Domains 1–2
cCl	Lagrange (Linear)	mol/m³	Concentration	Spatial	Domains 1–2

Weak Expressions

Weak expression	Integration order	Integration frame	Selection
2*(tds.dflux_cNar*test(cNar)+tds.dflux_cNaz*test(cNaz))*tds.d*pi*r	2	Spatial	Domains 1–2
2*(tds.dflux_cClr*test(cClr)+tds.dflux_cClz*test(cClz))*tds.d*pi*r	2	Spatial	Domains 1–2
2*tds.z_cNa*F_const*cNa*((-tds.um_cNarr*d(tds.V,r)-tds.um_cNarz*d(tds.V,z))*test(cNar)+(-tds.um_cNazr*d(tds.V,r)-tds.um_cNazz*d(tds.V,z))*test(cNaz))*tds.d*pi*r	2	Spatial	Domains 1–2
2*tds.z_cCl*F_const*cCl*((-tds.um_cClrr*d(tds.V,r)-tds.um_cClrz*d(tds.V,z))*test(cClr)+(-tds.um_cClzr*d(tds.V,r)-tds.um_cClzz*d(tds.V,z))*test(cClz))*tds.d*pi*r	2	Spatial	Domains 1–2
2*tds.streamline*(isScalingSystemDomain==0)*tds.d*pi*r	2	Spatial	Domains 1–2
2*tds.crosswind*(isScalingSystemDomain==0)*tds.d*pi*r	4	Spatial	Domains 1–2

2.3.5 Axial Symmetry 1

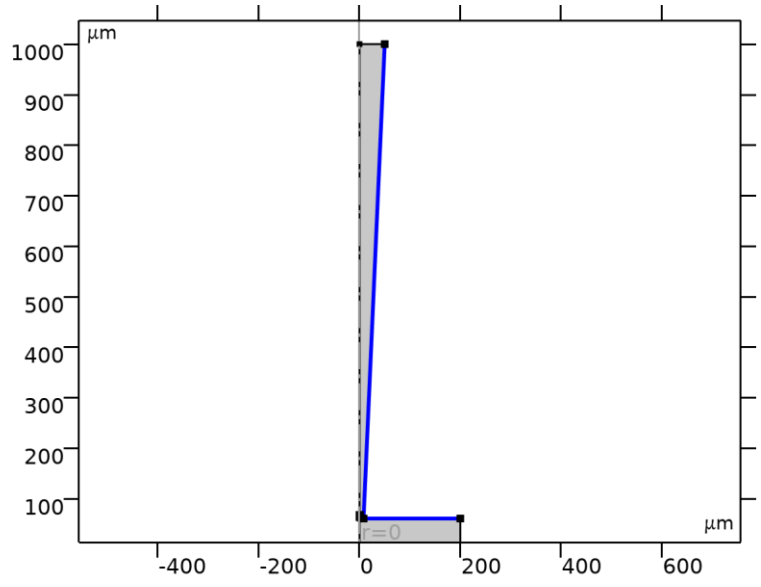


Axial Symmetry 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: All boundaries

2.3.6 No Flux 1



No Flux 1

SELECTION

Geometric entity level	Boundary

Selection	Geometry geom1: Dimension 1: All boundaries

EQUATIONS

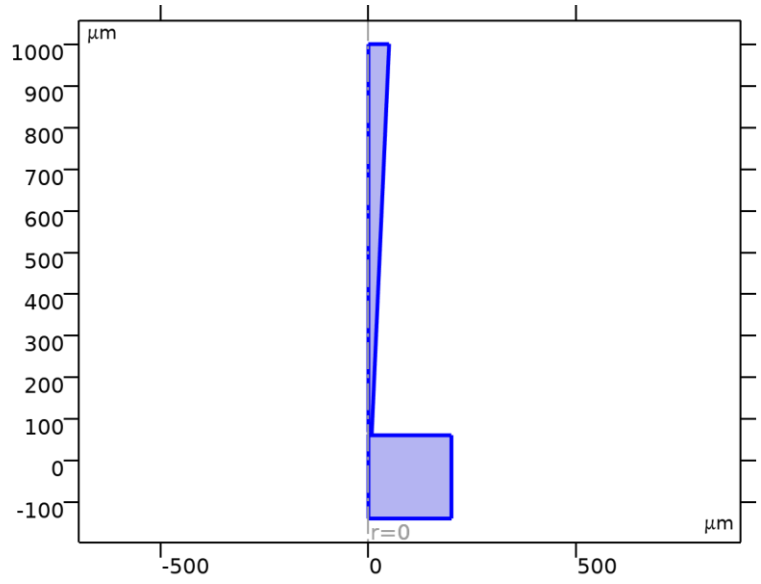
$$-\mathbf{n}\cdot(\mathbf{J}_i+\mathbf{u}c_i)=0$$

Convection

SETTINGS

Description	Value
Include	On

2.3.7 Initial Values 1



Initial Values 1

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

Initial Values

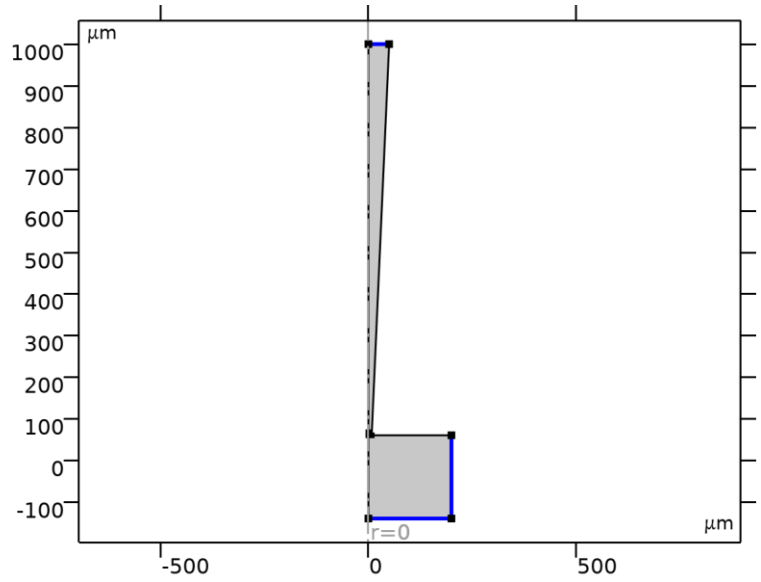
SETTINGS

Description	Value	Unit
Concentration	{150, 150}	mol/m³

Variables

Name	Expression	Unit	Description	Selection	Details
tds.c0_cNa	150	mol/m³	Concentration	Domains 1–2	+ operation
tds.c0_cCl	150	mol/m³	Concentration	Domains 1–2	+ operation

2.3.8 Concentration 1



Concentration 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundaries 2, 8, 12

EQUATIONS

$c_i = c_{0,i}$

Concentration

SETTINGS

Description	Value	Unit
Species cNa	On	
Species cCl	On	
Concentration	{150, 150}	mol/m³

Variables

Name	Expression	Unit	Description	Selection	Details
tds.c0_cNa	150	mol/m³	Concentration	Boundaries 2, 8, 12	+ operation
tds.c0_cCl	150	mol/m³	Concentration	Boundaries 2, 8, 12	+ operation
tds.conc1.nmflow_cNa	tds.conc1.int(2*tds.ntflux_cNa*pi*r)*td	mol/s	Normal molar flow rate	Global	

Name	Expression	Unit	Description	Selection	Details
	s.d				
tds.conc1.nmflow_cCl	tds.conc1.int(2*tds.ntflux_cCl*pi*r)*tds.d	mol/s	Normal molar flow rate	Global	

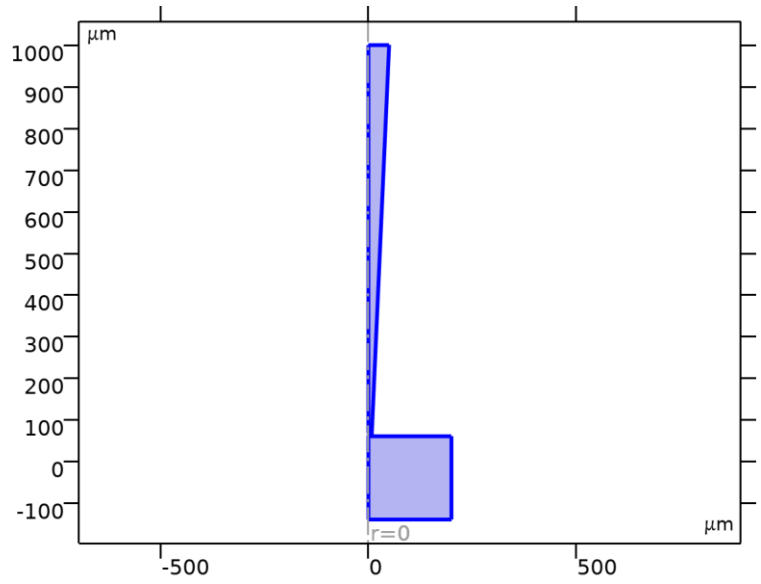
Constraints

Constraint	Constraint force	Shape function	Selection	Details
-tds.cVar_cNa+tds.c0_cNa	test(-tds.cVar_cNa+tds.c0_cNa)	Lagrange (Linear)	Boundaries 2, 8, 12	Elemental
-tds.cVar_cCl+tds.c0_cCl	test(-tds.cVar_cCl+tds.c0_cCl)	Lagrange (Linear)	Boundaries 2, 8, 12	Elemental

2.4 ELECTROSTATICS

USED PRODUCTS

COMSOL Multiphysics



Electrostatics

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

EQUATIONS

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\mathbf{E} = -\nabla V$$

2.4.1 Interface Settings

Discretization

SETTINGS

Description	Value
Electric potential	Quadratic

Manual Terminal Sweep Settings

SETTINGS

Description	Value	Unit
Use manual terminal sweep	Off	
Reference impedance	50	Ω

2.4.2 Variables

Name	Expression	Unit	Description	Selection	Details
es.d	1	1	Contribution	Domains 1–2	
es.nr	nr		Normal vector, r-component	Boundary 4	
es.nphi	0		Normal vector, phi-component	Boundary 4	
es.nz	nz		Normal vector, z-component	Boundary 4	
es.nr	dnr		Normal vector, r-component	Boundaries 1–3, 5–12	
es.nphi	0		Normal vector, phi-component	Boundaries 1–3, 5–12	
es.nz	dnz		Normal vector, z-component	Boundaries 1–3, 5–12	
es.nmeshr	nrmesh		Mesh normal vector, r-component	Boundary 4	
es.nmeshphi	0		Mesh normal vector, phi-component	Boundary 4	
es.nmeshz	nzmesh		Mesh normal vector, z-component	Boundary 4	
es.nmeshr	dnrmesh		Mesh normal vector, r-	Boundaries 1–3, 5–12	

Name	Expression	Unit	Description	Selection	Details
			component		
es.nmeshphi	0		Mesh normal vector, phi-component	Boundaries 1–3, 5–12	
es.nmeshz	dnzmesh		Mesh normal vector, z-component	Boundaries 1–3, 5–12	
es.unmeshr	unrmesh		Mesh normal vector, upside, r-component	Boundaries 1–12	
es.unmeshphi	0		Mesh normal vector, upside, phi-component	Boundaries 1–12	
es.unmeshz	unzmesh		Mesh normal vector, upside, z-component	Boundaries 1–12	
es.dnmeshr	dnrmesh		Mesh normal vector, downside, r-component	Boundaries 1–12	
es.dnmeshphi	0		Mesh normal vector, downside, phi-component	Boundaries 1–12	
es.dnmeshz	dnzmesh		Mesh normal vector, downside, z-component	Boundaries 1–12	
es.I_sRR	(spatial.invF11*(spatial.invF11*es.I_srr+spatial.invF31*es.I_szr)+spatial.invF31*(spatial.invF11*es.I_srz+spatial.invF31*es.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, RR-component	Domains 1–2	
es.I_sPHIR	if(Rg>0.001*h,R/r,Rr)*(spatial.invF11*es.I_sphir+spatial.invF31*es.I_sphiz)*spatial.detF	1	Spatial identity matrix, material frame, PHIR-component	Domains 1–2	
es.I_sZR	(spatial.invF11*(spatial.invF13*es.I_srr+spatial.invF33*es.I_szr)+spatial.invF31*(spatial.invF13*es.I_srz+spatial.inv	1	Spatial identity matrix, material frame, ZR-component	Domains 1–2	

Name	Expression	Unit	Description	Selection	Details
	F33*es.l_szz))*spatial.detF				
es.l_sRPHI	if(Rg>0.001*h,R/r,Rr)*(spatial.invF11*es.l_srphi+spatial.invF31*es.l_szphi)*spatial.detF	1	Spatial identity matrix, material frame, RPHI-component	Domains 1–2	
es.l_sPHIPHI	if(Rg>0.001*h,R/r,Rr)^2*es.l_sphiphi*spatial.detF	1	Spatial identity matrix, material frame, PHIPHI-component	Domains 1–2	
es.l_sZPHI	if(Rg>0.001*h,R/r,Rr)*(spatial.invF13*es.l_srphi+spatial.invF33*es.l_szphi)*spatial.detF	1	Spatial identity matrix, material frame, ZPHI-component	Domains 1–2	
es.l_sRZ	(spatial.invF13*(spatial.invF11*es.l_srr+spatial.invF31*es.l_szr)+spatial.invF33*(spatial.invF11*es.l_srz+spatial.invF31*es.l_szz))*spatial.detF	1	Spatial identity matrix, material frame, RZ-component	Domains 1–2	
es.l_sPHIZ	if(Rg>0.001*h,R/r,Rr)*(spatial.invF13*es.l_sphir+spatial.invF33*es.l_sphiz)*spatial.detF	1	Spatial identity matrix, material frame, PHIZ-component	Domains 1–2	
es.l_sZZ	(spatial.invF13*(spatial.invF13*es.l_srr+spatial.invF33*es.l_szr)+spatial.invF33*(spatial.invF13*es.l_srz+spatial.invF33*es.l_szz))*spatial.detF	1	Spatial identity matrix, material frame, ZZ-component	Domains 1–2	
es.l_srr	1	1	Spatial identity matrix, rr-component	Domains 1–2	
es.l_sphir	0	1	Spatial identity matrix, phir-component	Domains 1–2	
es.l_szr	0	1	Spatial identity matrix, zr-component	Domains 1–2	
es.l_srphi	0	1	Spatial identity matrix, rphi-component	Domains 1–2	

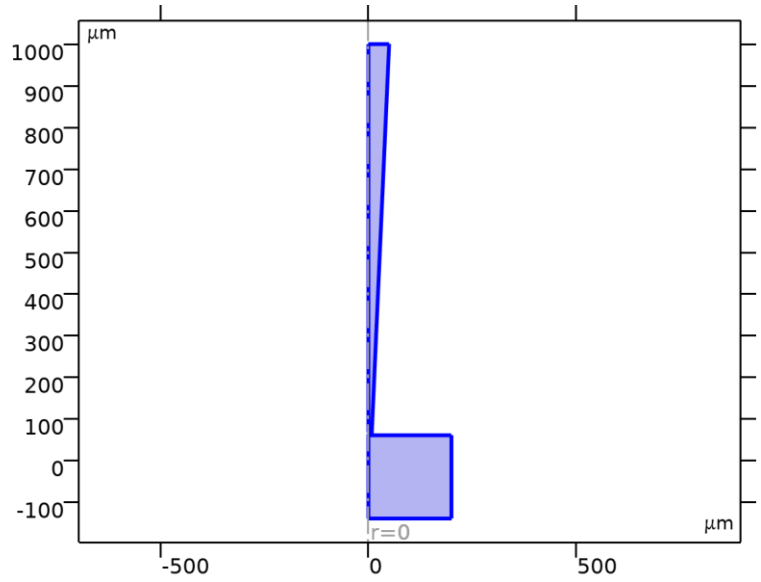
Name	Expression	Unit	Description	Selection	Details
es.I_sphiphi	1	1	Spatial identity matrix, phiphi-component	Domains 1–2	
es.I_szphi	0	1	Spatial identity matrix, zphi-component	Domains 1–2	
es.I_srz	0	1	Spatial identity matrix, rz-component	Domains 1–2	
es.I_sphiz	0	1	Spatial identity matrix, phiz-component	Domains 1–2	
es.I_szz	1	1	Spatial identity matrix, zz-component	Domains 1–2	
es.unTr	es.unTer	Pa	Maxwell upward surface stress tensor, r-component	Boundaries 1–12	
es.unTphi	es.unTephi	Pa	Maxwell upward surface stress tensor, phi-component	Boundaries 1–12	
es.unTz	es.unTez	Pa	Maxwell upward surface stress tensor, z-component	Boundaries 1–12	
es.dnTr	es.dnTer	Pa	Maxwell downward surface stress tensor, r-component	Boundaries 1–12	
es.dnTphi	es.dnTephi	Pa	Maxwell downward surface stress tensor, phi-component	Boundaries 1–12	
es.dnTz	es.dnTez	Pa	Maxwell downward surface stress tensor, z-component	Boundaries 1–12	
es.unr	unr		Normal vector up direction, r-	Boundaries 1–	

Name	Expression	Unit	Description	Selection	Details
			component	12	
es.unphi	0		Normal vector up direction, phi-component	Boundaries 1–12	
es.unz	unz		Normal vector up direction, z-component	Boundaries 1–12	
es.dnr	dnr		Normal vector down direction, r-component	Boundaries 1–12	
es.dnphi	0		Normal vector down direction, phi-component	Boundaries 1–12	
es.dnz	dnz		Normal vector down direction, z-component	Boundaries 1–12	
es.unTer	-0.5*es.dnr*(real(up(es.Dr))*real(up(es.Er))+real(up(es.Dphi))*real(up(es.Ephi))+real(up(es.Dz))*real(up(es.Ez)))+real(up(es.Dr))*(real(up(es.Er))*es.dnr+real(up(es.Ephi))*es.dnphi+real(up(es.Ez))*es.dnz)	Pa	Maxwell upward electric surface stress tensor, r-component	Boundary 4	
es.unTephi	-0.5*es.dnphi*(real(up(es.Dr))*real(up(es.Er))+real(up(es.Dphi))*real(up(es.Ephi))+real(up(es.Dz))*real(up(es.Ez)))+real(up(es.Dphi))*(real(up(es.Er))*es.dnr+real(up(es.Ephi))*es.dnphi+real(up(es.Ez))*es.dnz)	Pa	Maxwell upward electric surface stress tensor, phi-component	Boundary 4	
es.unTez	-0.5*es.dnz*(real(up(es.Dr))*real(up(es.Er))+real(up(es.Dphi))*real(up(es.Ephi))+real(up(es.Dz))*real(up(es.Ez)))+real(up(es.Dz))*(real(up(es.Er))*es.dnr+real(up	Pa	Maxwell upward electric surface stress tensor, z-component	Boundary 4	

Name	Expression	Unit	Description	Selection	Details
	(es.Ephi))*es.dnphi+real(up(es.Ez))*es.dnz)				
es.unTer	0	Pa	Maxwell upward electric surface stress tensor, r-component	Boundaries 1–3, 5–12	
es.unTephi	0	Pa	Maxwell upward electric surface stress tensor, phi-component	Boundaries 1–3, 5–12	
es.unTez	0	Pa	Maxwell upward electric surface stress tensor, z-component	Boundaries 1–3, 5–12	
es.dnTer	-0.5*es.unr*(real(down(es.Dr))*real(down(es.Er))+real(down(es.Dphi))*real(down(es.Ephi))+real(down(es.Dz))*real(down(es.Ez)))+real(down(es.Dr))*(real(down(es.Er))*es.unr+real(down(es.Ephi))*es.unphi+real(down(es.Ez))*es.unz)	Pa	Maxwell downward electric surface stress tensor, r-component	Boundaries 1–12	
es.dnTephi	-0.5*es.unphi*(real(down(es.Dr))*real(down(es.Er))+real(down(es.Dphi))*real(down(es.Ephi))+real(down(es.Dz))*real(down(es.Ez)))+real(down(es.Dphi))*(real(down(es.Er))*es.unr+real(down(es.Ephi))*es.unphi+real(down(es.Ez))*es.unz)	Pa	Maxwell downward electric surface stress tensor, phi-component	Boundaries 1–12	
es.dnTez	-0.5*es.unz*(real(down(es.Dr))*real(down(es.Er))+real(down(es.Dphi))*real(down(es.Ephi))+real(down(es.Dz))*real(down(es.Ez)))+real(	Pa	Maxwell downward electric surface stress tensor, z-component	Boundaries 1–12	

Name	Expression	Unit	Description	Selection	Details
	down(es.Dz))*(real(down(es.Er))*es.unr+real(down(es.Ephi))*es.unphi+real(down(es.Ez))*es.unz)				
es.intWe	es.int_We(es.d*es.dWe)	J	Total electric energy	Global	+ operation
es.zref	50[ohm]	Ω	Reference impedance	Global	

2.4.3 Charge Conservation 1



Charge Conservation 1

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

EQUATIONS

$$\mathbf{E} = -\nabla V$$
$$\nabla \cdot (\epsilon_0 \epsilon_r \mathbf{E}) = \rho_v$$

Constitutive Relation D-E

SETTINGS

Description	Value
Dielectric model	Relative permittivity
Relative permittivity	User defined

Description	Value
Relative permittivity	81

Coordinate System Selection

SETTINGS

Description	Value
Coordinate system	Global coordinate system

Variables

Name	Expression	Unit	Description	Selection	Details
es.nD	0	C/m^2	Surface charge density	Boundaries 1–12	+ operation
es.epsilonrrr	81	1	Relative permittivity, rr-component	Domains 1–2	
es.epsilonrphir	0	1	Relative permittivity, phir-component	Domains 1–2	
es.epsilonrzr	0	1	Relative permittivity, zr-component	Domains 1–2	
es.epsilonrrphi	0	1	Relative permittivity, rphi-component	Domains 1–2	
es.epsilonrphiphi	81	1	Relative permittivity, phiphi-component	Domains 1–2	
es.epsilonrzphi	0	1	Relative permittivity, zphi-component	Domains 1–2	
es.epsilonrrz	0	1	Relative permittivity, rz-component	Domains 1–2	
es.epsilonrphiz	0	1	Relative permittivity, phiz-component	Domains 1–2	
es.epsilonrzz	81	1	Relative permittivity, zz-component	Domains 1–2	
es.epsilonr_iso	81	1	Relative permittivity, isotropic value	Domains 1–2	

Name	Expression	Unit	Description	Selection	Details
es.DrR	0	C/m²	Remanent electric displacement, R-component	Domains 1–2	
es.DrPHI	0	C/m²	Remanent electric displacement, PHI-component	Domains 1–2	
es.DrZ	0	C/m²	Remanent electric displacement, Z-component	Domains 1–2	
es.Dr	epsilon0_const*es.l_srr*es.Er+epsilon0_const*es.l_srphi*es.Ephi+epsilon0_const*es.l_srz*es.Ez+es.Pr+es.Per+es.Phr	C/m²	Electric displacement field, r-component	Domains 1–2	
es.Dphi	epsilon0_const*es.l_sphir*es.Er+epsilon0_const*es.l_sphiphi*es.Ephi+epsilon0_const*es.l_sphiz*es.Ez+es.Pphi+es.Pephi+es.Phphi	C/m²	Electric displacement field, phi-component	Domains 1–2	
es.Dz	epsilon0_const*es.l_szr*es.Er+epsilon0_const*es.l_szphi*es.Ephi+epsilon0_const*es.l_szz*es.Ez+es.Pz+es.Pez+es.Phz	C/m²	Electric displacement field, z-component	Domains 1–2	
es.Pr	epsilon0_const*(es.chirr*es.Er+es.chirphi*es.Ephi+es.chirz*es.Ez)	C/m²	Polarization, r-component	Domains 1–2	
es.Pphi	epsilon0_const*(es.chiphir*es.Er+es.chiphiphi*es.Ephi+es.chiphiz*es.Ez)	C/m²	Polarization, phi-component	Domains 1–2	
es.Pz	epsilon0_const*(es.chizr*es.Er+es.chizphi*es.Ephi+es.chizz*es.Ez)	C/m²	Polarization, z-component	Domains 1–2	
es.normD	sqrt(realdot(es.Dr,es.	C/m²	Electric	Domains 1–2	

Name	Expression	Unit	Description	Selection	Details
	Dr)+realdot(es.Dphi, es.Dphi)+realdot(es.Dz,es.Dz))		displacement field norm		
es.normP	sqrt(realdot(es.Pr,es.Pr)+realdot(es.Pphi,es.Pphi)+realdot(es.Pz,es.Pz))	C/m²	Polarization norm	Domains 1–2	
es.Per	0	C/m²	Polarization contribution, r-component	Domains 1–2	+ operation
es.Pephi	0	C/m²	Polarization contribution, phi-component	Domains 1–2	+ operation
es.Pez	0	C/m²	Polarization contribution, z-component	Domains 1–2	+ operation
es.Phr	0	C/m²	Polarization contribution, r-component	Domains 1–2	+ operation
es.Phphi	0	C/m²	Polarization contribution, phi-component	Domains 1–2	+ operation
es.Phz	0	C/m²	Polarization contribution, z-component	Domains 1–2	+ operation
es.chirr	-1+es.epsilonrrr	1	Electric susceptibility, rr-component	Domains 1–2	
es.chiphir	es.epsilonrphir	1	Electric susceptibility, phir-component	Domains 1–2	
es.chizr	es.epsilonrzr	1	Electric susceptibility, zr-component	Domains 1–2	
es.chirphi	es.epsilonrrphi	1	Electric susceptibility, rphi-component	Domains 1–2	
es.chiphiphi	-1+es.epsilonrphiphi	1	Electric susceptibility, phiphi-component	Domains 1–2	
es.chizphi	es.epsilonrzphi	1	Electric susceptibility,	Domains 1–2	

Name	Expression	Unit	Description	Selection	Details
			zphi-component		
es.chirz	es.epsilonrrz	1	Electric susceptibility, rz-component	Domains 1–2	
es.chiphiz	es.epsilonrphiz	1	Electric susceptibility, phiz-component	Domains 1–2	
es.chizz	-1+es.epsilonrzz	1	Electric susceptibility, zz-component	Domains 1–2	
es.Er	-Vr	V/m	Electric field, r-component	Domains 1–2	
es.Ephi	0	V/m	Electric field, phi-component	Domains 1–2	
es.Ez	-Vz	V/m	Electric field, z-component	Domains 1–2	
es.tEr	-VTr	V/m	Tangential electric field, r-component	Boundaries 1–12	
es.tEphi	0	V/m	Tangential electric field, phi-component	Boundaries 1–12	
es.tEz	-VTz	V/m	Tangential electric field, z-component	Boundaries 1–12	
es.normE	sqrt(realdot(es.Er,es.Er)+realdot(es.Ephi,es.Ephi)+realdot(es.Ez,es.Ez))	V/m	Electric field norm	Domains 1–2	
es.Jr	es.Jdr	A/m²	Current density, r-component	Domains 1–2	+ operation
es.Jphi	es.Jdphi	A/m²	Current density, phi-component	Domains 1–2	+ operation
es.Jz	es.Jdz	A/m²	Current density, z-component	Domains 1–2	+ operation
es.JR	(spatial.invF11*es.Jdr+spatial.invF31*es.Jdz)*spatial.detF	A/m²	Current density, R-component	Domains 1–2	+ operation
es.JPHI	if(Rg>0.001*h,R/r,Rr)*es.Jdphi*spatial.detF	A/m²	Current density, PHI-component	Domains 1–2	+ operation
es.JZ	(spatial.invF13*es.Jdr	A/m²	Current density,	Domains 1–2	+ operation

Name	Expression	Unit	Description	Selection	Details
	+spatial.invF33*es.Jdz)*spatial.detF		Z-component		
es.Jdr	0	A/m²	Displacement current density, r-component	Domains 1–2	
es.Jdphi	0	A/m²	Displacement current density, phi-component	Domains 1–2	
es.Jdz	0	A/m²	Displacement current density, z-component	Domains 1–2	
es.normJ	sqrt(realdot(es.Jr,es.Jr)+realdot(es.Jphi,es.Jphi)+realdot(es.Jz,es.Jz))	A/m²	Current density norm	Domains 1–2	
es.ccn1.nJ	es.unr*down(es.Jr)+es.unphi*down(es.Jphi)+es.unz*down(es.Jz)	A/m²	Inward current density	Boundaries 1–3, 5–12	
es.W	es.We	J/m³	Energy density	Domains 1–2	+ operation
es.dWe	2*es.We*pi*r	J/m²	Integrand for total electric energy	Domains 1–2	Meta
es.We	0.5*epsilon0_const*(((es.l_srr+es.chirr)*es.Er+(es.l_srphi+es.chirphi)*es.Ephi+(es.l_srz+es.chirz)*es.Ez)*es.Er+((es.l_sphir+es.chiphir)*es.Er+(es.l_sphiphi+es.chiphiphi)*es.Ephi+(es.l_sphiz+es.chiphiz)*es.Ez)*es.Ephi+((es.l_szr+es.chizr)*es.Er+(es.l_szphi+es.chizphi)*es.Ephi+(es.l_szz+es.chizz)*es.Ez)*es.Ez)	J/m³	Electric energy density	Domains 1–2	

Shape functions

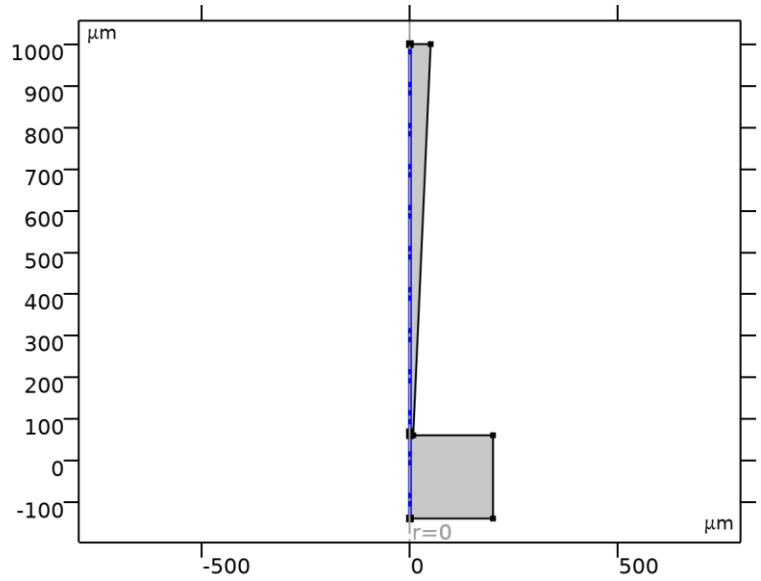
Name	Shape function	Unit	Description	Shape frame	Selection
V	Lagrange (Quadratic)	V	Electric potential	Spatial	Domains 1–2
V	Lagrange (Quadratic)	V	Electric potential	Material	Domains 1–2

Name	Shape function	Unit	Description	Shape frame	Selection
V	Lagrange (Quadratic)	V	Electric potential	Geometry	Domains 1–2
V	Lagrange (Quadratic)	V	Electric potential	Mesh	Domains 1–2

Weak Expressions

Weak expression	Integration order	Integration frame	Selection
-2*(es.Dr*test(Vr)+es.Dz*test(Vz))*es.d*pi*r	4	Spatial	Domains 1–2

2.4.4 Axial Symmetry 1

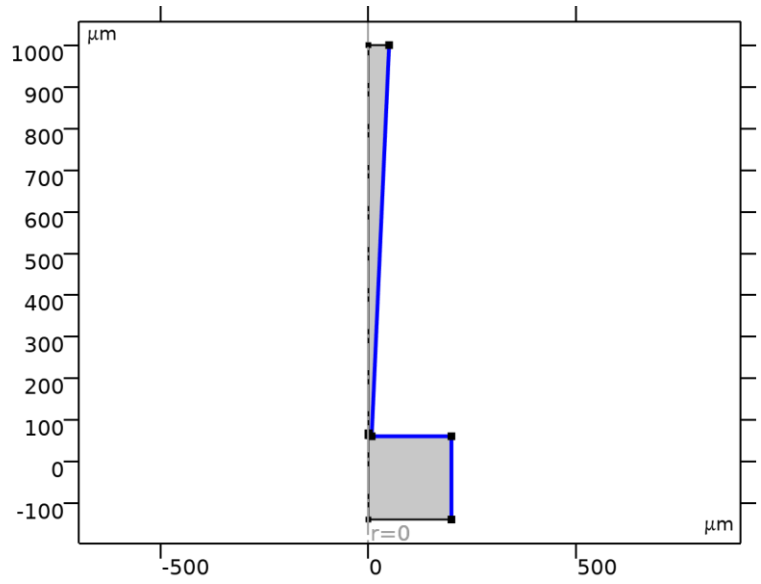


Axial Symmetry 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: All boundaries

2.4.5 Zero Charge 1



Zero Charge 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: All boundaries

EQUATIONS

$$\mathbf{n} \cdot \mathbf{D} = 0$$

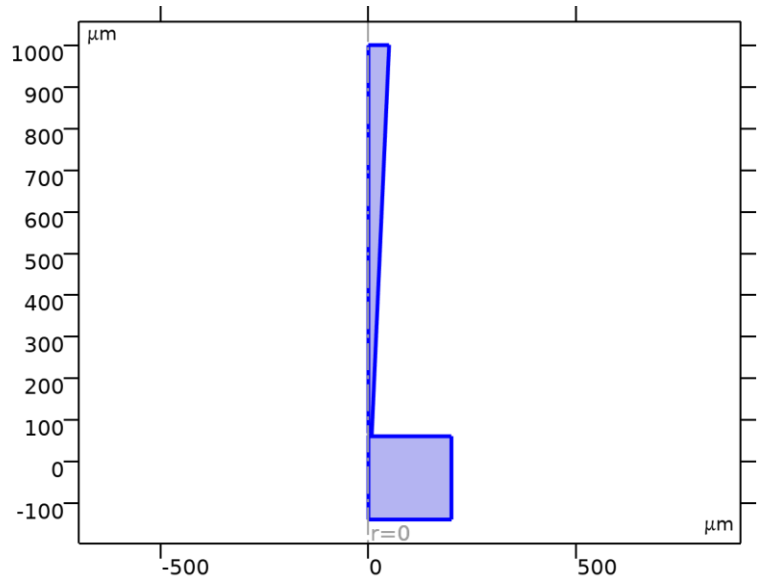
Variables

Name	Expression	Unit	Description	Selection	Details
es.nD	0	C/m²	Surface charge density	Boundaries 5, 7, 9–12	+ operation

Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection	Details
V	Lagrange (Quadratic)	V	Electric potential	Spatial	No boundaries	Slit
V	Lagrange (Quadratic)	V	Electric potential	Material	No boundaries	Slit
V	Lagrange (Quadratic)	V	Electric potential	Geometry	No boundaries	Slit
V	Lagrange (Quadratic)	V	Electric potential	Mesh	No boundaries	Slit

2.4.6 Initial Values 1



Initial Values 1

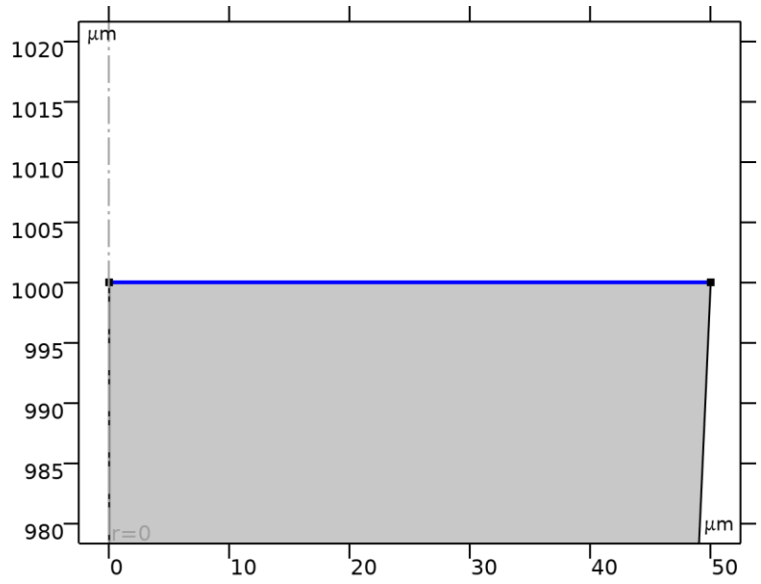
SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

SETTINGS

Description	Value	Unit
Electric potential	0	V

2.4.7 Electric Potential 1



Electric Potential 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundary 8

EQUATIONS

$$V = V_0$$

Electric Potential

SETTINGS

Description	Value	Unit
Electric potential	0.1	V

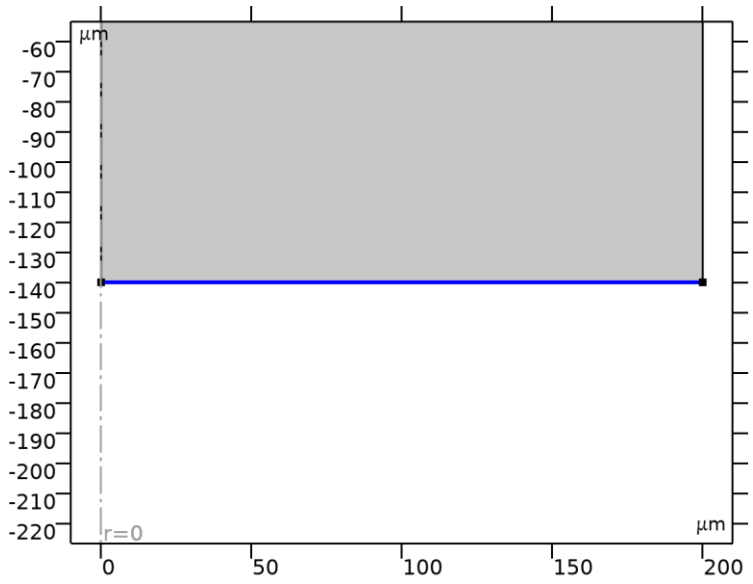
Variables

Name	Expression	Unit	Description	Selection	Details
es.nD	es.unr*down(es.Dr)+es.unphi*down(es.Dphi)+es.unz*down(es.Dz)	C/m²	Surface charge density	Boundary 8	+ operation
es.V0	0.1	V	Electric potential	Boundary 8	

Constraints

Constraint	Constraint force	Shape function	Selection	Details
es.V0-V	test(es.V0-V)	Lagrange (Quadratic)	Boundary 8	Elemental

2.4.8 Ground 1



Ground 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundary 2

EQUATIONS

$$V = 0$$

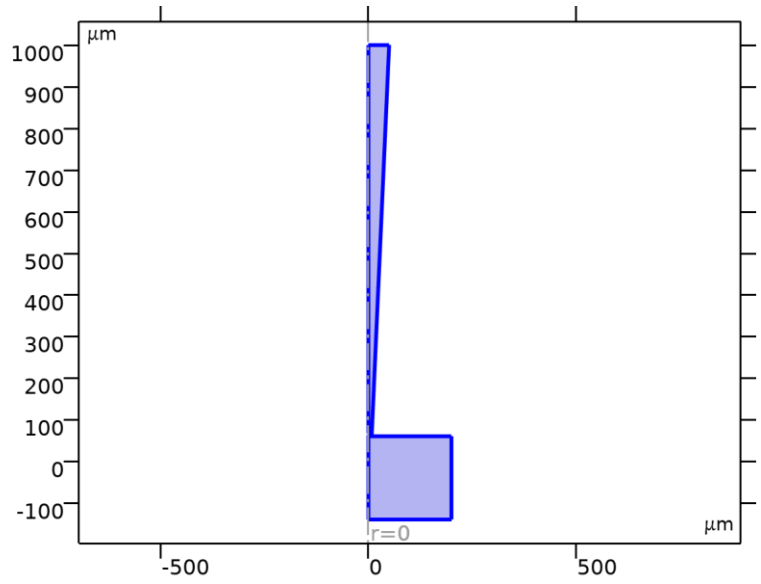
Variables

Name	Expression	Unit	Description	Selection	Details
es.nD	es.unr*down(es.Dr)+es.unphi*down(es.Dphi)+es.unz*down(es.Dz)	C/m²	Surface charge density	Boundary 2	+ operation

Constraints

Constraint	Constraint force	Shape function	Selection	Details
-V	test(-V)	Lagrange (Quadratic)	Boundary 2	Elemental

2.4.9 Space Charge Density 1



Space Charge Density 1

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: Domains 1–2

EQUATIONS

$$\nabla \cdot \mathbf{D} = \rho_v$$

Coordinate System Selection

SETTINGS

Description	Value
Coordinate system	Global coordinate system

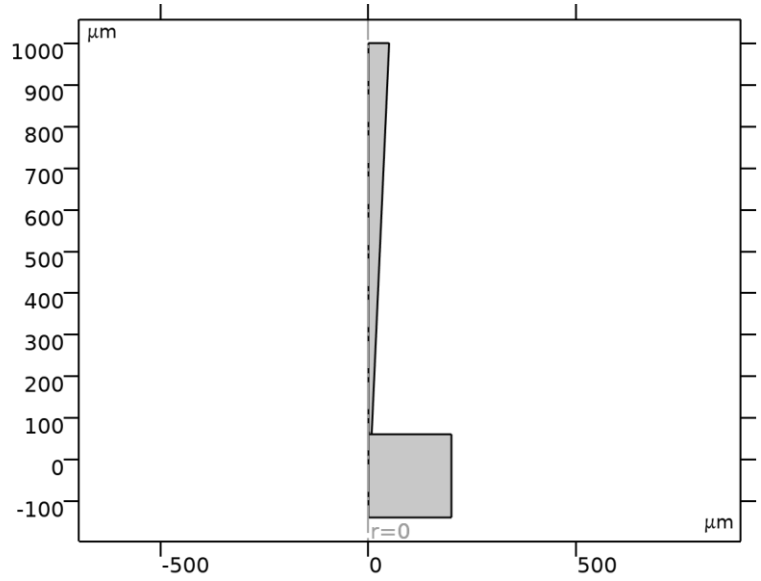
Variables

Name	Expression	Unit	Description	Selection	Details
es.scd1.rhoq	(cNa-cCl)*96480[C/mol]	C/m³	Space charge density	Domains 1–2	
es.rhoq	es.scd1.rhoq	C/m³	Space charge density	Domains 1–2	+ operation

Weak Expressions

Weak expression	Integration order	Integration frame	Selection
-2*es.scd1.rhoq*test(V)*es.d*pi*r	4	Spatial	Domains 1–2

2.5 MESH 1



Mesh 1

MESH STATISTICS

Description	Value
Status	Empty mesh
Mesh vertices	0

2.5.1 Size (size)

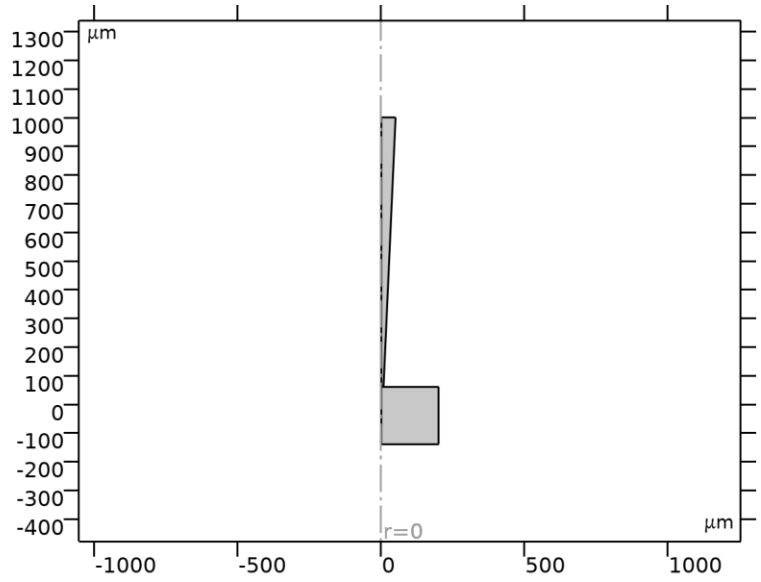
SETTINGS

Description	Value
Calibrate for	Fluid dynamics
Maximum element size	1.34
Minimum element size	0.004
Curvature factor	0.2
Maximum element growth rate	1.05
Predefined size	Extremely fine

2.5.2 Free Triangular 1 (ftri1)

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1



Free Triangular 1

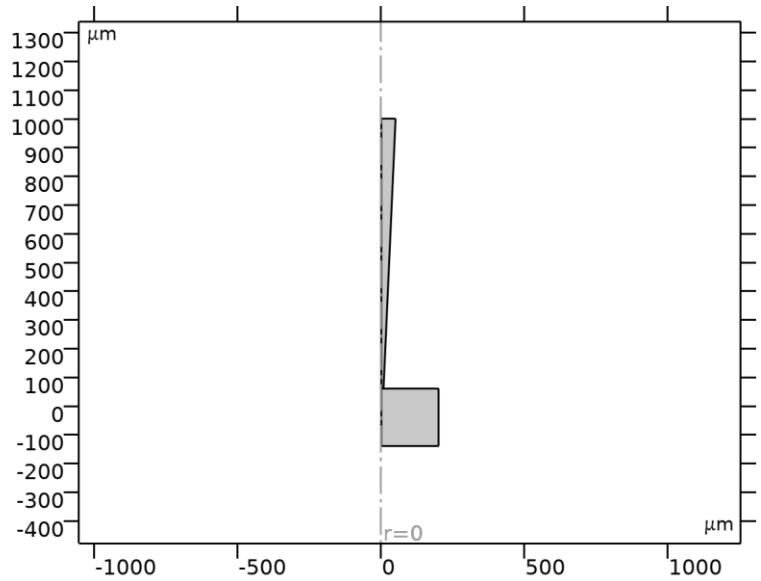
INFORMATION

Description	Value
Last build time	Not built

Size 1 (size1)

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1



Size 1

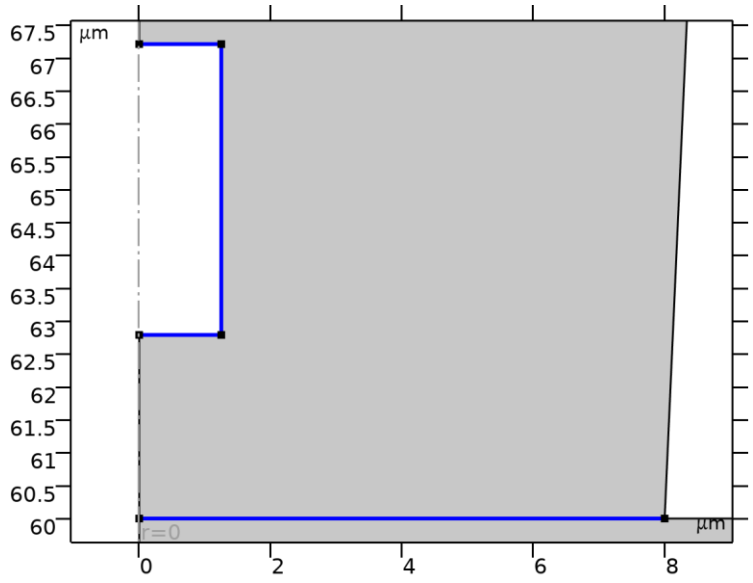
SETTINGS

Description	Value
Calibrate for	Fluid dynamics
Maximum element size	1.34
Minimum element size	0.004
Curvature factor	0.2
Maximum element growth rate	1.05
Predefined size	Extremely fine

Size 2 (size2)

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundaries 4–5, 7, 9



Size 2

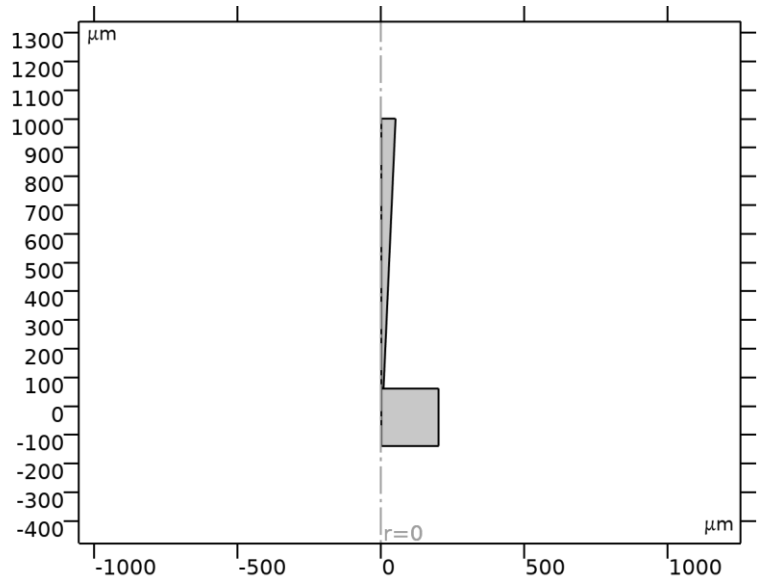
SETTINGS

Description	Value
Calibrate for	Semiconductor
Maximum element size	0.456
Minimum element size	0.0912
Curvature factor	0.2
Maximum element growth rate	1.05
Predefined size	Extremely fine

Corner Refinement 1 (cr1)

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1

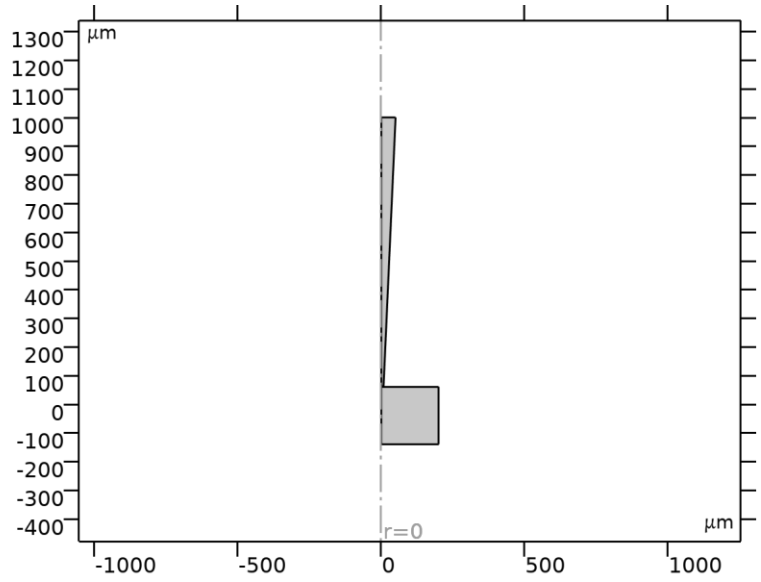


Corner Refinement 1

2.5.3 Boundary Layers 1 (bl1)

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1



Boundary Layers 1

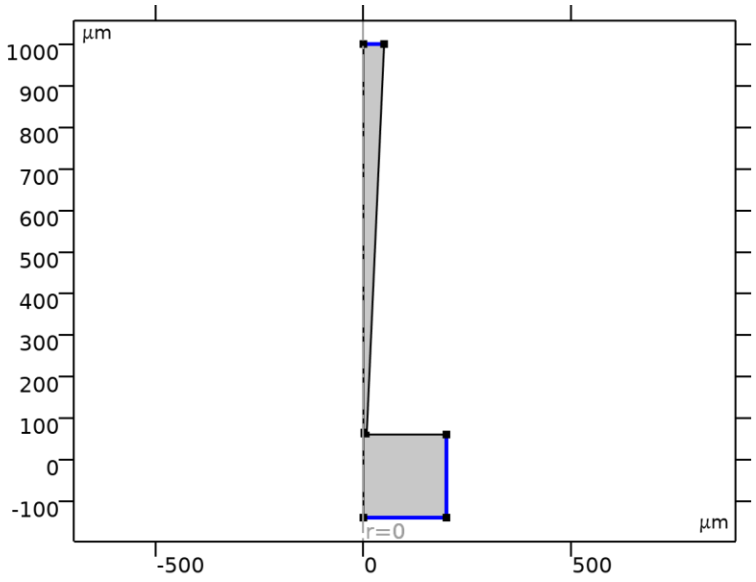
INFORMATION

Description	Value
Method	Legacy version 6.0
Last build time	Not built

Boundary Layer Properties (blp)

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundaries 2, 8, 12



Boundary Layer Properties

3 Study 1

COMPUTATION INFORMATION

Computation time	16 s

3.1 STATIONARY

STUDY SETTINGS

Description	Value
Include geometric nonlinearity	Off

PHYSICS AND VARIABLES SELECTION

Physics interface	Solve for	Equation form
Transport of Diluted Species (tds)	On	Automatic (Stationary)
Electrostatics (es)	On	Automatic (Stationary)

STORE IN OUTPUT

Interface	Output	Selection
Transport of Diluted Species (tds)	Physics controlled	
Electrostatics (es)	Physics controlled	

MESH SELECTION

Component	Mesh
Component 1	Mesh 1

3.2 SOLVER CONFIGURATIONS

3.2.1 Solution 1

Compile Equations: Stationary (st1)

STUDY AND STEP

Description	Value
Use study	Study 1
Use study step	Stationary

LOG

```
<---- Compile Equations: Stationary in Study 1/Solution 1 (sol1) ----------
Started at 12-Feb-2024 14:05:51.
Geometry shape order: Linear
Running on Intel64 Family 6 Model 183 Stepping 1, GenuineIntel.
Using 1 socket with 32 cores in total on DS034675.
Available memory: 65.22 GB.
Time: 1 s.
Physical memory: 1.56 GB
Virtual memory: 1.67 GB
Ended at 12-Feb-2024 14:05:52.
----- Compile Equations: Stationary in Study 1/Solution 1 (sol1) ---------->
```

Dependent Variables 1 (v1)

GENERAL

Description	Value
Defined by study step	Step 1: Stationary

Concentration (comp1.cCl) (comp1_cCl)

GENERAL

Description	Value
Field components	comp1.cCl
Internal variables	{comp1.uflux.cCl, comp1.dflux.cCl}

Concentration (comp1.cNa) (comp1_cNa)

GENERAL

Description	Value
Field components	comp1.cNa
Internal variables	{comp1.uflux.cNa, comp1.dflux.cNa}

Inverse time step measure (comp1.tds.dt2Inv_cCl) (comp1_tds_dt2Inv_cCl)

GENERAL

Description	Value
Field components	comp1.tds.dt2Inv_cCl
Solve for this field	Off

Inverse time step measure (comp1.tds.dt2Inv_cNa) (comp1_tds_dt2Inv_cNa)

GENERAL

Description	Value
Field components	comp1.tds.dt2Inv_cNa
Solve for this field	Off

Electric potential (comp1.V) (comp1_V)

GENERAL

Description	Value
Field components	comp1.V

Stationary Solver 1 (s1)

GENERAL

Description	Value
Defined by study step	Step 1: Stationary

LOG

```
<---- Stationary Solver 1 in Study 1/Solution 1 (sol1) ------------------------
Started at 12-Feb-2024 14:05:53.
Nonlinear solver
Number of degrees of freedom solved for: 322149 (plus 4590 internal DOFs).
Nonsymmetric matrix found.
Scales for dependent variables:
Concentration (comp1.cCl): 1.5e+002
Concentration (comp1.cNa): 1.9e+002
Electric potential (comp1.V): 99
Orthonormal null-space function used.
Iter      SolEst      ResEst     Damping    Stepsize #Res #Jac #Sol   LinErr   LinRe
s
   1        0.32    9.4e+006   0.0100000        0.33    2    1    2 9.2e-008 3.4e-
015
   2         0.3          64   0.1000000        0.34    3    2    4 2.2e-007 3.5e-
015
   3    8.2e-008    5.3e+003   1.0000000         0.3    4    3    6 6.7e-008 5.9e-
012
   4    5.2e-009          13   1.0000000    8.6e-008    5    4    8 1.3e-006 7.8e-
010
Solution time: 14 s.
Physical memory: 2.85 GB
Virtual memory: 3.57 GB
Ended at 12-Feb-2024 14:06:07.
----- Stationary Solver 1 in Study 1/Solution 1 (sol1) ----------------------->
```

Fully Coupled 1 (fc1)

GENERAL

Description	Value
Linear solver	Direct 1

METHOD AND TERMINATION

Description	Value
Initial damping factor	0.01
Minimum damping factor	1E-6
Maximum number of iterations	50

Direct 1 (d1)

GENERAL

Description	Value
Pivot threshold	0.1

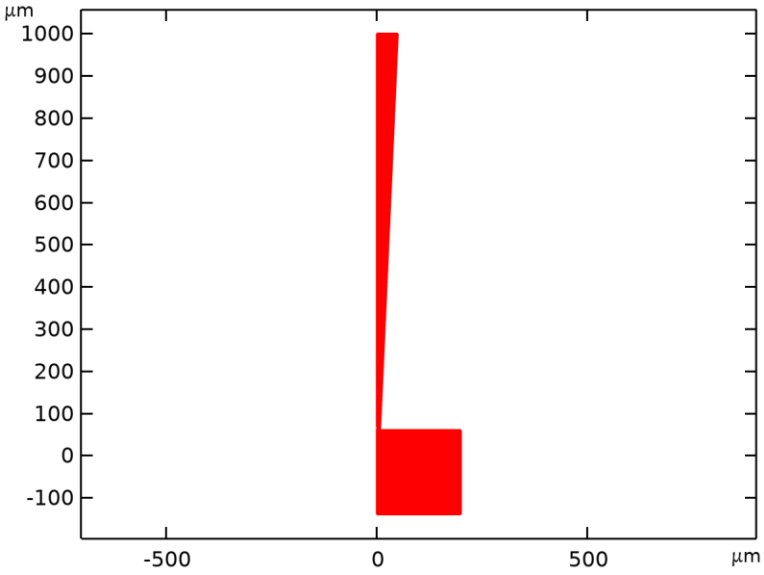
4 Results

4.1 DATA SETS

4.1.1 Study 1/Solution 1

SOLUTION

Description	Value
Solution	Solution 1 (sol1)
Component	Component 1 (comp1)



Dataset: Study 1/Solution 1

4.1.2 Revolution 2D

DATA

Description	Value
Dataset	Study 1/Solution 1 (sol1)

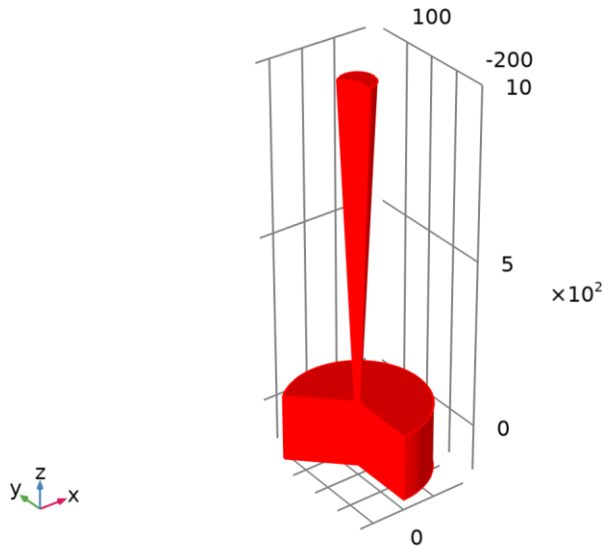
AXIS DATA

Description	Value
Axis entry method	Two points
Points	{{0, 0}, {0, 1}}

REVOLUTION LAYERS

Description	Value
Start angle	-90

Description	Value
Revolution angle	225



Dataset: Revolution 2D

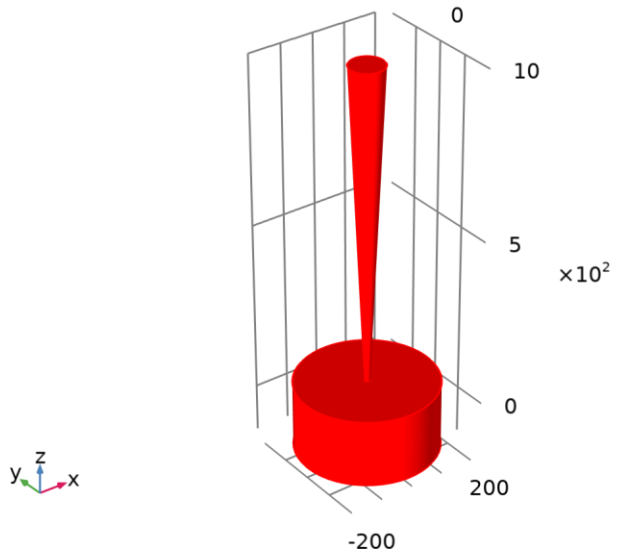
4.1.3 Revolution 2D 2

DATA

Description	Value
Dataset	Study 1/Solution 1 (sol1)

AXIS DATA

Description	Value
Axis entry method	Two points
Points	{{0, 0}, {0, 1}}



Dataset: Revolution 2D 2

4.2 DERIVED VALUES

4.2.1 Line Integration 1

OUTPUT

Evaluated in	Table 1

DATA

Description	Value
Dataset	Study 1/Solution 1 (sol1)

EXPRESSIONS

Expression	Unit	Description
(tds.bndFlux_cNa-tds.bndFlux_cCl)*96480[C/mol]	A	

INTEGRATION SETTINGS

Description	Value
Integration order	4
Compute surface integral	On

4.3 TABLES

4.3.1 Evaluation 2D

Interactive 2D values

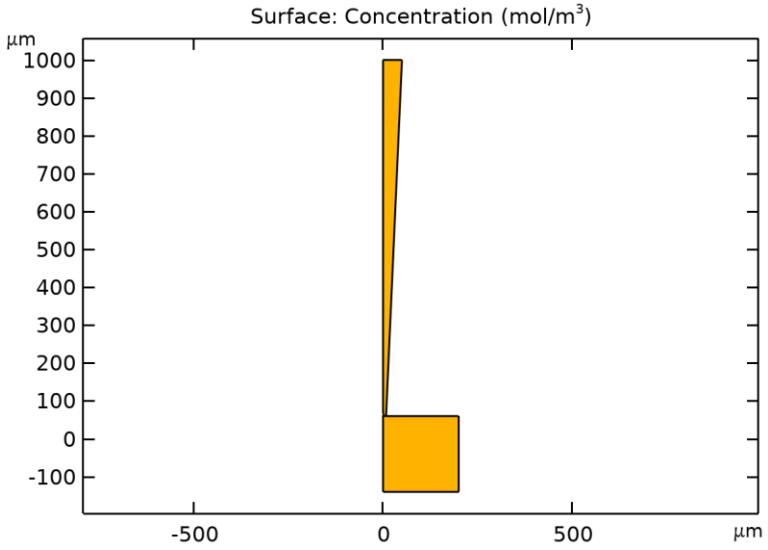
4.3.2 Table 1

Line Integration 1 ((tds.bndFlux_cNa-tds.bndFlux_cCl)*96480[C/mol])

(tds.bndFlux_cNa-tds.bndFlux_cCl)*96480[C/mol] (A)
2.4643E-7

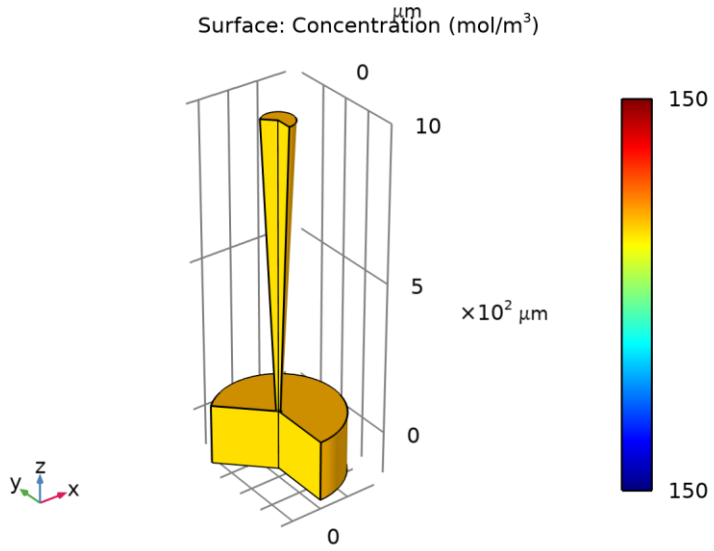
4.4 PLOT GROUPS

4.4.1 Concentration (tds)



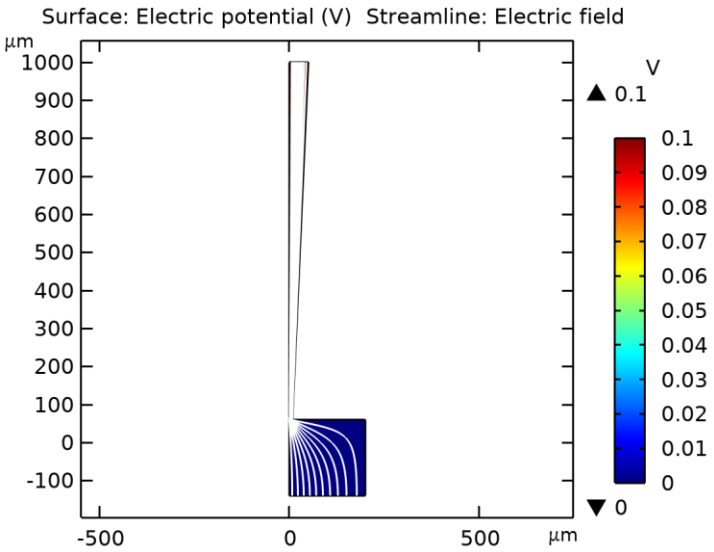
Surface: Concentration (mol/m³)

4.4.2 Concentration (tds) 1



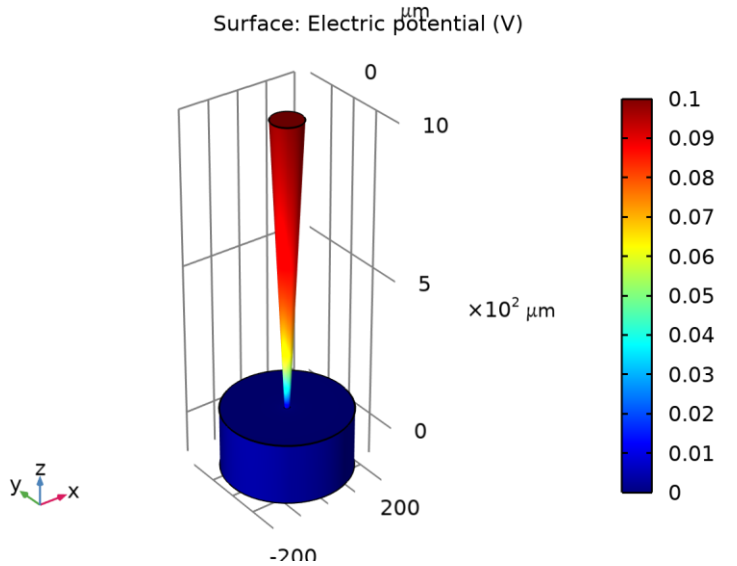
Surface: Concentration (mol/m³)

4.4.3 Electric Potential (es)



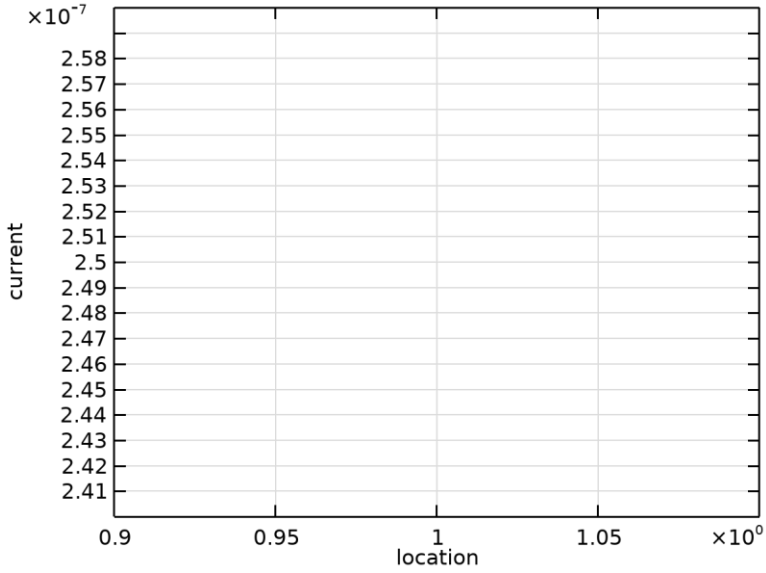
Surface: Electric potential (V) Streamline: Electric field

4.4.4 Electric Potential, Revolved Geometry (es)



Surface: Electric potential (V)

4.4.5 1D Plot Group 5



Point Graph: Concentration (mol/m^3)