

Supplementary Information for:  
Transferable Machine Learning Interatomic Potential for  
Carbon Hydrogen Systems

Somayeh Faraji<sup>1</sup> and Mingjie Liu<sup>1\*</sup>

<sup>1</sup>Department of Chemistry, University of Florida, Gainesville, FL 32611, United States.

\*mingjieliu@ufl.edu

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## 1 Impact of training data energy span on potential accuracy

Despite increasing the training dataset size, we observed minimal improvement in the accuracy of the trained potential during the last training cycles. We hypothesized that this could be attributed to the complexity and significant diversity in energy among the data points. Our dataset displayed a wide range of energy differences between structures, ranging from lower to higher energy values, with a magnitude of 4.52 eV/atom. This broad spectrum of energy, coupled with variations in the ratios of C and H atoms within the structures, poses a formidable and complex challenge for the training process. Regarding this complexity, we conducted multiple training sessions at different energy range values ( $\Delta E$ ), including 1.0, 1.5, 2.0, 2.5, and 3.0 eV/atom above the minimum energy in the final dataset. Figure S1 displays the configuration analysis of data across each of these energy ranges. For each  $\Delta E$ , we evaluated the RMSE of energy and forces in the training and validation sets for the best-selected potentials. As depicted in Figures S2 and S3, the potentials constructed with  $\Delta E$  set to 1.0, 1.5, and 2.5 eV/atom exhibited overfitting, evidenced by larger errors in the validation than in the training sets. However, for  $\Delta E = 2.0$  eV/atom, the RMSE values were lower compared to the previous ranges, with instances of overfitting but also cases without it. For  $\Delta E = 2.5$  eV/atom, overfitting was again observed. On the contrary,  $\Delta E = 3.0$  eV/atom showed no overfitting (i.e. the error of valid is smaller than that of training), but yielded higher RMSE values. Therefore, we found that the training dataset within the energy range of 2.0 eV/atom produced more accurate potentials, thereby becoming our focus.

Focusing on this energy range revealed outlier structures both in training and validation datasets, resulting large errors in the produced potentials. we visualized the outliers and noticed that they were structures with atoms that had no bonds with each other. By excluding these outliers from

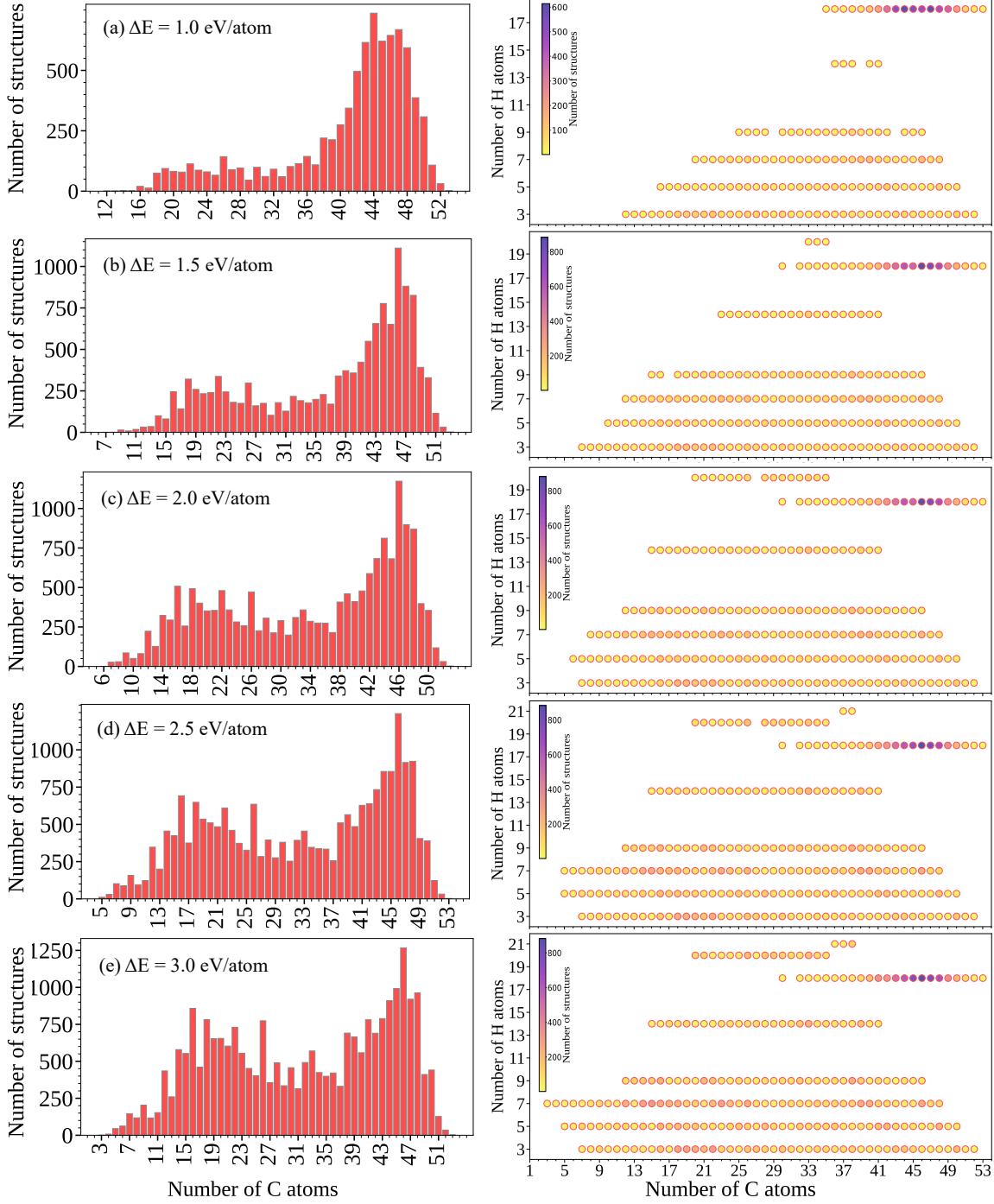


Figure S1: The configuration analysis of data at different energy ranges tested for selecting the energy changes for train. The first column illustrates the distribution of structures based on the count of carbon (C) atoms at different energy ranges ( $\Delta E$ ), ranging from 1.0 to 3.0 eV/atom. The second column displays the scatter plots showcase the frequency of structures for distinct C-H atom ratios, depicted by a colormap (color intensity) representing the count of structures within each ratio. The color bar provides a quantitative representation of the structure count for the observed C-H compositions.

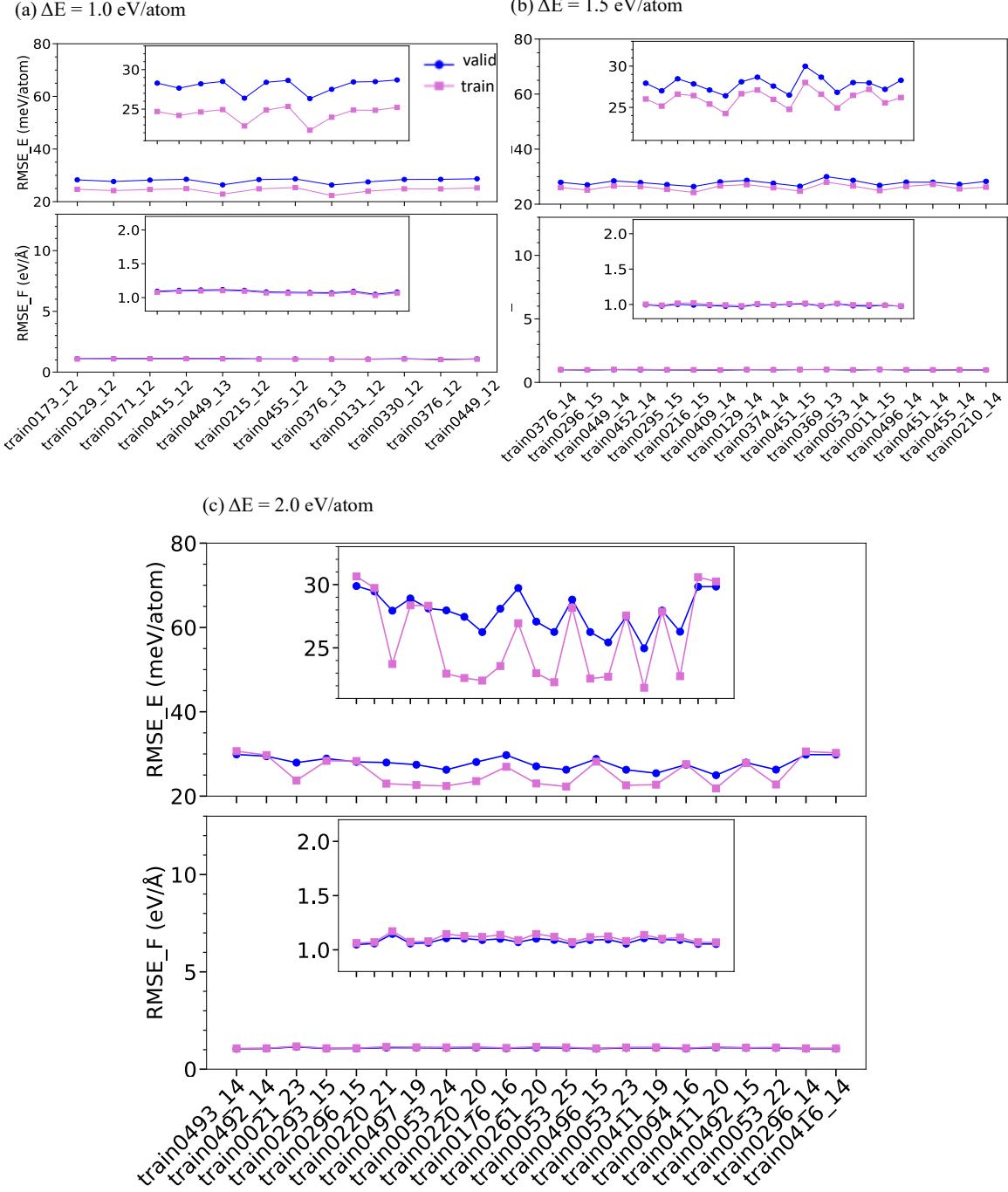


Figure S2: The RMSE of energy and forces of different trial potentials at different energy ranges (part I).

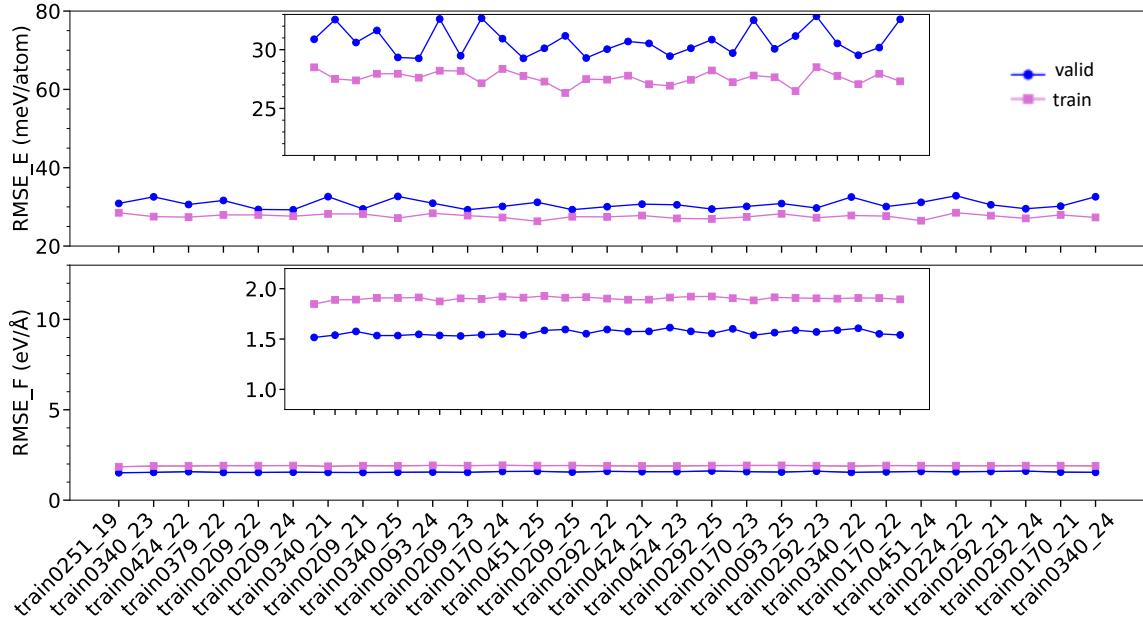
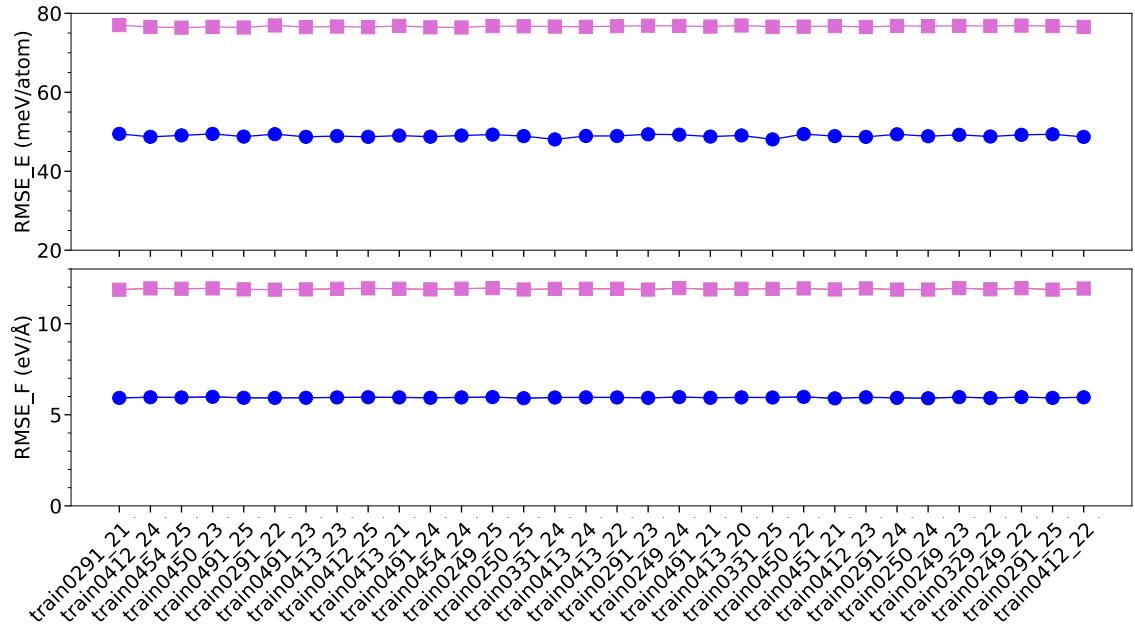
(d)  $\Delta E = 2.5$  eV/atom(e)  $\Delta E = 3.0$  eV/atom

Figure S3: The RMSE of energy and forces of different trial potentials at different energy ranges (part2).

both the training and validation datasets, we achieved a reduction in large errors in the produced potentials. Figures S2-c and S4 illustrate the reduction in RMSEs before and after excluding these outliers from both training and validation.

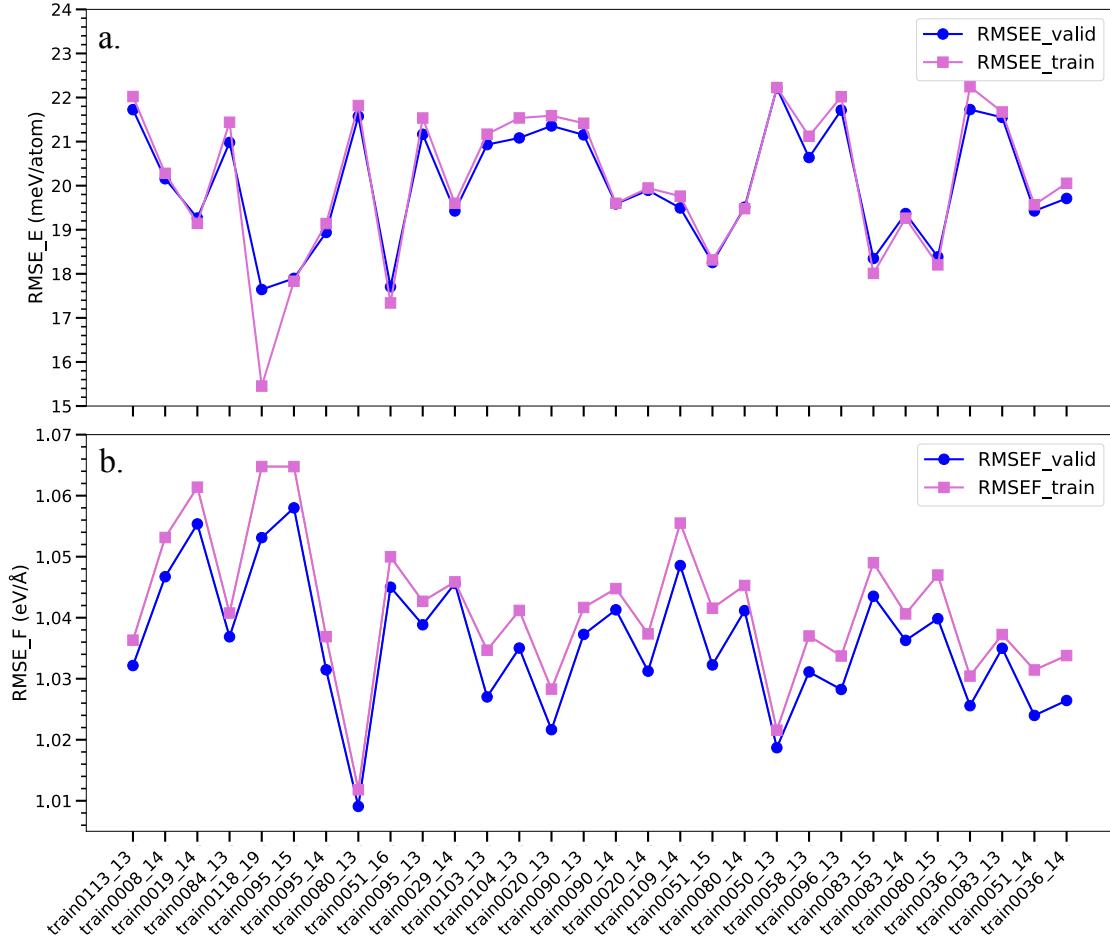


Figure S4: The RMSE of energy and forces of the selected potentials at energy range of 2.0 eV/atom.

Figure S5 illustrates the distribution of energy contributions across the dataset within the specified energy window of 2.0 eV/atom. This visual representation provides insights into how the energy values are spread across the range.

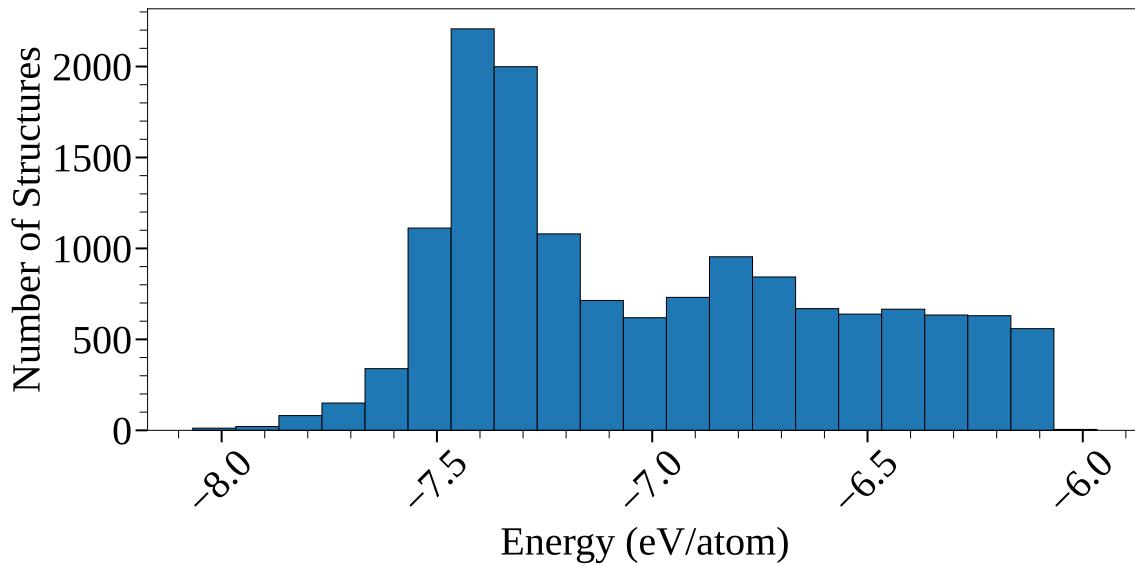


Figure S5: The energy contribution of dataset at energy windows of 2.0 eV/atom.

## 2 Impact of spin considerations on open-shell systems

As mentioned in the method section, the initial dataset was obtained by creating 1 – 24 C vacancies in  $C_{52}H_{18}$  zigzag flake. Therefore, all data initially had an even number of H atoms. To make the dataset more diverse, we added datapoints with an odd number of H atoms (i.e.  $H_x$ ,  $x = 3, 5, 7, 9$ ) and considered different numbers of C atoms for each odd number of H atoms during the data augmentation process. This resulted in having clusters with an odd number of H atoms, which are open-shell systems requiring spin-polarized calculations.

To assess whether the non-spin-polarized calculations can correctly compute the energies for these open-shell systems, we selected sample systems with 3, 5, 7 and 9 H atoms, along with varying odd and even numbers of C atoms and performed single-point calculations to compare the results of energies and forces. The results of spin- and non-spin-polarized DFT calculations with the PBE functional on these systems are summarized in Table S1. The results show that the total energies obtained from spin-polarized calculations are lower than those from non-spin-polarized calculations, with an MAE of total energy of 0.42 eV. However, it results in an MAE of 0.0188 eV/atom in  $E_f$ , which is smaller than the error in the ANN potential (0.02 eV/atom). Additionally, this test shows that the maximum RMSE of force components is 0.53 eV/Å, which is also a small value compared to the errors in our training and validation. Therefore, non-spin-polarized calculations of the open-shell systems do not introduce considerable error in the reference energies and forces of the dataset.

Table S1:  $\Delta E_{tot}$  is the  $E_{tot,spin} - E_{tot,nonspin}$ ,  $MAE_F$  and  $mAE_F$  are the maximum and minimum absolute error in forces from spin and nonspin polarized single-point calculations, respectively,  $RMSE_F$  is the root mean absolute error of forces,  $E_f$  is the formation energy,  $\Delta E_f$  is the energy difference in formation energy obtained from spin and nonspin polarized calculations.

|        | $\Delta E_{tot}$<br>(eV) | $MAE_F$<br>(eV/Å) | $mAE_F$<br>(eV/Å) | $RMSE_F$<br>(eV/Å) | $E_{f,spin}$<br>(eV/atom) | $E_{f,nonspin}$<br>(eV/atom) | $\Delta E_f$<br>(eV/atom) |
|--------|--------------------------|-------------------|-------------------|--------------------|---------------------------|------------------------------|---------------------------|
| H07C18 | -0.22551                 | 0.371842          | 0.004620          | 0.140027           | -11.72947                 | -11.72045                    | -0.00902                  |
| H07C19 | -0.16484                 | 0.731991          | 0.012878          | 0.249873           | -11.96006                 | -11.95372                    | -0.00634                  |
| H07C42 | -0.03888                 | 1.250212          | 0.034761          | 0.503975           | -13.80533                 | -13.80453                    | -0.00080                  |
| H07C43 | -0.22167                 | 1.533610          | 0.013094          | 0.398164           | -14.27994                 | -14.27551                    | -0.00443                  |
| H03C09 | -0.22580                 | 0.327090          | 0.003396          | 0.172941           | -12.46157                 | -12.44275                    | -0.01882                  |
| H03C18 | -0.12670                 | 0.560343          | 0.010566          | 0.186549           | -14.18570                 | -14.17966                    | -0.00604                  |
| H03C27 | -0.14312                 | 0.303562          | 0.025608          | 0.137181           | -14.00647                 | -14.00170                    | -0.00477                  |
| H03C42 | -0.11255                 | 0.275119          | 0.004444          | 0.086380           | -15.42721                 | -15.42471                    | -0.00250                  |
| H05C12 | -0.24764                 | 0.417436          | 0.006959          | 0.188259           | -11.72844                 | -11.71387                    | -0.01457                  |
| H05C29 | -0.10720                 | 0.145316          | 0.007962          | 0.073648           | -14.21146                 | -14.20831                    | -0.00315                  |
| H05C41 | -0.22839                 | 1.191151          | 0.011326          | 0.437105           | -14.77498                 | -14.77002                    | -0.00496                  |
| H09C22 | -0.18302                 | 1.375544          | 0.028908          | 0.496279           | -12.07224                 | -12.06634                    | -0.00590                  |
| H09C23 | -0.31419                 | 0.429440          | 0.002054          | 0.141359           | -12.41900                 | -12.40918                    | -0.00982                  |
| H09C38 | -0.42436                 | 0.822324          | 0.003721          | 0.232690           | -13.61539                 | -13.60636                    | -0.00903                  |
| H09C46 | -0.30921                 | 1.583654          | 0.016955          | 0.396462           | -14.12526                 | -14.11964                    | -0.00562                  |

To investigate the smoothness of PES in the transition from closed- to open-shell systems, we conducted two tests. Firstly, we examined the C-C bond dissociation in  $C_2H_2$ . As shown in Figure S6-(a), using spin polarized wavefunctions in the calculations did not change the equilibrium C-C bond length and energy. The energy difference begins to diverge from the non-spin calculations at distances greater than 2 Å. At larger distances,  $\Delta E_{tot}$  reaches up to 1.19 eV, with the spin-polarized energy being more favorable. However, there is no abrupt change or discontinuous in the transition from close to open shell systems. Secondly, we focused on the geometry of  $C_7H_5$  and performed geometry optimization using PBE and PBE0, each with and without including spin-polarized calculations. Then, one H atom was removed from the optimized geometry and the remaining geometry was optimized like the previous step. This step was repeated until all the H atoms were removed. Figure S6-(b) shows the total energy versus the number of removed H atoms. As depicted in the figure, there is a smooth transition between closed- and open-shell systems. The absence of sharp changes or discontinuities indicates that the PBE can handle both states effectively. Additionally, the geometries obtained from PBE without spin polarization (blue cross) and those from PBE0 with spin polarization (green circle) shown in the figure are similar. Comparing the absolute values of energies reveals that both PBE and PBE0 exhibit a similar slope, with a noticeable shift in the absolute values, indicating that the energies obtained from PBE are reliable.

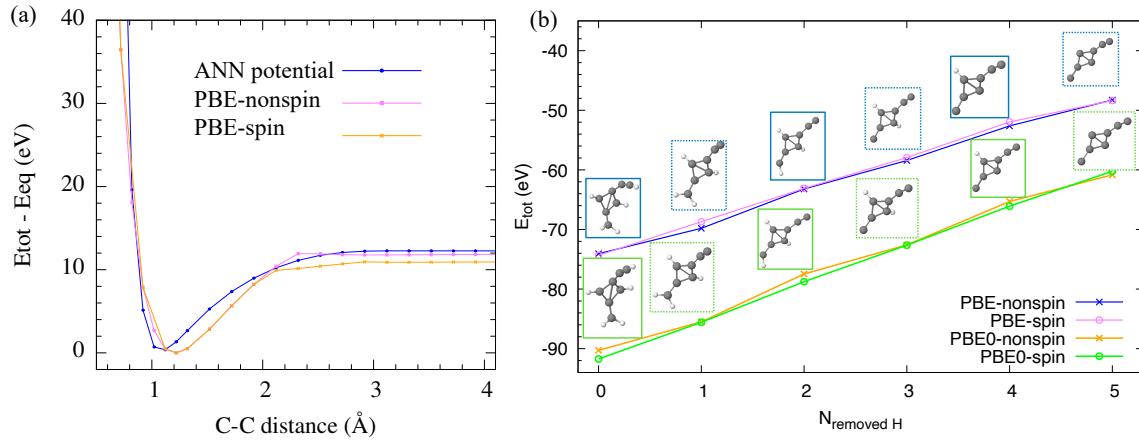


Figure S6: (a) Potential energy curves for  $\text{C}_2\text{H}_2$  calculated by the ANN potential, PBE functional without and with including spin. (b) Total energies obtained by PBE and PBE0 functionals without and with including spin versus the number of removed H atoms from  $\text{C}_7\text{H}_5$ . The optimized geometries in the blue and green boxes show the results from PBE without spin and PBE0 with spin, respectively.

### 3 0D test cases

Table S2: Chemical name, number of H ( $n_H$ ) and C ( $n_C$ ) atoms in the geometry, total energies ( $E_{tot}$ ), formation energies ( $E_f$ ), and formation energy differences of DFT and the ANN potential ( $E_{f,diff}$ ) of the examined 0D test cases. All the energies are in eV/atom.

| ID | system                 | $n_H$ | $n_C$ | $E_{tot,DFT}$ | $E_{tot,ANN}$ | $E_{f,DFT}$ | $E_{f,ANN}$ | $E_{f,diff}$ |
|----|------------------------|-------|-------|---------------|---------------|-------------|-------------|--------------|
| 01 | methane                | 4     | 1     | -4.80779      | -4.83077      | -0.280812   | -0.325548   | 0.022971     |
| 02 | ethane                 | 6     | 2     | -5.06129      | -5.05908      | -0.24895    | -0.24675    | -0.00221     |
| 03 | cyclopropane           | 6     | 3     | -5.38114      | -5.44119      | -0.09321    | -0.15326    | 0.06005      |
| 04 | propane                | 8     | 3     | -5.18301      | -5.17456      | -0.24097    | -0.23251    | -0.00845     |
| 05 | cyclobutane            | 8     | 4     | -5.41396      | -5.46376      | -0.12603    | -0.17583    | 0.04980      |
| 06 | butane                 | 10    | 4     | -5.25342      | -5.24421      | -0.23726    | -0.22805    | -0.00921     |
| 07 | isobutane              | 10    | 4     | -5.25624      | -5.23825      | -0.24008    | -0.22209    | -0.01799     |
| 08 | propellane             | 6     | 5     | -5.79979      | -5.66478      | 0.17992     | 0.31492     | -0.13501     |
| 09 | cyclopentane           | 10    | 5     | -5.48841      | -5.50076      | -0.20048    | -0.21283    | 0.01235      |
| 10 | dimethylcyclopropane-c | 10    | 5     | -5.43828      | -5.45257      | -0.15035    | -0.16464    | 0.01429      |
| 11 | dimethylcyclopropane-t | 10    | 5     | -5.44229      | -5.44797      | -0.15436    | -0.16004    | 0.00569      |
| 12 | methylbutane           | 12    | 5     | -5.29781      | -5.28984      | -0.23369    | -0.22572    | -0.00797     |
| 13 | pentane                | 12    | 5     | -5.29449      | -5.28881      | -0.23037    | -0.22468    | -0.00568     |
| 14 | cyclohexane-boat       | 12    | 6     | -5.48763      | -5.49273      | -0.19970    | -0.20480    | 0.00510      |
| 15 | cyclohexane-chair      | 12    | 6     | -5.50365      | -5.49537      | -0.21572    | -0.20744    | -0.00828     |
| 16 | hexane                 | 14    | 6     | -5.32871      | -5.32045      | -0.23101    | -0.22275    | -0.00826     |
| 17 | norbornane             | 12    | 7     | -5.66376      | -5.67171      | -0.17558    | -0.18353    | 0.00795      |

| ID | system               | $n_H$ | $n_C$ | $E_{tot,DFT}$ | $E_{tot,ANN}$ | $E_{f,DFT}$ | $E_{f,ANN}$ | $E_{f,diff}$ |
|----|----------------------|-------|-------|---------------|---------------|-------------|-------------|--------------|
| 18 | cycloheptane         | 14    | 7     | -5.49117      | -5.47987      | -0.20324    | -0.19194    | -0.01130     |
| 19 | heptane              | 16    | 7     | -5.35278      | -5.34379      | -0.23027    | -0.22128    | -0.00899     |
| 20 | cyclooctane          | 16    | 8     | -5.48280      | -5.46976      | -0.19487    | -0.18183    | -0.01304     |
| 21 | isooctane            | 18    | 8     | -5.36537      | -5.36285      | -0.22377    | -0.22126    | -0.00251     |
| 22 | octane               | 18    | 8     | -5.36901      | -5.36175      | -0.22742    | -0.22016    | -0.00726     |
| 23 | spirononane          | 16    | 9     | -5.62866      | -5.64692      | -0.18854    | -0.20680    | 0.01826      |
| 24 | cyclononane          | 18    | 9     | -5.48065      | -5.46853      | -0.19272    | -0.18060    | -0.01213     |
| 25 | nonane               | 20    | 9     | -5.38289      | -5.37600      | -0.22616    | -0.21927    | -0.00689     |
| 26 | adamantane           | 16    | 10    | -5.77559      | -5.77587      | -0.19499    | -0.19527    | 0.00028      |
| 27 | decalin-cis          | 18    | 10    | -5.62940      | -5.61818      | -0.20559    | -0.19437    | -0.01122     |
| 28 | decalin-trans        | 18    | 10    | -5.63438      | -5.63268      | -0.21057    | -0.20886    | -0.00171     |
| 29 | cyclodecane          | 20    | 10    | -5.48136      | -5.47265      | -0.19343    | -0.18472    | -0.00871     |
| 30 | decane               | 22    | 10    | -5.39417      | -5.38757      | -0.22514    | -0.21854    | -0.00660     |
| 31 | cetane               | 34    | 16    | -5.43340      | -5.42785      | -0.22157    | -0.21602    | -0.00555     |
| 32 | dodecahedrane        | 20    | 20    | -6.36100      | -6.39322      | -0.12189    | -0.15411    | 0.03222      |
| 33 | anthracene           | 10    | 14    | -6.79043      | -6.83212      | -0.07573    | -0.11742    | 0.04169      |
| 34 | benzene              | 6     | 6     | -6.33352      | -6.38190      | -0.09440    | -0.14279    | 0.04839      |
| 35 | benzopyrene          | 12    | 20    | -7.03310      | -7.07485      | -0.08060    | -0.12234    | 0.04175      |
| 36 | buckminsterfullerene | 0     | 60    | -8.85075      | -8.83164      | 0.24192     | 0.26103     | -0.01911     |
| 37 | butadiene-13         | 6     | 4     | -5.68233      | -5.69387      | -0.01392    | -0.02547    | 0.01154      |
| 38 | butene-1             | 8     | 4     | -5.42971      | -5.42078      | -0.14178    | -0.13285    | -0.00893     |
| 39 | butene-2c            | 8     | 4     | -5.43878      | -5.46266      | -0.15085    | -0.17473    | 0.02388      |
| 40 | butene-2t            | 8     | 4     | -5.44292      | -5.44838      | -0.15499    | -0.16046    | 0.00547      |
| 41 | butyne-1             | 6     | 4     | -5.63181      | -5.59669      | 0.03660     | 0.07172     | -0.03512     |
| 42 | butyne-2             | 6     | 4     | -5.66271      | -5.62300      | 0.00570     | 0.04541     | -0.03971     |
| 43 | $C_{70}$             | 0     | 70    | -8.88897      | -8.88035      | 0.20370     | 0.21232     | -0.00862     |
| 44 | camphene             | 16    | 10    | -5.72713      | -5.74576      | -0.14653    | -0.16516    | 0.01862      |
| 45 | caryophyllene        | 24    | 15    | -5.70729      | -5.73241      | -0.12669    | -0.15181    | 0.02512      |
| 46 | cyclobutadiene       | 4     | 4     | -5.82136      | -6.09382      | 0.41775     | 0.14530     | 0.27245      |
| 47 | cyclobutene          | 6     | 4     | -5.65483      | -5.72055      | 0.01357     | -0.05214    | 0.06571      |
| 48 | cyclohexene          | 10    | 6     | -5.65311      | -5.69548      | -0.12738    | -0.16975    | 0.04237      |
| 49 | cyclooctatetraene    | 8     | 8     | -6.14917      | -6.29209      | 0.08994     | -0.05297    | 0.14291      |
| 50 | cyclooctene-cis      | 14    | 8     | -5.60596      | -5.61822      | -0.14509    | -0.15734    | 0.01225      |
| 51 | cyclooctene-trans    | 14    | 8     | -5.58386      | -5.60775      | -0.12299    | -0.14687    | 0.02388      |
| 52 | cyclopentene         | 8     | 5     | -5.70147      | -5.74831      | -0.12087    | -0.16771    | 0.04684      |
| 53 | cyclopropene         | 4     | 3     | -5.56636      | -5.75567      | 0.26511     | 0.07580     | 0.18931      |
| 54 | dibenzanthracene     | 14    | 22    | -6.95236      | -6.99128      | -0.07912    | -0.11804    | 0.03892      |
| 55 | ethene               | 4     | 2     | -5.32868      | -5.30135      | -0.04075    | -0.01342    | -0.02733     |
| 56 | ethyne               | 2     | 2     | -5.73412      | -5.97970      | 0.50500     | 0.25942     | 0.24558      |
| 57 | farnesene            | 24    | 15    | -5.70273      | -5.72768      | -0.12213    | -0.14708    | 0.02495      |

| ID | system                    | $n_H$ | $n_C$ | $E_{tot,DFT}$ | $E_{tot,ANN}$ | $E_{f,DFT}$ | $E_{f,ANN}$ | $E_{f,diff}$ |
|----|---------------------------|-------|-------|---------------|---------------|-------------|-------------|--------------|
| 58 | isoprene                  | 8     | 5     | -5.66527      | -5.66707      | -0.08467    | -0.08647    | 0.00180      |
| 59 | limonene                  | 16    | 10    | -5.72337      | -5.74715      | -0.14277    | -0.16654    | 0.02378      |
| 60 | lycopen                   | 56    | 40    | -5.87820      | -5.90102      | -0.11468    | -0.13749    | 0.02281      |
| 61 | muscalure                 | 46    | 23    | -5.49219      | -5.48873      | -0.20426    | -0.20080    | -0.00346     |
| 62 | myrcene                   | 16    | 10    | -5.68687      | -5.70678      | -0.10627    | -0.12618    | 0.01992      |
| 63 | naphthalene               | 8     | 10    | -6.64218      | -6.68297      | -0.08600    | -0.12679    | 0.04079      |
| 64 | phellandrene-a            | 16    | 10    | -5.72407      | -5.73884      | -0.14347    | -0.15824    | 0.01477      |
| 65 | phellandrene-b            | 16    | 10    | -5.72288      | -5.76458      | -0.14227    | -0.18398    | 0.04171      |
| 66 | phenanthrene              | 10    | 14    | -6.79829      | -6.83943      | -0.08358    | -0.12473    | 0.04114      |
| 67 | pinene-a                  | 16    | 10    | -5.70997      | -5.76748      | -0.12937    | -0.18688    | 0.05751      |
| 68 | pinene-b                  | 16    | 10    | -5.70421      | -5.74235      | -0.12361    | -0.16174    | 0.03814      |
| 69 | propadiene-12             | 4     | 3     | -5.70084      | -5.67288      | 0.13062     | 0.15858     | -0.02796     |
| 70 | propene                   | 6     | 3     | -5.40625      | -5.40136      | -0.11832    | -0.11343    | -0.00489     |
| 71 | pyrene                    | 10    | 16    | -6.98371      | -7.02201      | -0.08608    | -0.12438    | 0.03830      |
| 72 | selinene                  | 24    | 15    | -5.73372      | -5.75062      | -0.15312    | -0.17002    | 0.01690      |
| 73 | styrene                   | 8     | 8     | -6.30484      | -6.33580      | -0.06573    | -0.09669    | 0.03096      |
| 74 | thujene                   | 16    | 10    | -5.70572      | -5.73221      | -0.12511    | -0.15160    | 0.02649      |
| 75 | toluene                   | 8     | 7     | -6.17476      | -6.21434      | -0.12588    | -0.16546    | 0.03958      |
| 76 | xylene-m                  | 10    | 8     | -6.06909      | -6.10267      | -0.14704    | -0.18061    | 0.03357      |
| 77 | xylene-o                  | 10    | 8     | -6.06902      | -6.10645      | -0.14697    | -0.18439    | 0.03742      |
| 78 | xylene-p                  | 10    | 8     | -6.06856      | -6.10224      | -0.14651    | -0.18019    | 0.03368      |
| 79 | C <sub>11</sub> -008      | 0     | 11    | -7.80387      | -7.88095      | 1.28880     | 1.21172     | 0.07708      |
| 80 | C <sub>32</sub> -255      | 0     | 32    | -8.49852      | -8.41582      | 0.59415     | 0.67685     | -0.08270     |
| 81 | C <sub>46</sub> -377      | 0     | 46    | -7.98483      | -8.06876      | 1.10784     | 1.02391     | 0.08393      |
| 82 | C <sub>48</sub> -412      | 0     | 48    | -8.47599      | -8.45829      | 0.61668     | 0.63438     | -0.01770     |
| 83 | C <sub>60</sub> -isomer01 | 0     | 60    | -8.85062      | -8.83164      | 0.24205     | 0.26103     | -0.01897     |
| 84 | C <sub>60</sub> -isomer02 | 0     | 60    | -8.82463      | -8.81694      | 0.26804     | 0.27573     | -0.00769     |
| 85 | C <sub>60</sub> -isomer03 | 0     | 60    | -8.81194      | -8.81028      | 0.28072     | 0.28239     | -0.00166     |
| 86 | C <sub>60</sub> -isomer04 | 0     | 60    | -8.81220      | -8.80985      | 0.28047     | 0.28282     | -0.00235     |
| 87 | C <sub>60</sub> -isomer05 | 0     | 60    | -8.80369      | -8.81048      | 0.28898     | 0.28219     | 0.00679      |

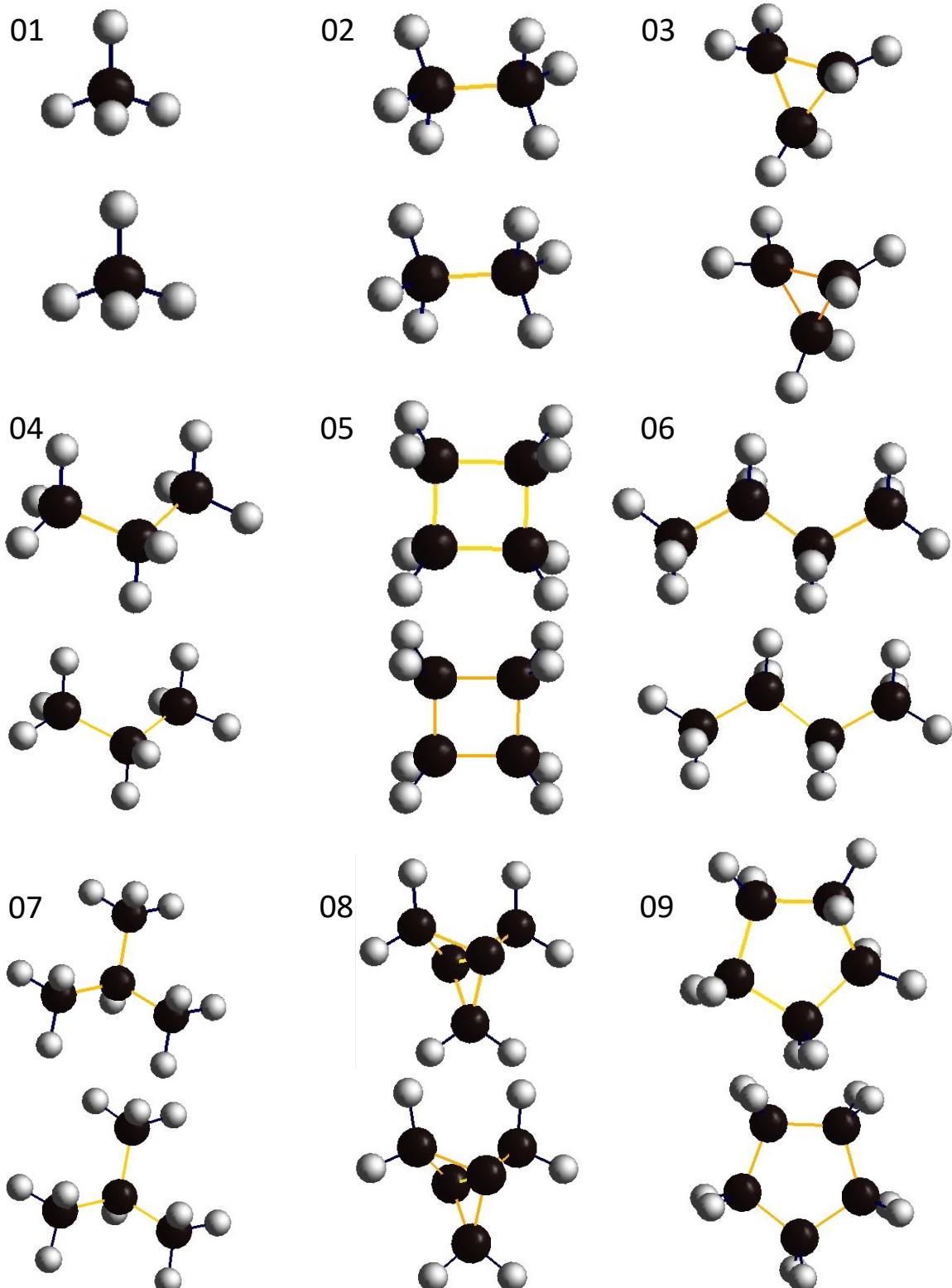


Figure S7: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure. (part 1)

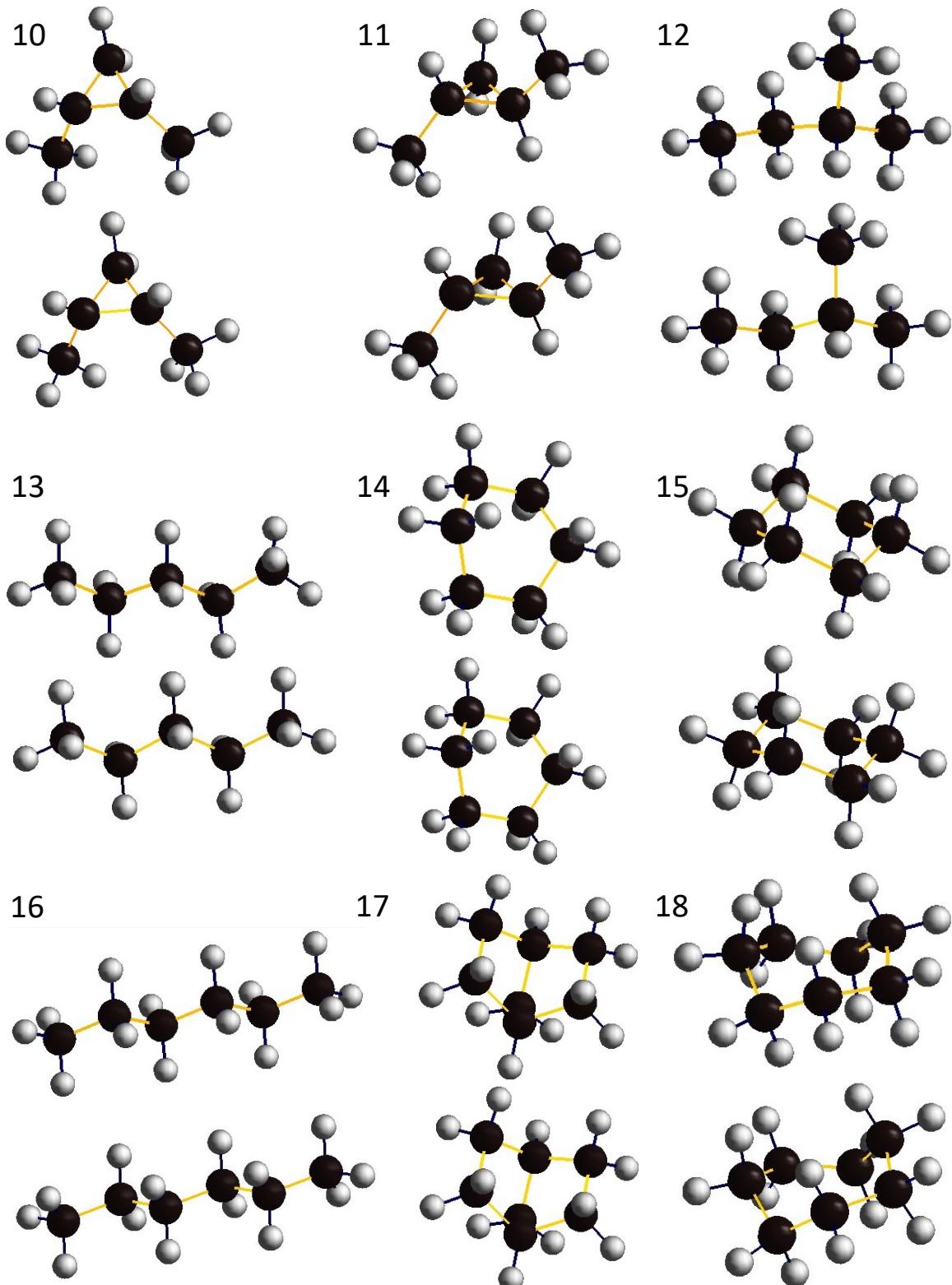


Figure S8: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 2).

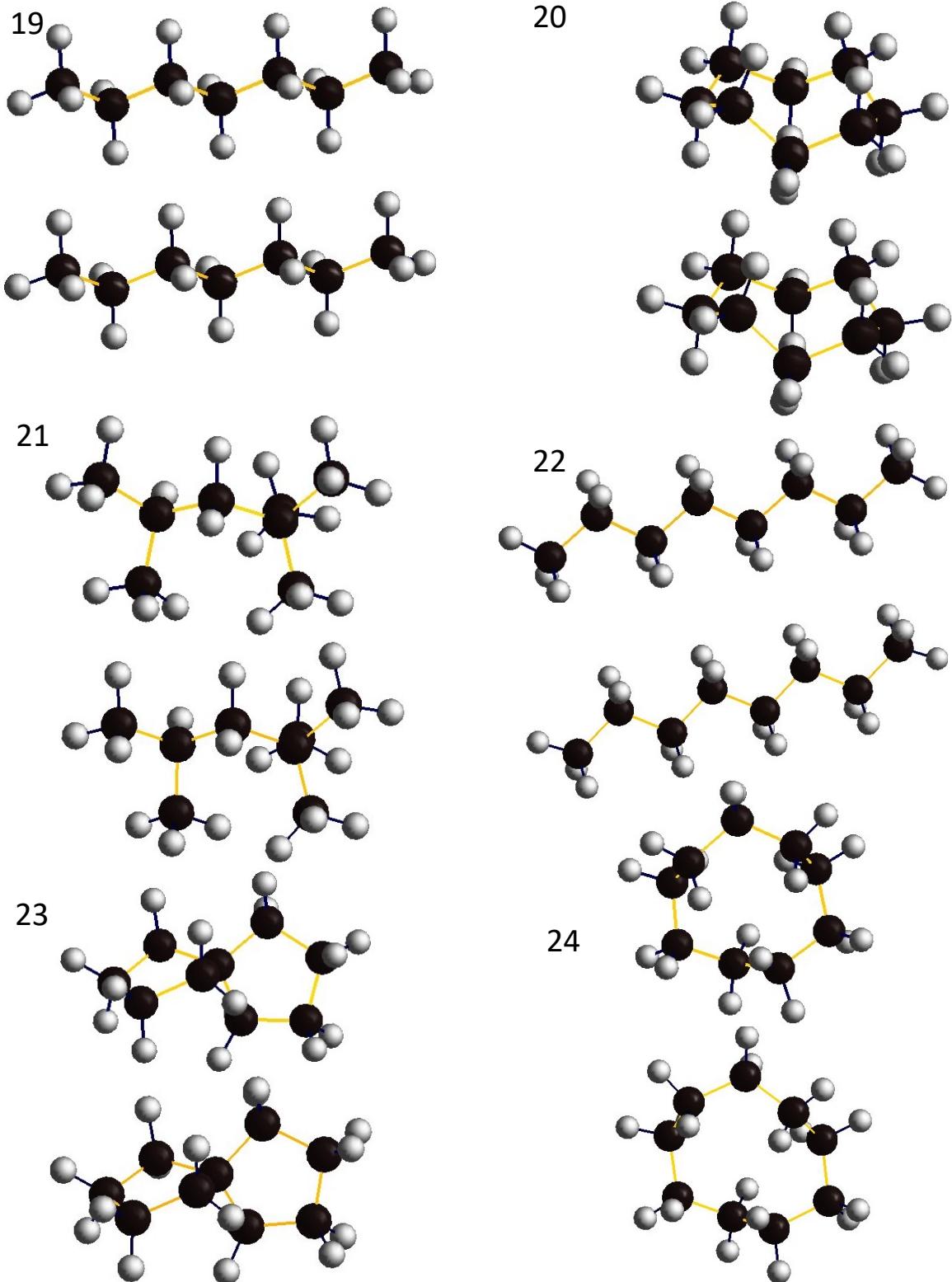


Figure S9: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 3).

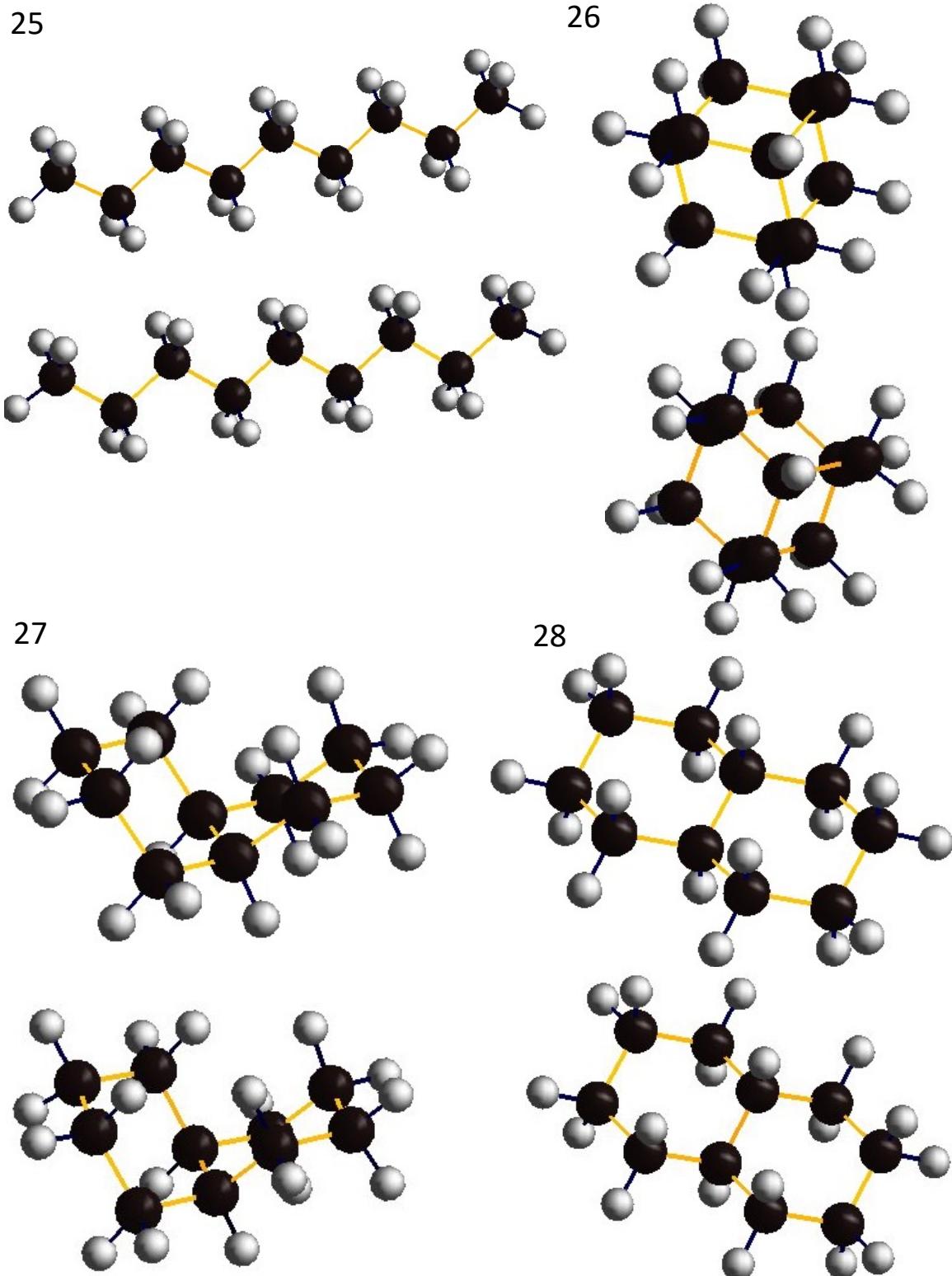


Figure S10: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 4).

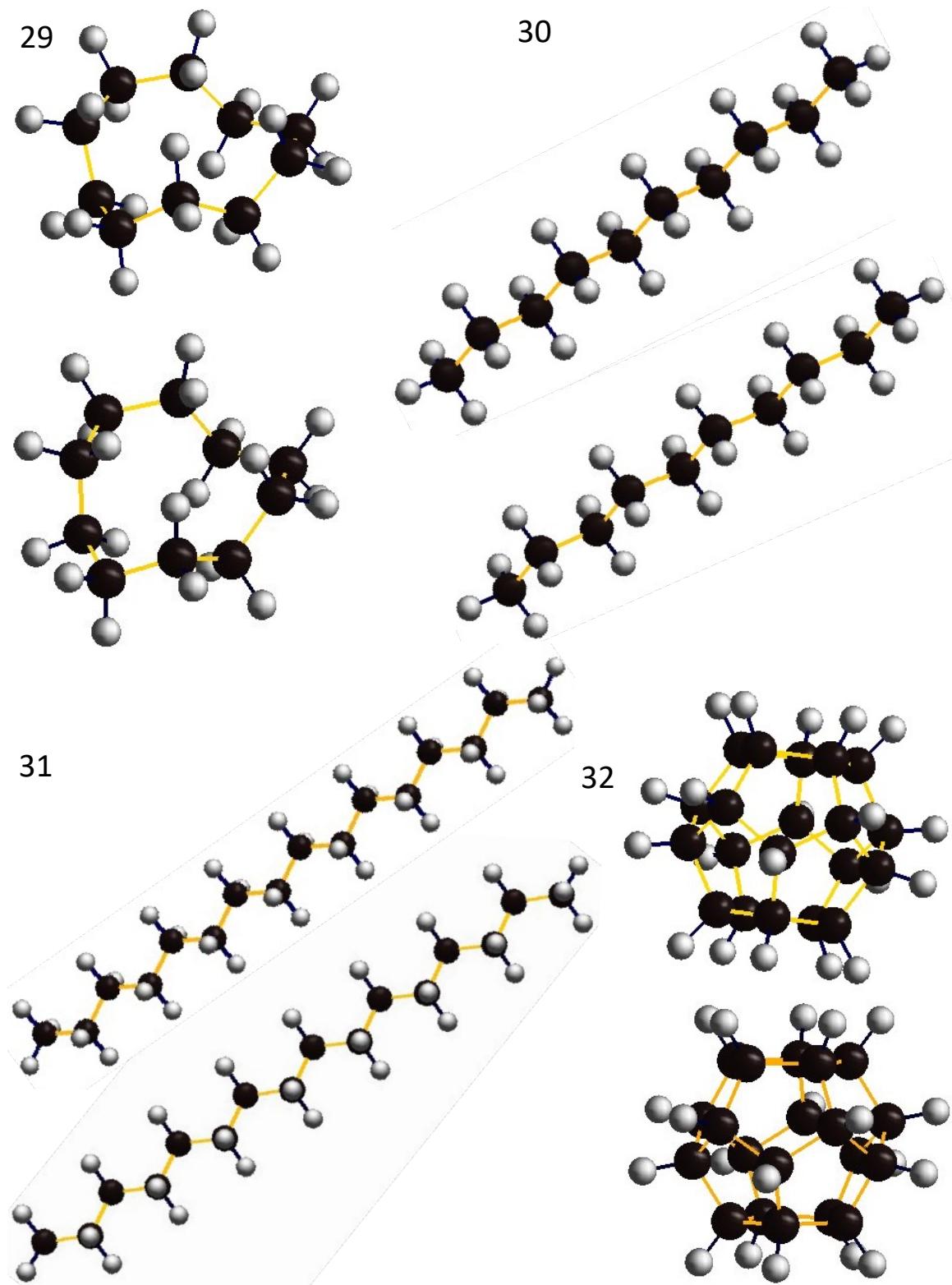


Figure S11: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 5).

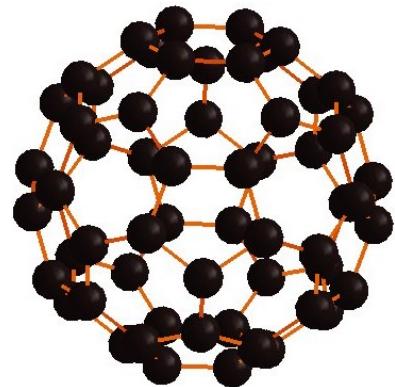
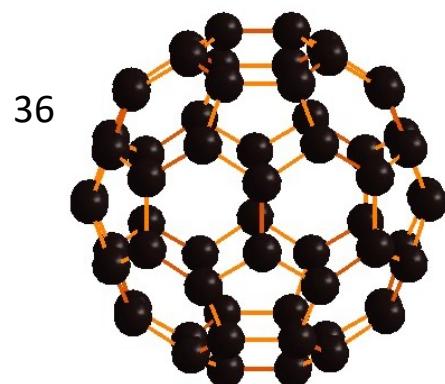
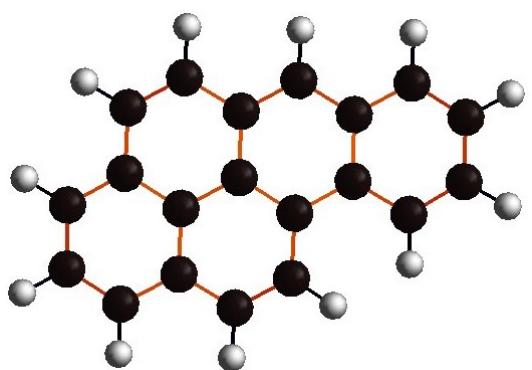
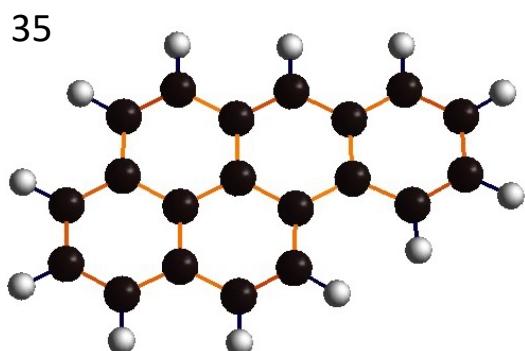
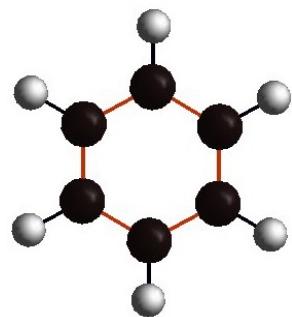
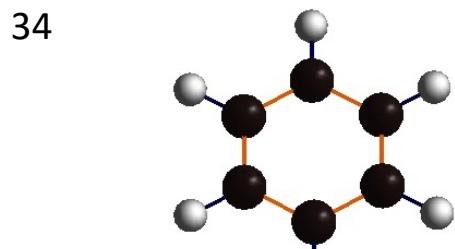
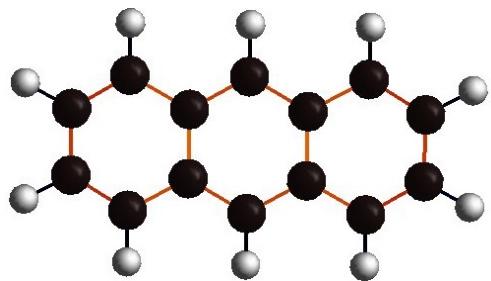
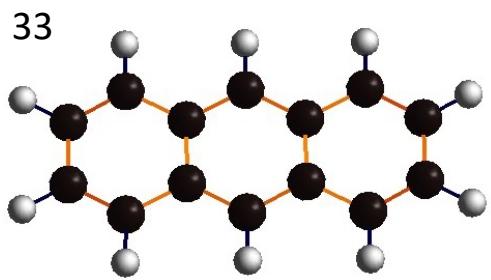


Figure S12: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 6).

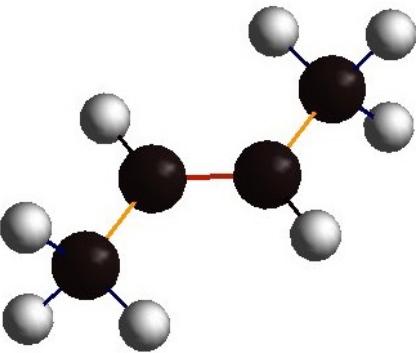
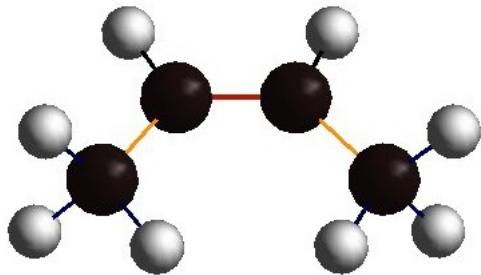
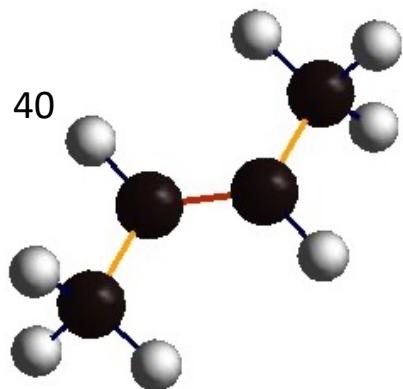
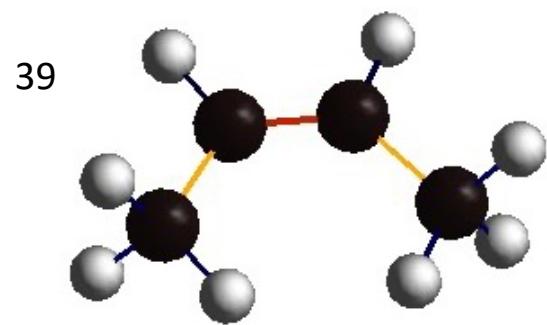
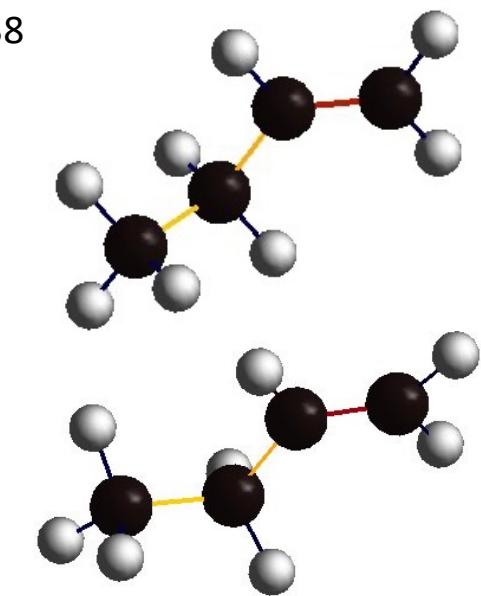
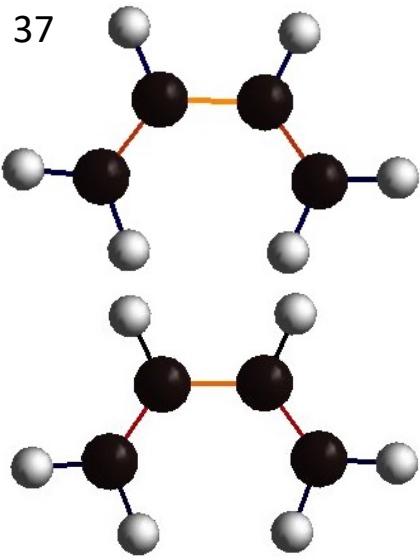


Figure S13: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 7).

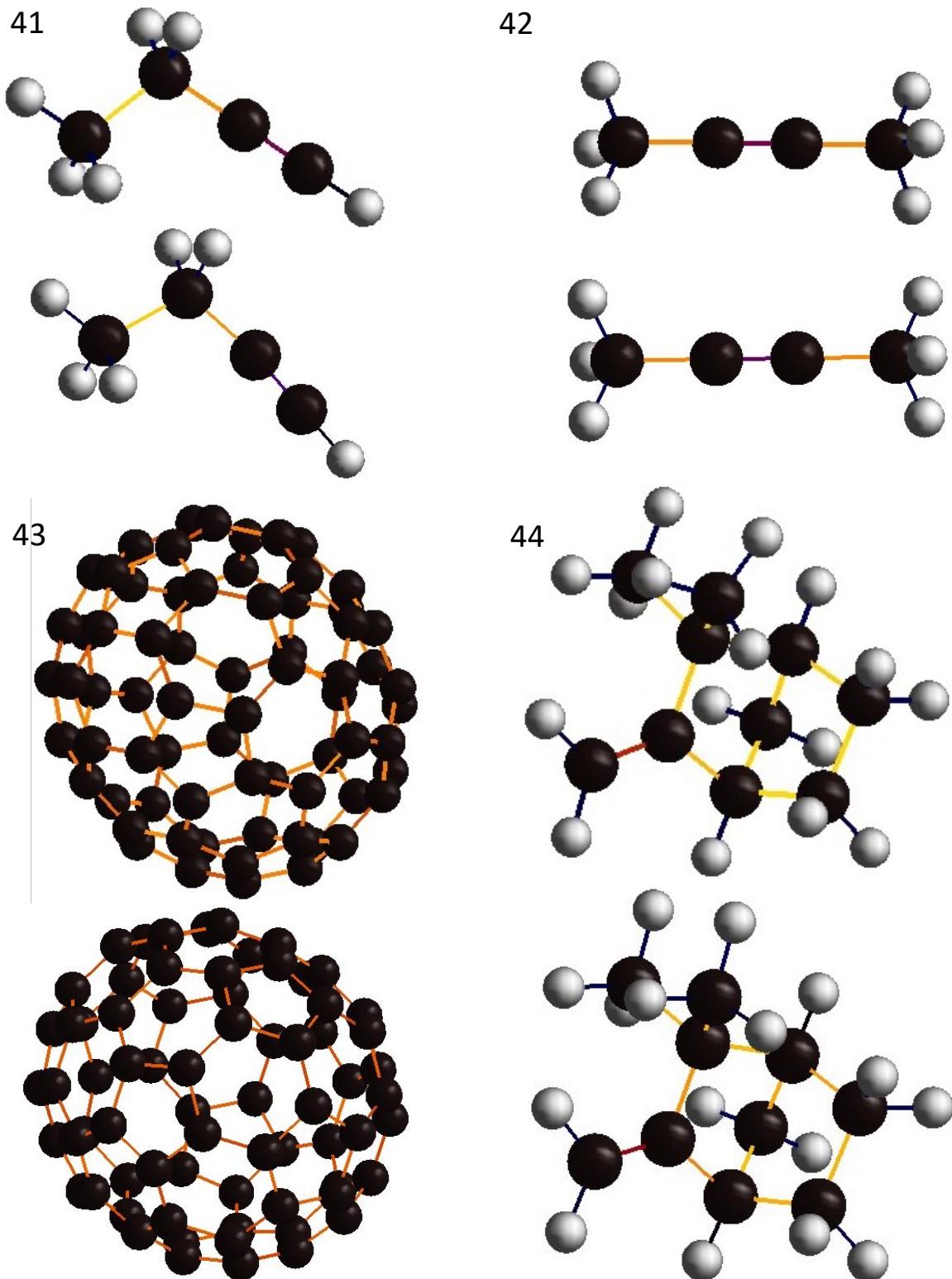


Figure S14: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 8).

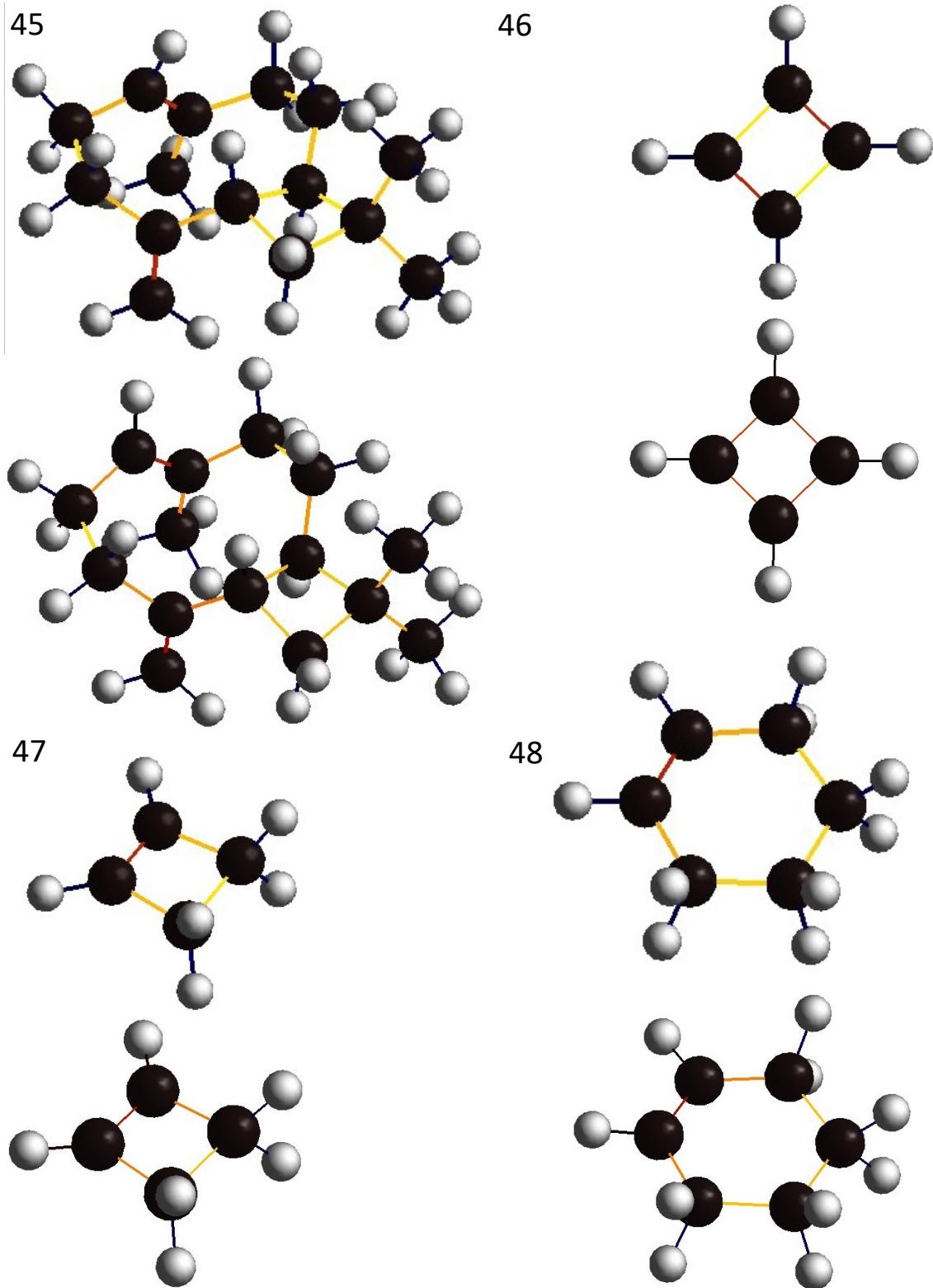


Figure S15: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 9).

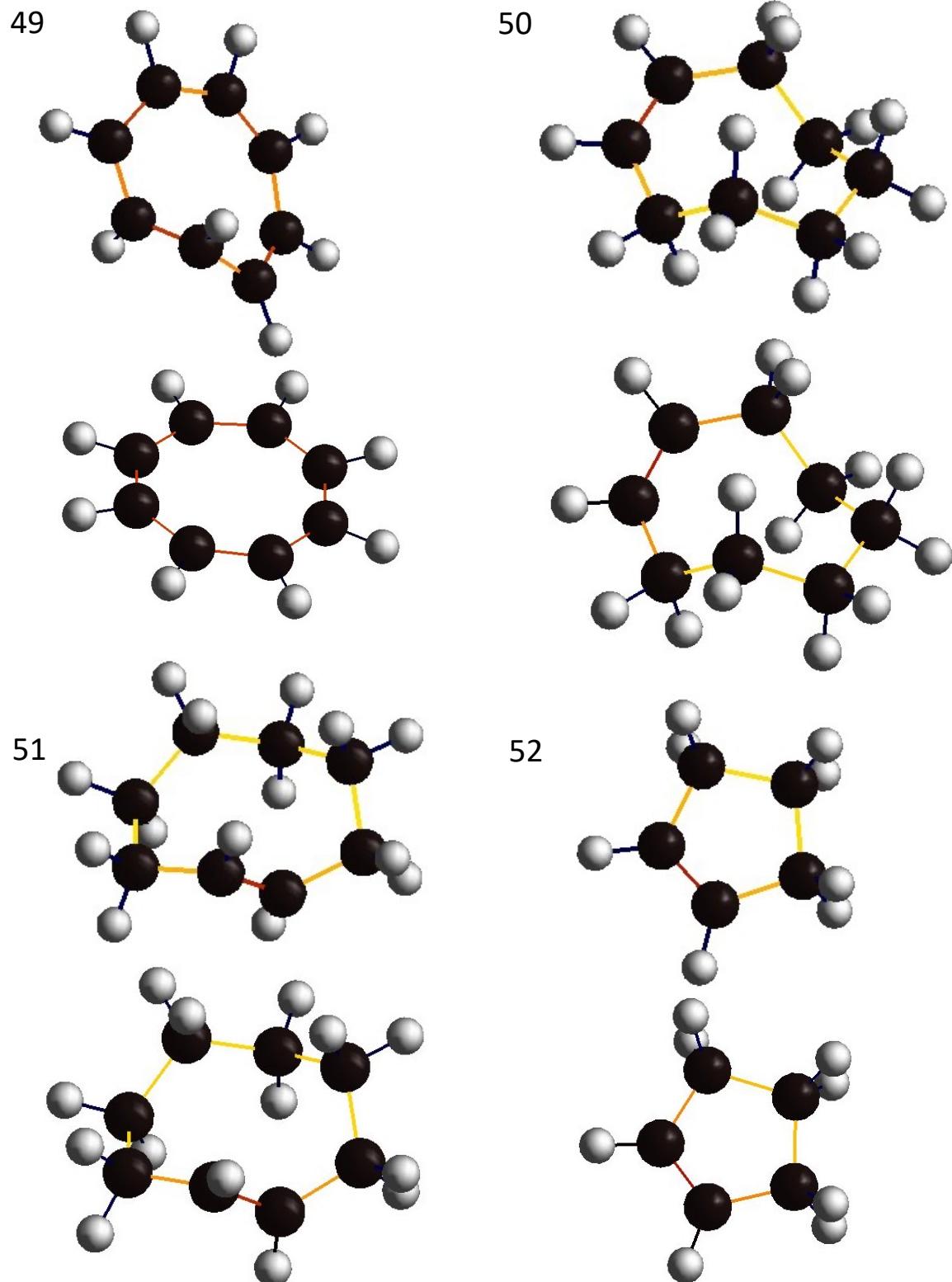


Figure S16: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 10).

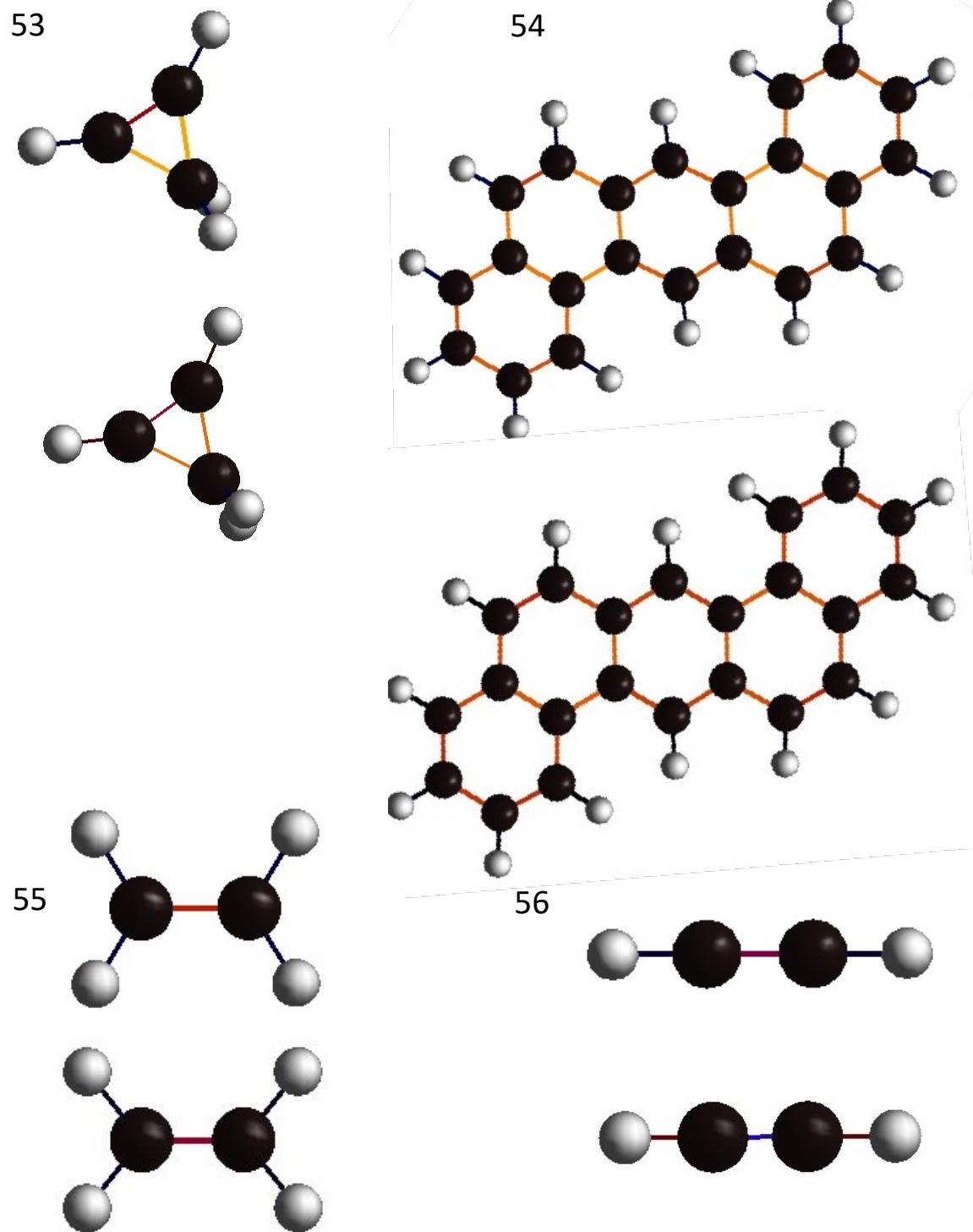


Figure S17: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 11).

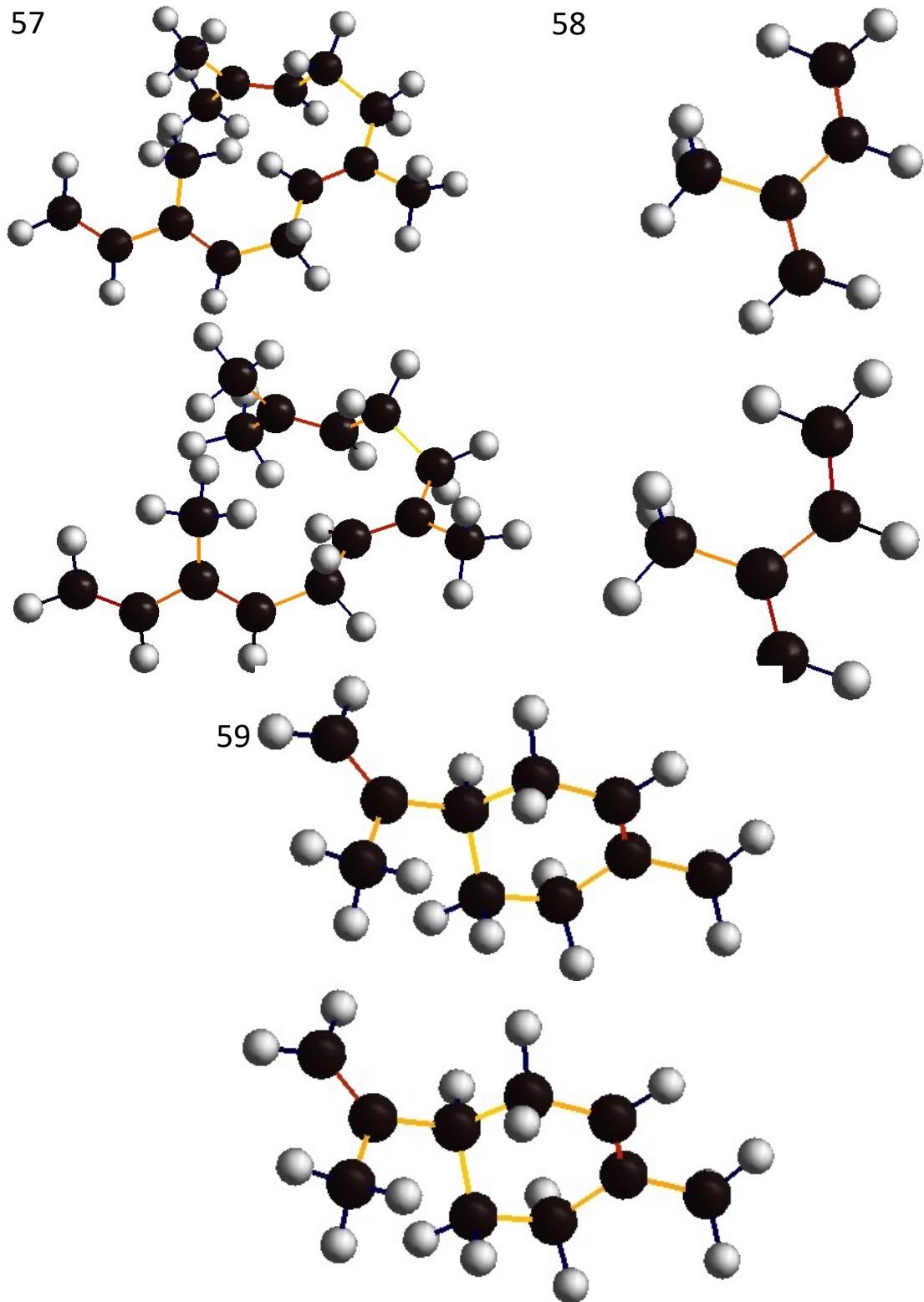


Figure S18: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 12).

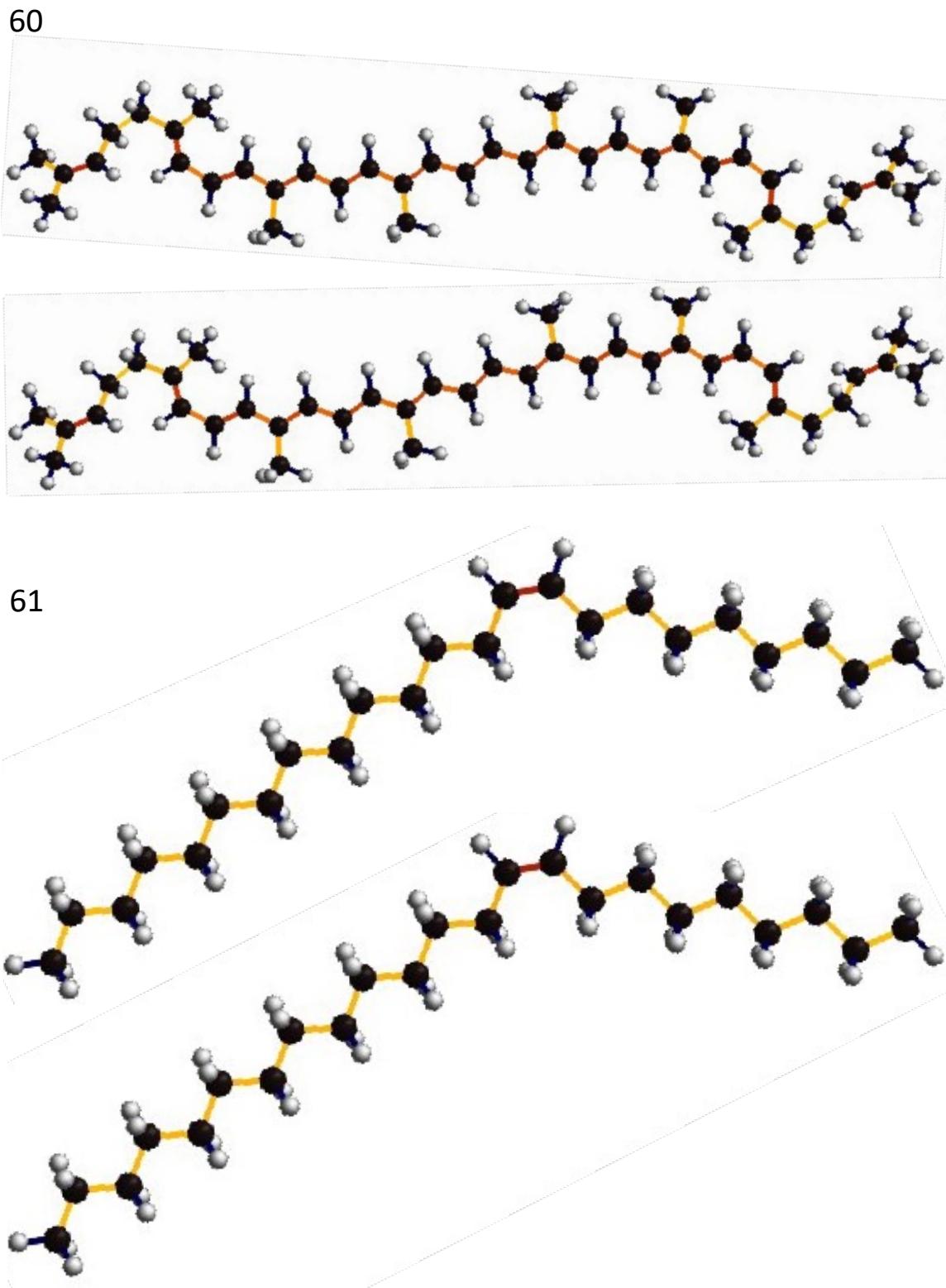
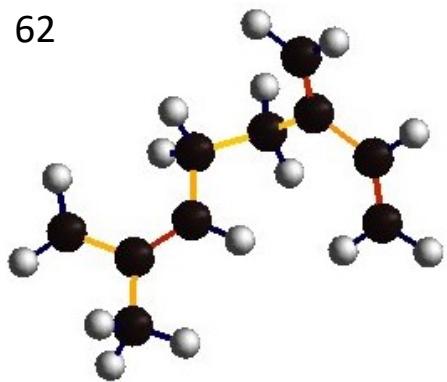
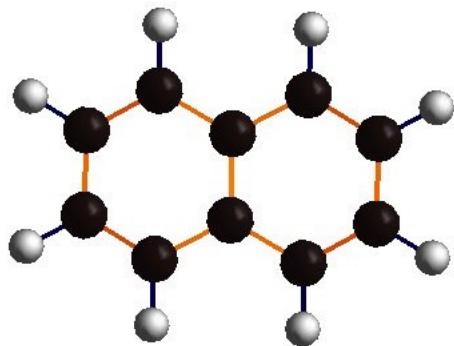


Figure S19: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 13).

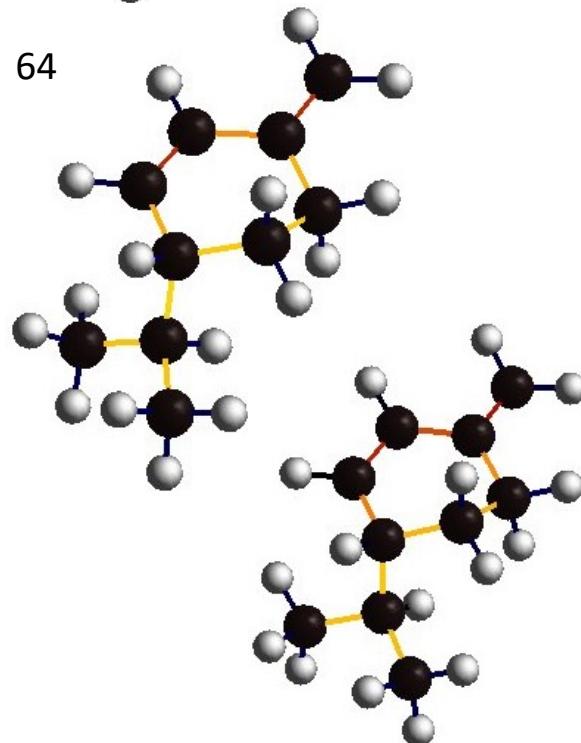
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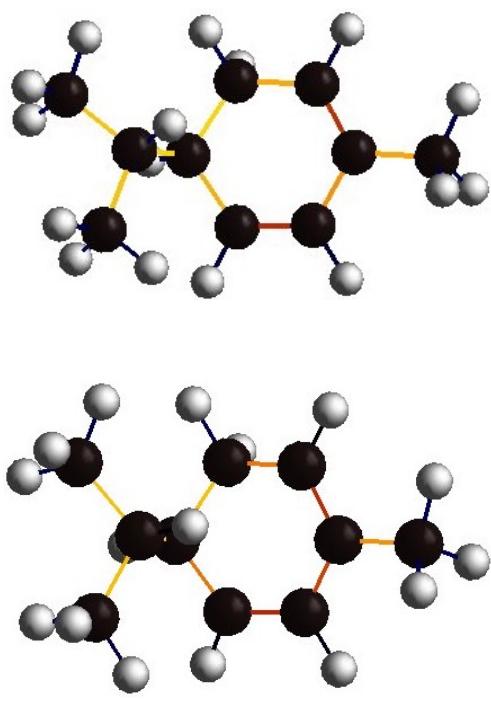


Figure S20: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 14).

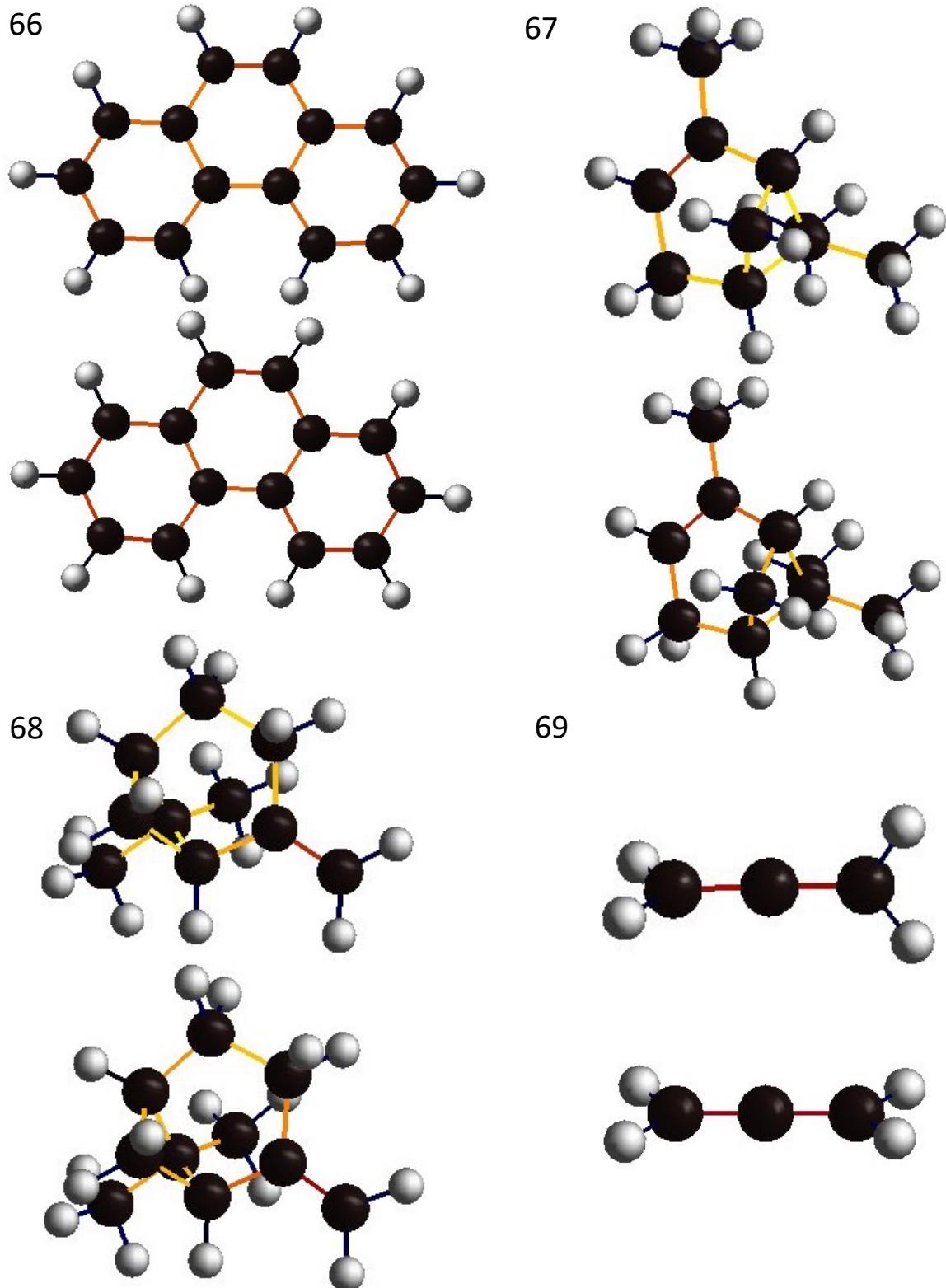


Figure S21: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 15).

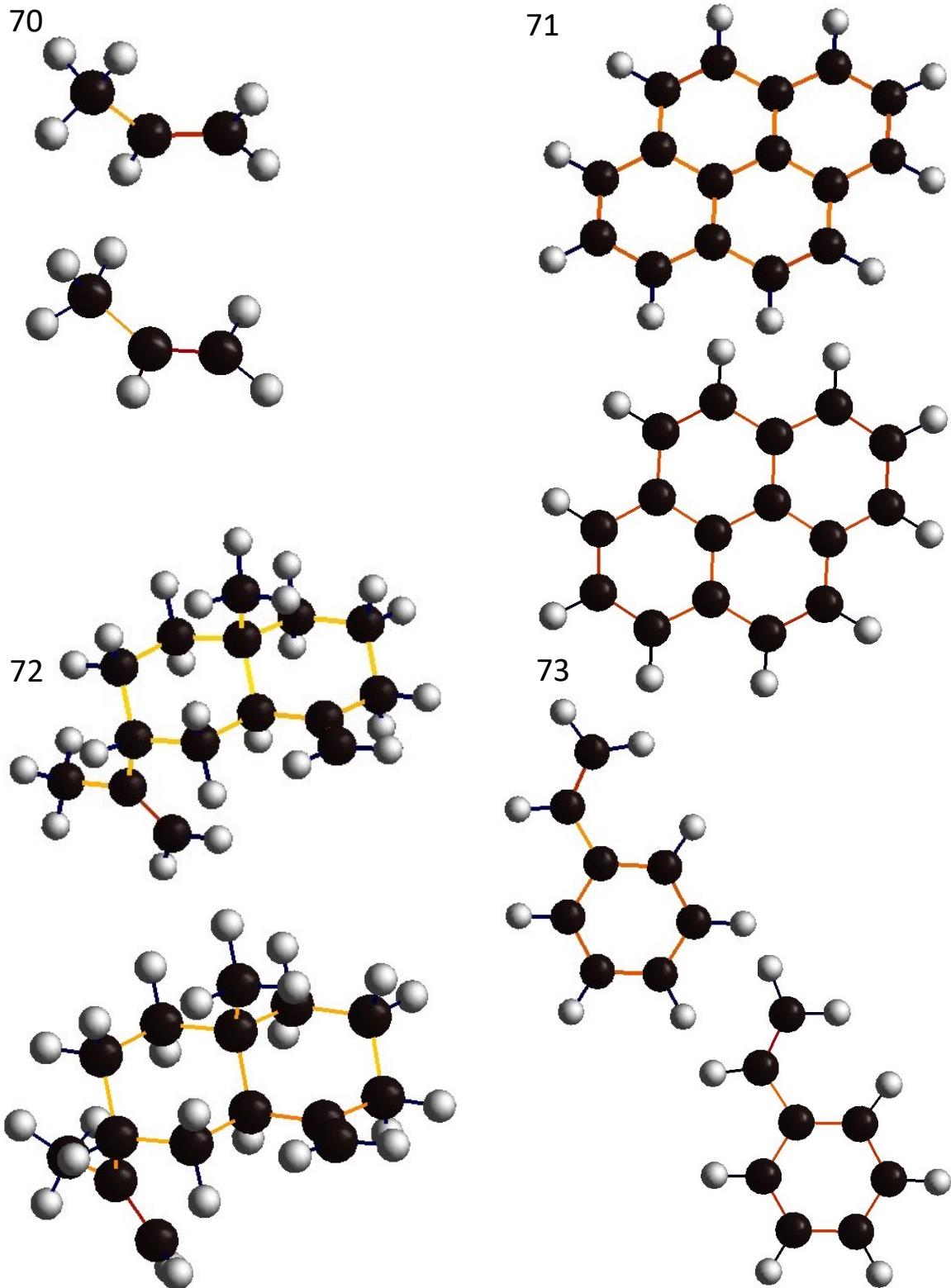


Figure S22: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 16).

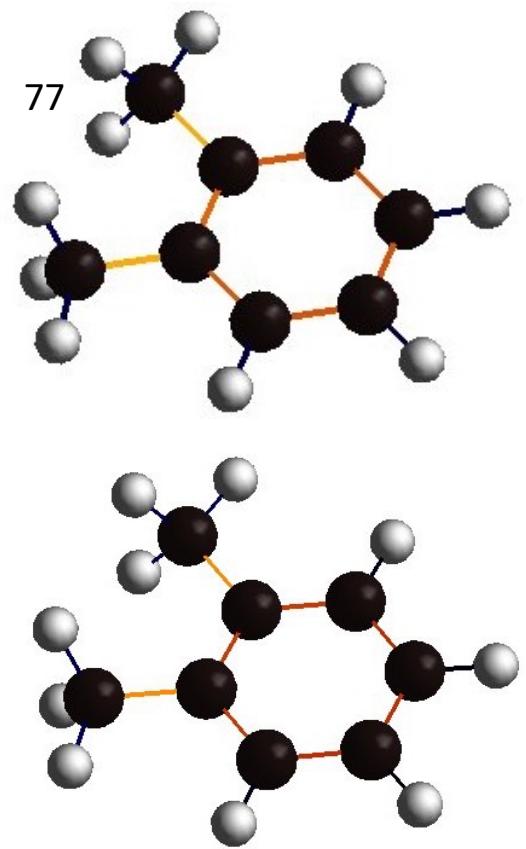
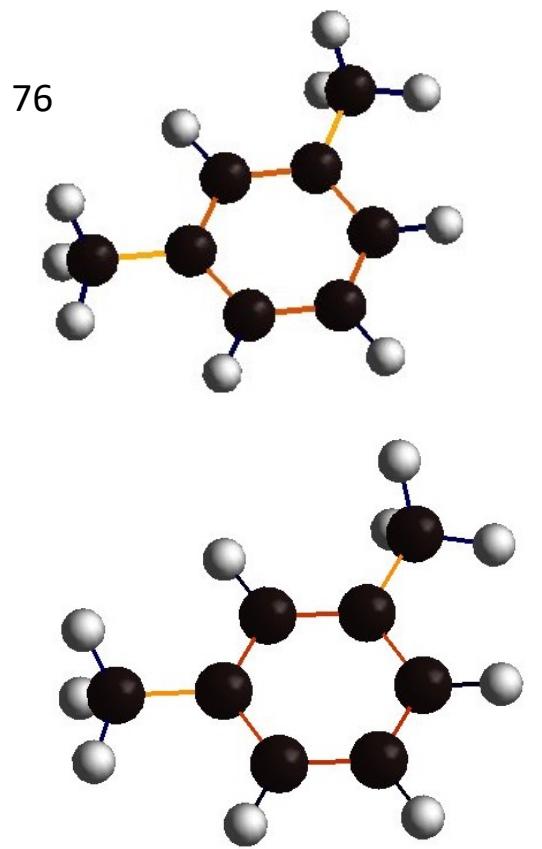
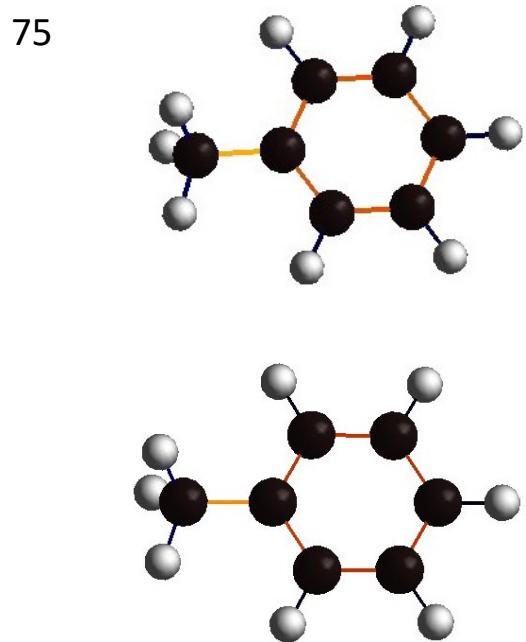
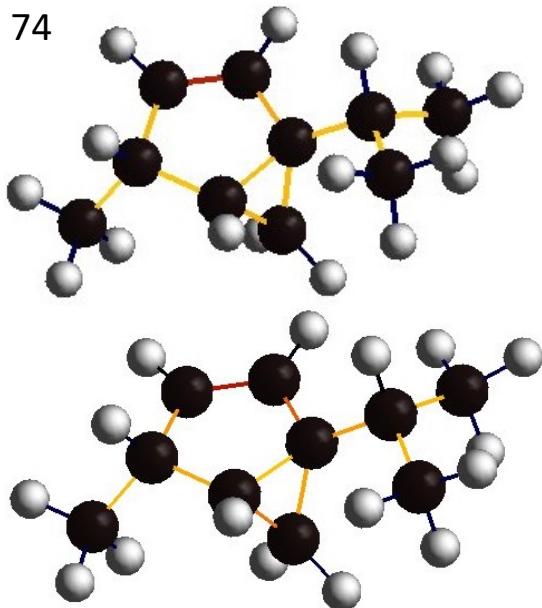


Figure S23: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 17).

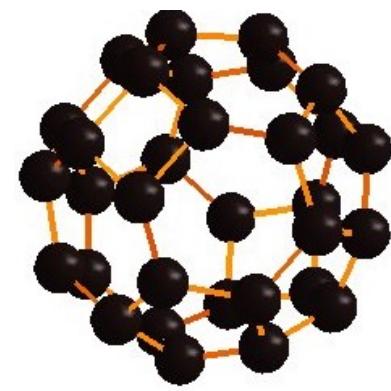
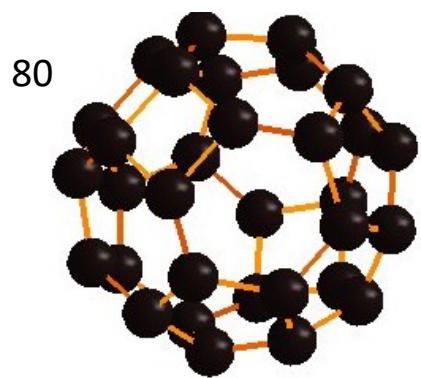
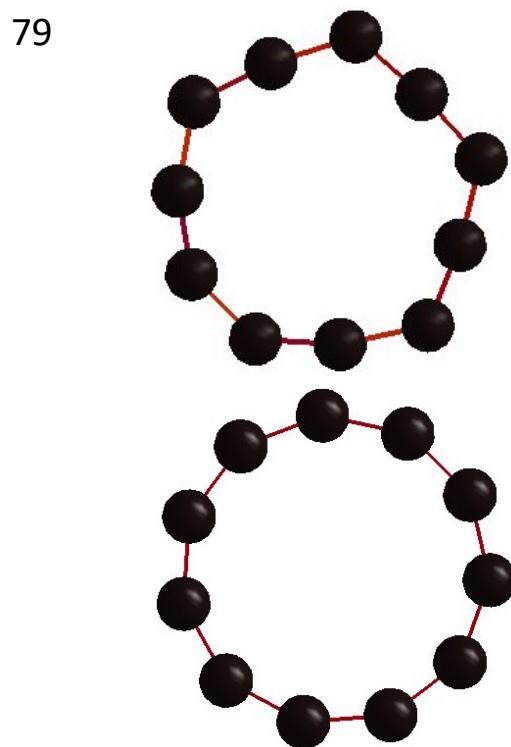
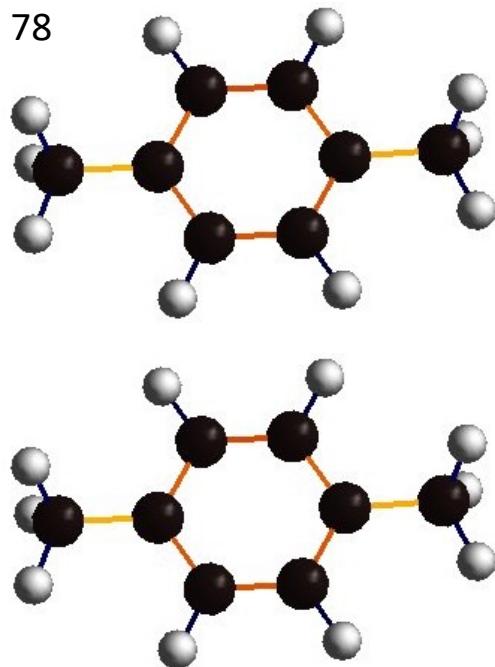


Figure S24: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 18).

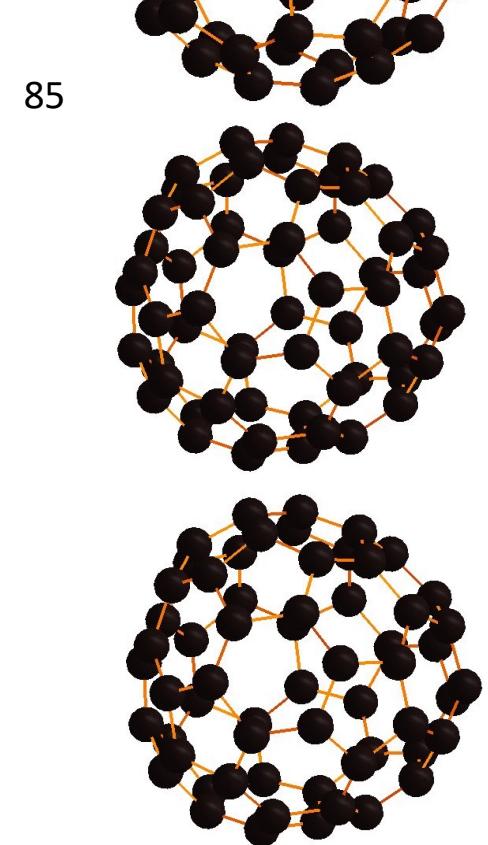
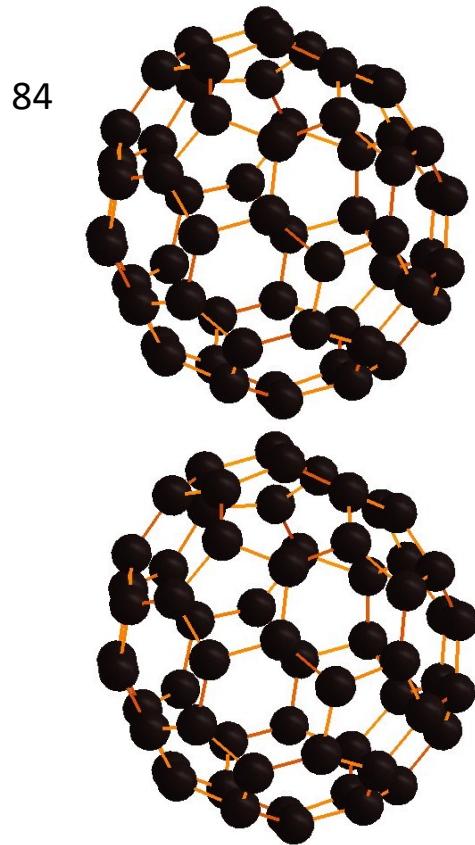
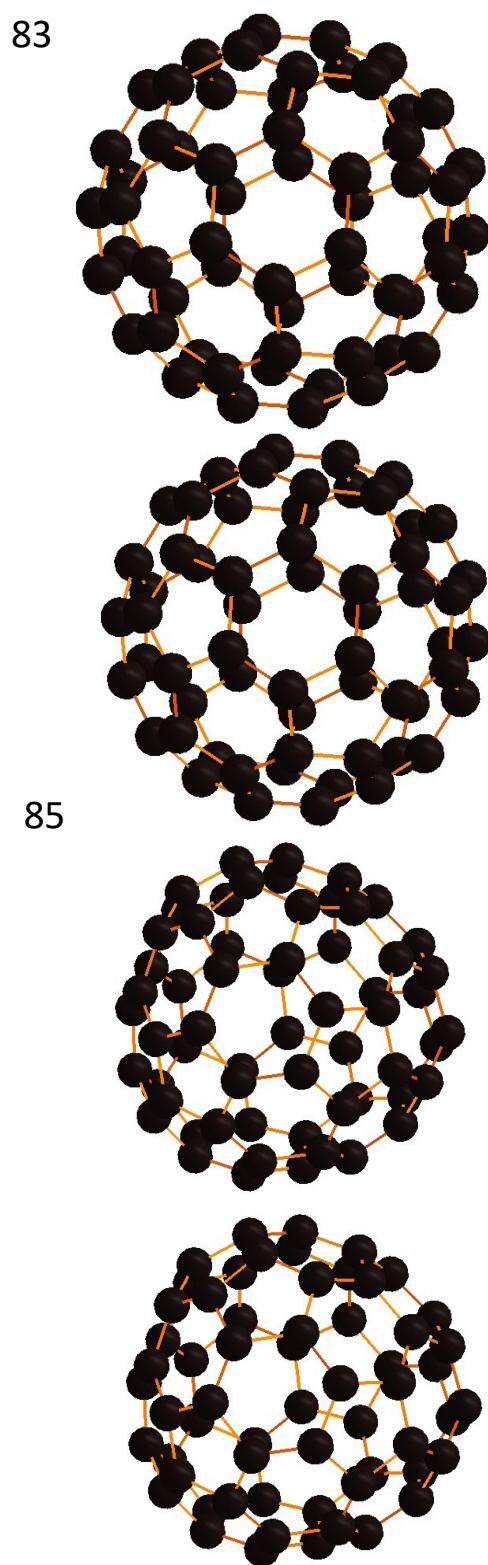
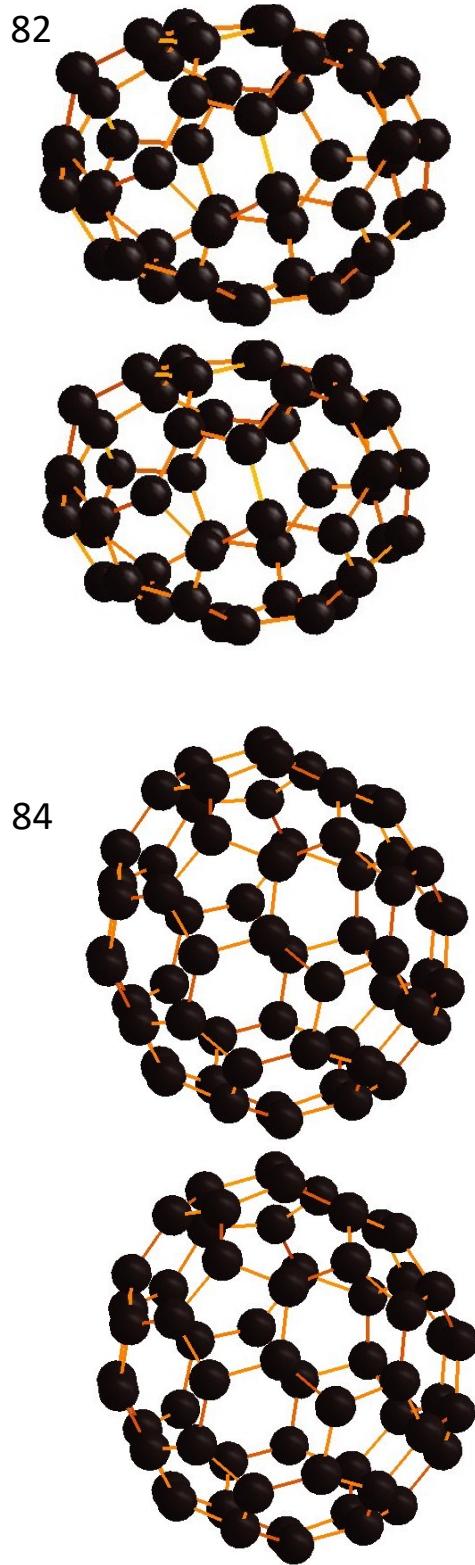
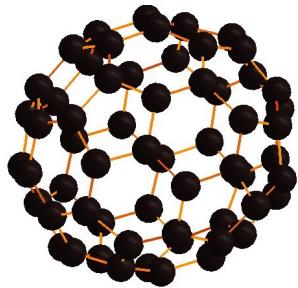


Figure S25: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 19).

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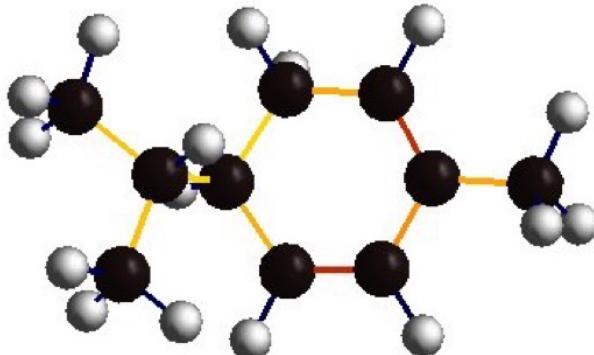
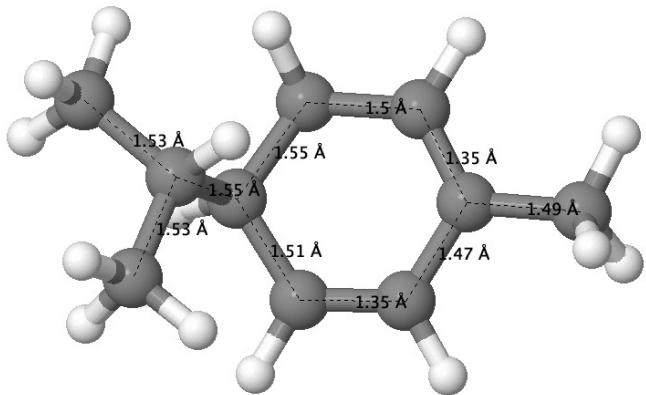
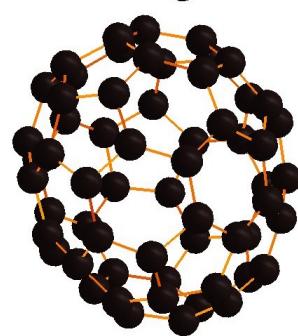
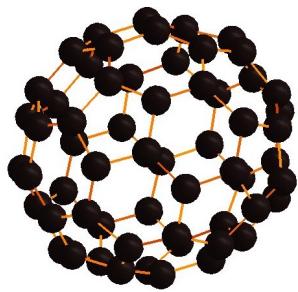
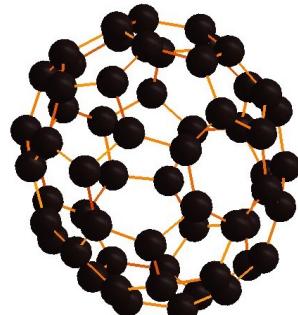


Figure S26: Optimized structures obtained from both DFT and the ANN potential. For each structural ID, the upper geometry is the DFT-optimized structure and the lower one is the ANN-optimized structure (part 20). The figure at the bottom is a sample showcasing bond lengths in one structure, offering insight into the color-coded representation denoting varying bond lengths.

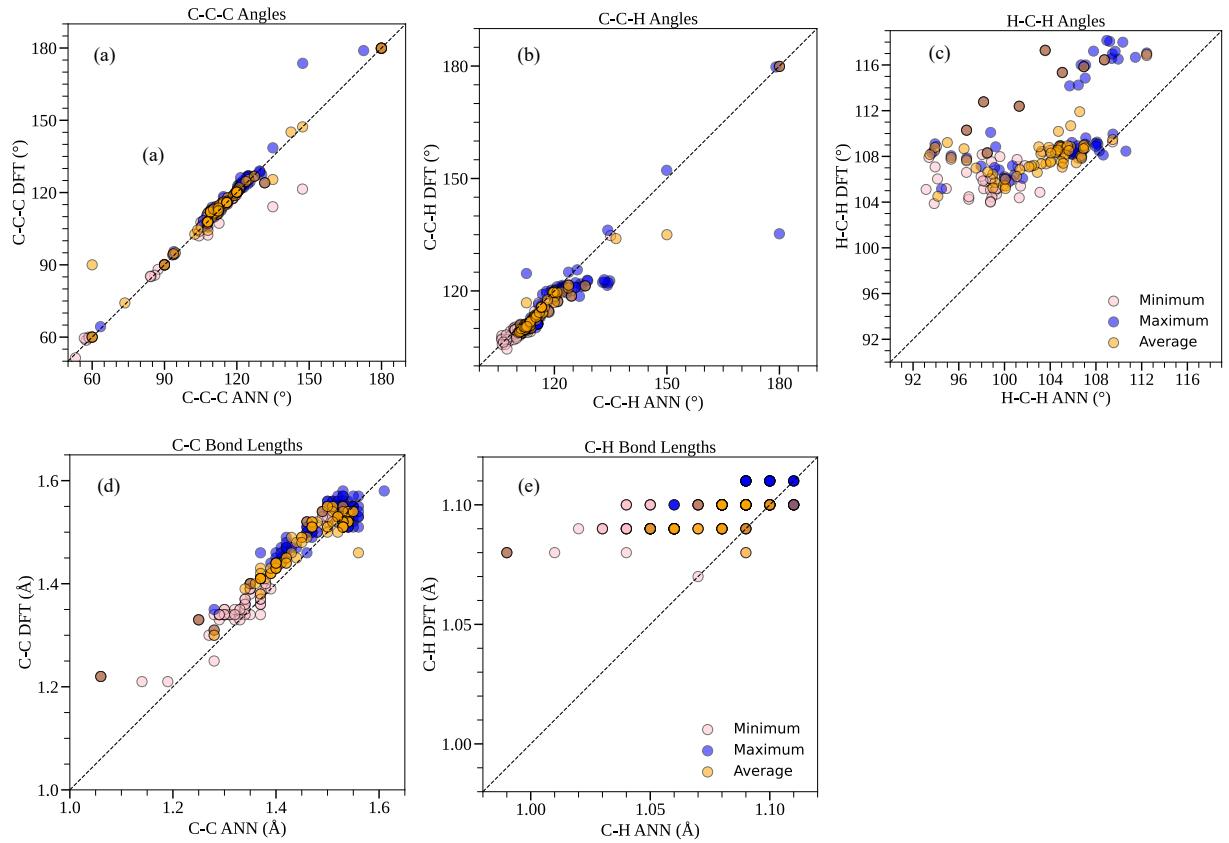


Figure S27: Bond length and bond angle analysis were conducted for the 0D test cases optimized by both DFT and the ANN potential. Panels (a)-(c) display the C-C-C, C-C-H, and H-C-H bond angles, while panels (d) and (e) represent the C-C and C-H bond lengths, respectively.

## 4 1D and 2D test cases

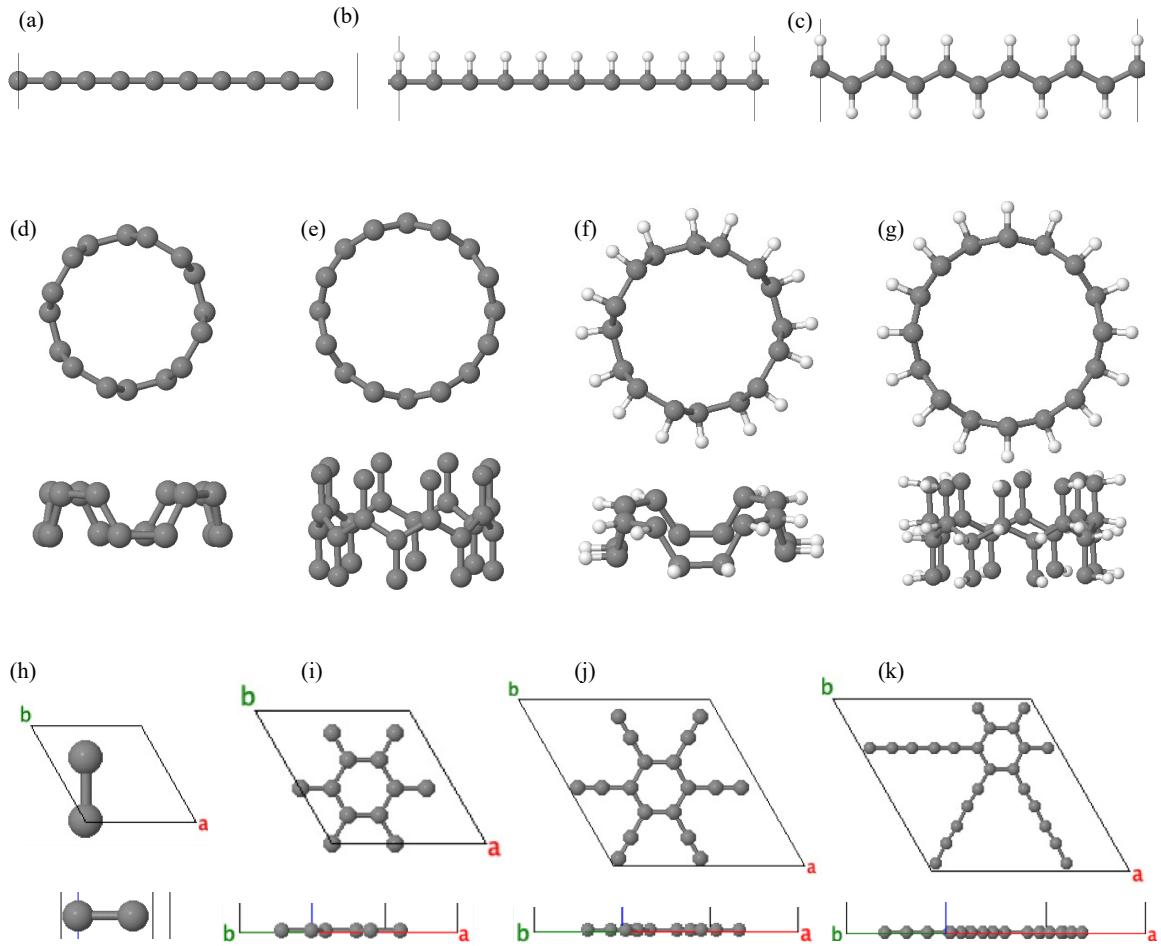


Figure S28: Top views and side views of the unitcells of the studied 1D and 2D systems: (a)-(c) 10-atom carbon chains, (d) SWCNT(4,4), (e) SWCNT(8,0), (f) H-saturated SWCNT(4,4), (g) H-saturated SWCNT(8,0), (h) graphene, (i) graphyne-1, (j) graphyne-2, and (k) graphyne-3.

## 5 3D test cases

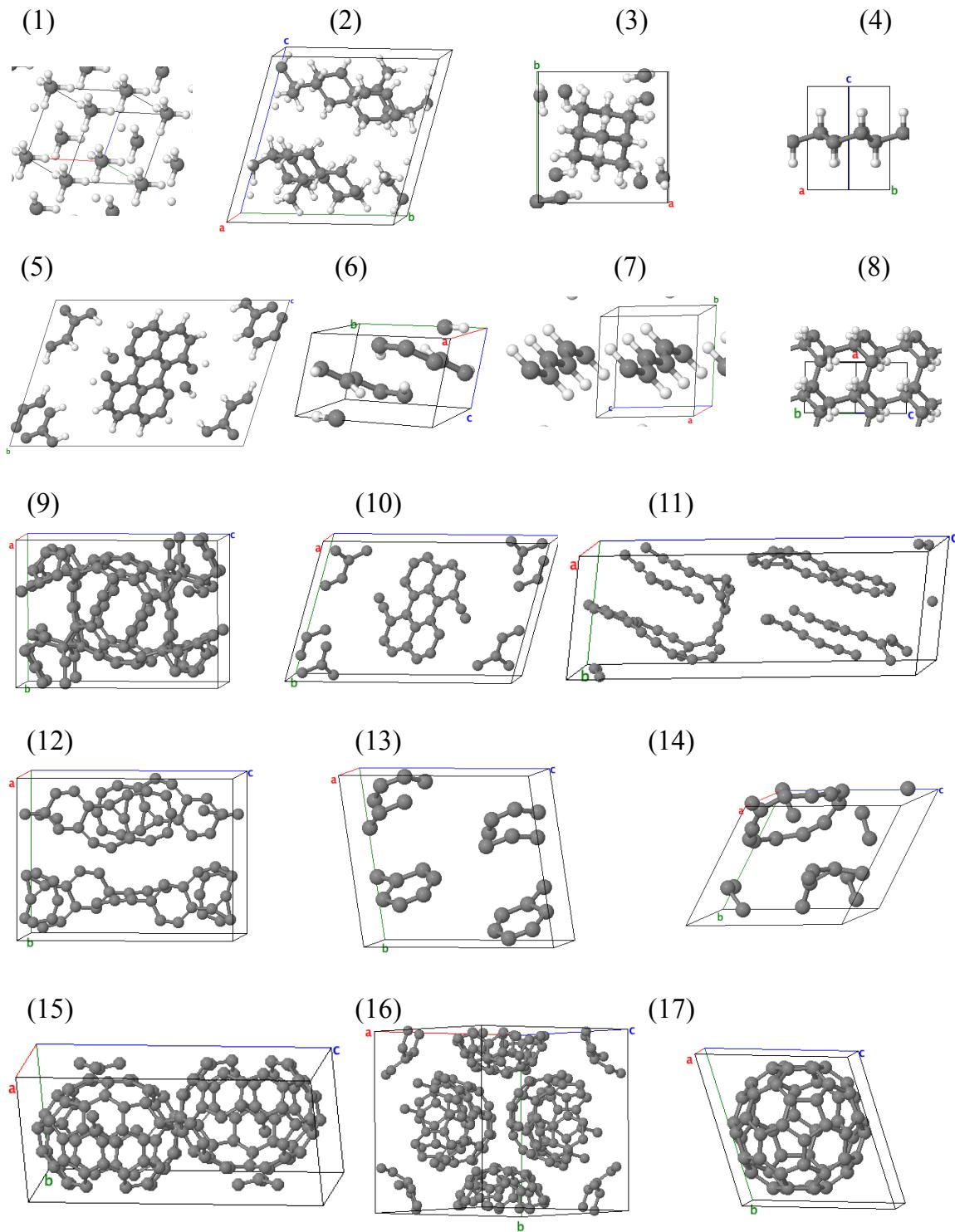


Figure S29: 3D C-H and C systems used as test case for geometry optimizations. The ID of each structure is the one reported in Table S2 and S3 (part 1).

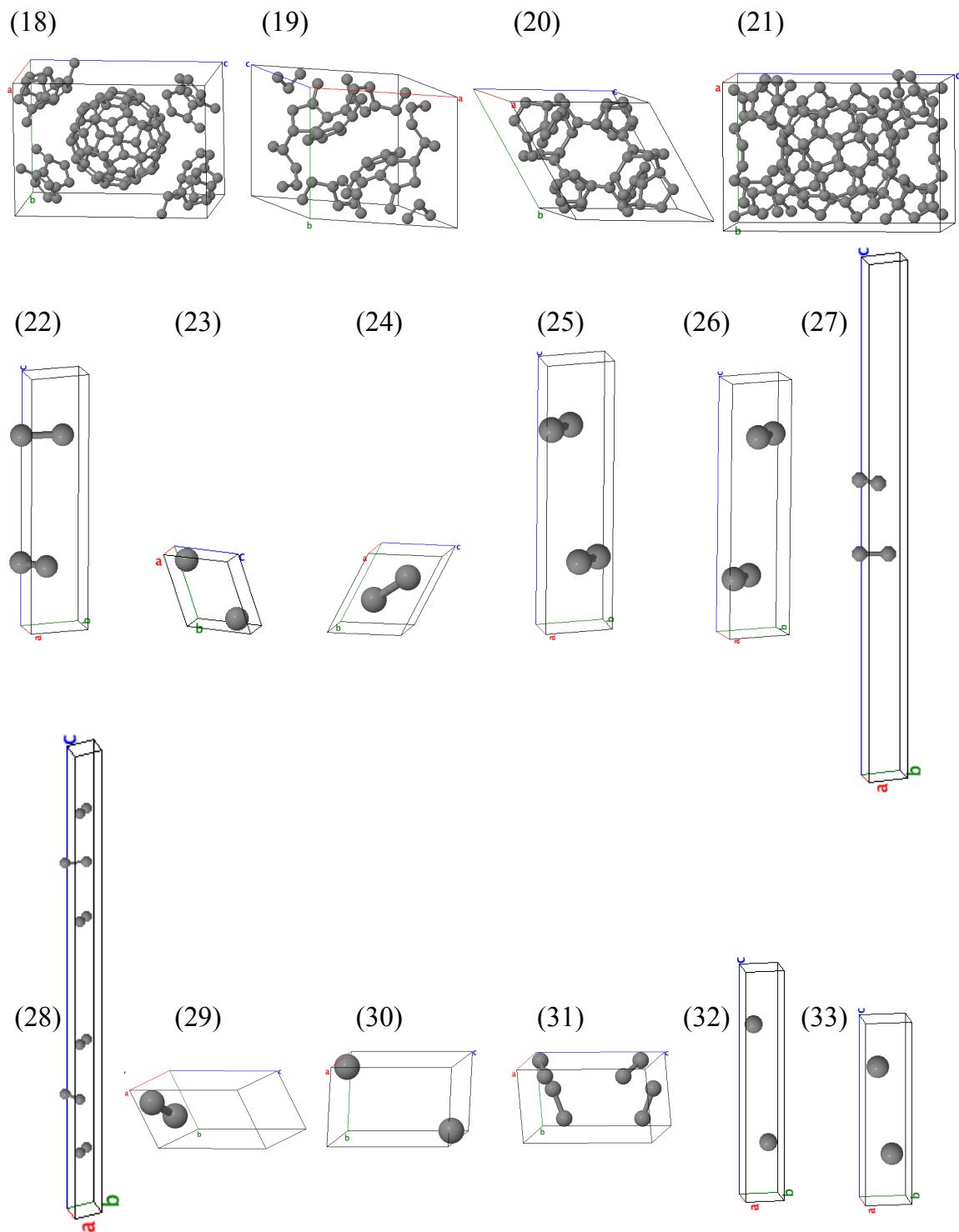


Figure S30: 3D C-H and C systems used as test case for geometry optimizations. The ID of each structure is the one reported in Table S2 and S3 (part 2).

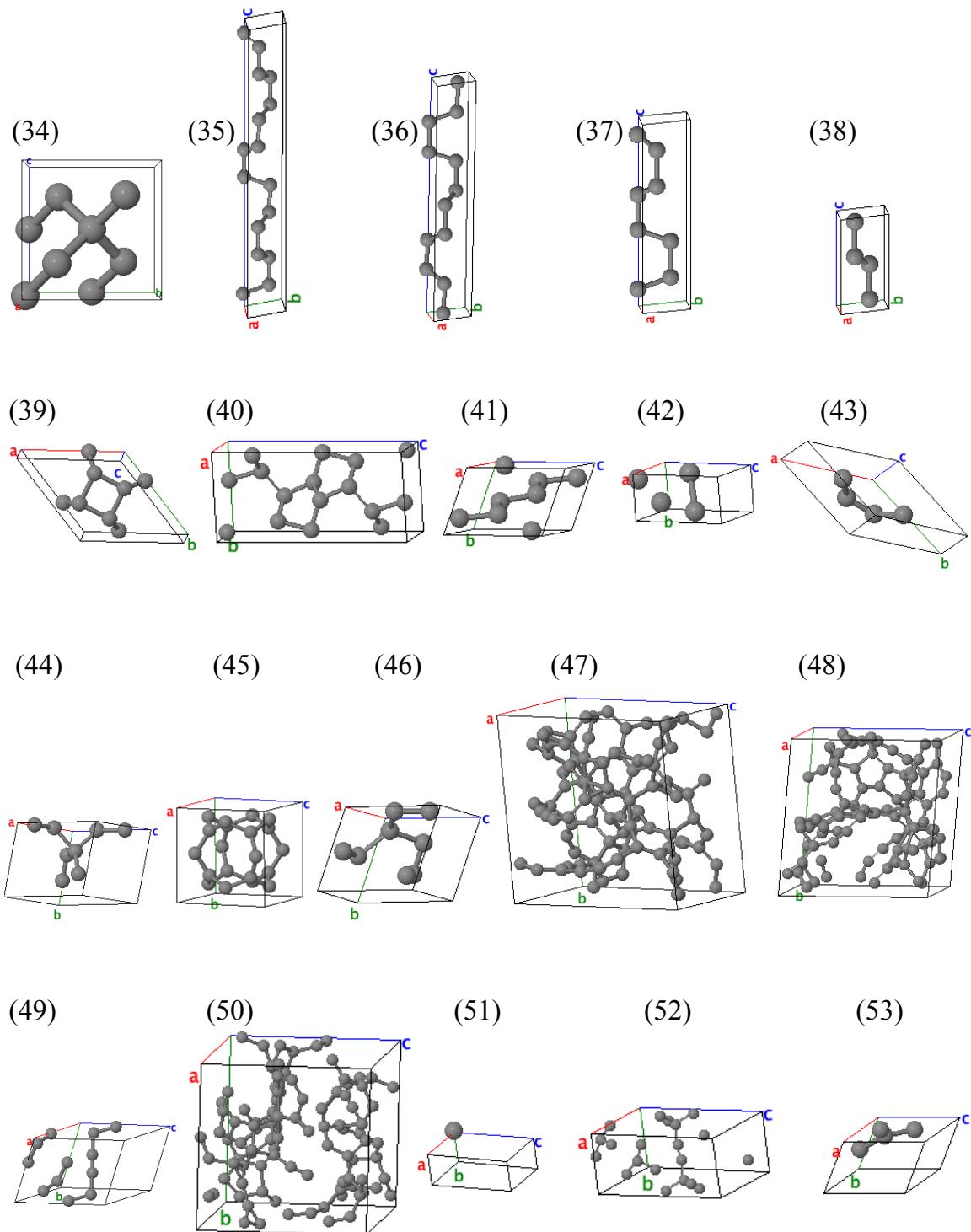


Figure S31: 3D C-H and C systems used as test case for geometry optimizations. The ID of each structure is the one reported in Table S2 and S3 (part 3).

Table S3: Chemical formula, space group (spg), total energies ( $E_{tot}$ ), total energies differences of DFT and ANN ( $\Delta E_{tot}$ ), formation energies ( $E_f$ ), and formation energy differences of DFT and ANN ( $\Delta E_f$ ) of the examined 3D systems. All the energies are in eV/atom. The Materials Project'IDs of the structures are listed in the last column. The experimentally reported structures are marked by \*.

| ID        | system         | spg            | $E_{tot,DFT}$ | $E_{tot,ANN}$ | $\Delta E_{tot}$ | $E_{f,DFT}$ | $E_{f,ANN}$ | $\Delta E_f$ | MP-ID       |
|-----------|----------------|----------------|---------------|---------------|------------------|-------------|-------------|--------------|-------------|
| 01        | $H_{04}C_{01}$ | $I\bar{4}3m$   | -4.81413      | -4.83068      | 0.01655          | -0.28714    | -0.30369    | 0.01655      | mp-1021328  |
| 02        | $H_{68}C_{38}$ | $P12_1\bar{1}$ | -5.63471      | -5.64953      | 0.01482          | -0.20320    | -0.21802    | 0.01482      | mp-30168*   |
| 03        | $H_{32}C_{20}$ | $P\bar{4}2_1c$ | -5.77756      | -5.77546      | -0.00210         | -0.19696    | -0.19486    | -0.00210     | mp-1195106* |
| 04        | $H_{02}C_{02}$ | $P\bar{3}m1$   | -6.40774      | -24.93188     | 18.5241          | -0.16862    | -18.6928    | 18.5242      | mp-1096986  |
| 05        | $H_{24}C_{48}$ | $P12_1/c1$     | -7.28579      | -7.31600      | 0.03021          | -0.09549    | -0.12570    | 0.03021      | mp-603334*  |
| 06        | $H_8C_{12}$    | $C12/c1$       | -6.88042      | -6.91393      | 0.03351          | -0.07060    | -0.10410    | 0.03350      | mp-995217   |
| 07        | $H_{02}C_{02}$ | $Cmcm$         | -6.30604      | -6.24579      | -0.06025         | -0.06693    | -0.00668    | -0.06025     | mp-995195   |
| 08        | $H_{04}C_{04}$ | $I2_1\bar{3}$  | -6.15054      | -6.17993      | 0.02939          | 0.08858     | 0.05919     | 0.02939      | mp-1079612  |
| group I   |                |                |               |               |                  |             |             |              |             |
| 09        | $C_{120}$      | $Pnnm$         | -8.53708      | -8.59123      | 0.05415          | 0.55559     | 0.50144     | 0.05414      | mp-1205283  |
| 10        | $C_{48}$       | $P2_1/c$       | -8.19050      | -8.13138      | -0.05912         | 0.90217     | 0.96128     | -0.05912     | mp-1203645* |
| 11        | $C_{80}$       | $P1$           | -8.02929      | -7.99525      | -0.03404         | 1.06338     | 1.09742     | -0.03404     | mp-1197903  |
| 12        | $C_{80}$       | $P2_12_12_1$   | -7.98126      | -7.78202      | -0.19924         | 1.11141     | 1.31065     | -0.19924     | mp-1182684  |
| 13        | $C_{28}$       | $P2_1$         | -7.72848      | -7.66183      | -0.06665         | 1.36419     | 1.43084     | -0.06665     | mp-1194362* |
| 14        | $C_{29}$       | $I\bar{4}3m$   | -7.30820      | -7.37988      | 0.07168          | 1.78447     | 1.71279     | 0.07168      | mp-1192619  |
| group II  |                |                |               |               |                  |             |             |              |             |
| 15        | $C_{140}$      | $Cmcm$         | -8.88456      | -8.87055      | -0.01401         | 0.20811     | 0.22212     | -0.01401     | mp-683919*  |
| 16        | $C_{240}$      | $Pa\bar{3}$    | -8.85087      | -8.83161      | -0.01926         | 0.24180     | 0.26105     | -0.01926     | mp-1196583* |
| 17        | $C_{60}$       | $Fm\bar{3}$    | -8.84961      | -8.83163      | -0.01798         | 0.24306     | 0.26104     | -0.01798     | mp-667273*  |
| 18        | $C_{120}$      | $Pnnm$         | -8.84576      | -8.81973      | -0.02603         | 0.24691     | 0.27293     | -0.02603     | mp-1147718  |
| 19        | $C_{60}$       | $Immm$         | -8.83945      | -8.80686      | -0.03256         | 0.25322     | 0.28581     | -0.03259     | mp-630227*  |
| 20        | $C_{60}$       | $R\bar{3}m$    | -8.82435      | -8.77838      | -0.04597         | 0.26831     | 0.31429     | -0.04597     | mp-680372*  |
| 21        | $C_{120}$      | $Pnnm$         | -8.69964      | -8.71730      | 0.01766          | 0.39303     | 0.37537     | 0.01766      | mp-568028*  |
| group III |                |                |               |               |                  |             |             |              |             |
| 22        | $C_4$          | $P6_3/mmc$     | -9.22515      | -9.28328      | 0.05813          | -0.13249    | -0.19061    | 0.05813      | mp-48*      |
| 23        | $C_2$          | $R\bar{3}m$    | -9.22459      | -9.28328      | 0.05869          | -0.13192    | -0.19061    | 0.05870      | mp-169*     |
| 24        | $C_2$          | $Fmmm$         | -9.22424      | -9.28328      | 0.05904          | -0.13157    | -0.19061    | 0.05904      | mp-937760   |
| 25        | $C_4$          | $Cmme$         | -9.22406      | -9.28328      | 0.05922          | -0.13139    | -0.19061    | 0.05923      | mp-568286   |
| 26        | $C_4$          | $Cmme$         | -9.22396      | -9.28328      | 0.05932          | -0.13129    | -0.19061    | 0.05933      | mp-568363   |
| 27        | $C_4$          | $P\bar{3}m1$   | -9.22303      | -9.28328      | 0.06025          | -0.13036    | -0.19061    | 0.06025      | mp-990424   |
| 28        | $C_{12}$       | $P6_3/mmc$     | -9.22292      | -9.28328      | 0.06036          | -0.13025    | -0.19061    | 0.06036      | mp-606949   |
| 29        | $C_2$          | $P6/mmm$       | -9.22073      | -9.28328      | 0.06255          | -0.12806    | -0.19061    | 0.06256      | mp-568806   |
| 30        | $C_2$          | $C2/m$         | -9.21831      | -9.28328      | 0.06497          | -0.12564    | -0.19061    | 0.06497      | mp-632329   |
| 31        | $C_8$          | $Cmmm$         | -8.28535      | -8.57616      | 0.29081          | 0.80732     | 0.51651     | 0.29082      | mp-579909   |
| 32        | $C_2$          | $Cmcm$         | -6.69705      | -7.20123      | 0.50418          | 2.39562     | 1.89144     | 0.50418      | mp-1097832  |

| ID       | system    | spg          | $E_{tot,DFT}$ | $E_{tot,ANN}$ | $\Delta E_{tot}$ | $E_{f,DFT}$ | $E_{f,ANN}$ | $\Delta E_f$ | MP-ID       |
|----------|-----------|--------------|---------------|---------------|------------------|-------------|-------------|--------------|-------------|
| 33       | $C_2$     | $Cmcm$       | -6.68170      | -7.20123      | 0.51953          | 2.41097     | 1.89144     | 0.51953      | mp-1056957* |
| group IV |           |              |               |               |                  |             |             |              |             |
| 34       | $C_8$     | $Fd\bar{3}m$ | -9.09267      | -8.90824      | -0.18443         | 0.00000     | 0.18442     | -0.18442     | mp-66*      |
| 35       | $C_{16}$  | $P6_3/mmc$   | -9.08823      | -8.91262      | -0.17561         | 0.00444     | 0.18005     | -0.17561     | mp-616440   |
| 36       | $C_{12}$  | $P6_3/mmc$   | -9.08682      | -8.91441      | -0.17241         | 0.00585     | 0.17826     | -0.17241     | mp-611448   |
| 37       | $C_8$     | $P6_3/mmc$   | -9.08352      | -8.91890      | -0.16462         | 0.00915     | 0.17376     | -0.16461     | mp-611426   |
| 38       | $C_4$     | $P6_3/mmc$   | -9.06806      | -8.93745      | -0.13061         | 0.02461     | 0.15522     | -0.13061     | mp-47*      |
| 39       | $C_8$     | $Cmmm$       | -8.96336      | -8.83272      | -0.13064         | 0.12930     | 0.25994     | -0.13064     | mp-1078845  |
| 40       | $C_{16}$  | $Pnma$       | -8.93867      | -8.86150      | -0.07717         | 0.15400     | 0.23117     | -0.07718     | mp-1190171  |
| 41       | $C_8$     | $C2/m$       | -8.93054      | -8.85154      | -0.07900         | 0.16213     | 0.24113     | -0.07900     | mp-1080826  |
| 42       | $C_4$     | $I4/mmm$     | -8.89623      | -8.74316      | -0.15307         | 0.19644     | 0.34950     | -0.15306     | mp-1008395  |
| 43       | $C_4$     | $Cmmm$       | -8.79138      | -8.63055      | -0.16083         | 0.30129     | 0.46212     | -0.16083     | mp-1008374  |
| 44       | $C_8$     | $Im\bar{3}m$ | -8.46581      | -8.79792      | 0.33211          | 0.62686     | 0.29475     | 0.33212      | mp-570002*  |
| 45       | $C_{20}$  | $Pm\bar{3}m$ | -8.46507      | -8.55512      | 0.09005          | 0.62760     | 0.53755     | 0.09005      | mp-1188817  |
| 46       | $C_8$     | $Ia\bar{3}$  | -8.39587      | -8.39042      | -0.00545         | 0.69680     | 0.70225     | -0.00544     | mp-24       |
| 47       | $C_{100}$ | $P1$         | -8.29905      | -8.43063      | 0.13158          | 0.79362     | 0.66203     | 0.13158      | mp-1244964  |
| 48       | $C_{100}$ | $P1$         | -8.23585      | -8.37504      | 0.13919          | 0.85682     | 0.71762     | 0.13920      | mp-1244913  |
| 49       | $C_{12}$  | $I4/mmm$     | -8.23249      | -8.00509      | -0.22740         | 0.86018     | 1.08758     | -0.22740     | mp-1095633  |
| 50       | $C_{100}$ | $P1$         | -8.21852      | -8.34462      | 0.12610          | 0.87415     | 0.74805     | 0.12610      | mp-1245190  |
| 51       | $C_1$     | $P2/m$       | -8.21158      | -8.38223      | 0.17065          | 0.88109     | 0.71044     | 0.17065      | mp-1182029  |
| 52       | $C_{20}$  | $I4/mmm$     | -8.06062      | -7.79945      | -0.26117         | 1.03205     | 1.29322     | -0.26117     | mp-1205417  |
| 53       | $C_4$     | $I4_132$     | -7.91997      | -8.68112      | 0.76115          | 1.17270     | 0.41155     | 0.76115      | mp-1018088  |

Table S4: Chemical formula, space group (spg), lattice constants (a, b, and c in Å), volumes (in Å<sup>3</sup>/atom), volume differences (Δ V in Å<sup>3</sup>/atom) obtained from DFT and ANN.

| ID        | system                          | spg   | a <sub>DFT</sub> | a <sub>ANN</sub> | b <sub>DFT</sub> | b <sub>ANN</sub> | c <sub>DFT</sub> | c <sub>ANN</sub> | V <sub>DFT</sub> | V <sub>ANN</sub> | Δ V    |
|-----------|---------------------------------|---|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|--------|
| 01        | H <sub>04</sub> C <sub>01</sub> | <i>I</i> 4̄3 <i>m</i>                                 | 4.31             | 4.29             | 4.31             | 4.30             | 4.31             | 4.30             | 12.303           | 12.219           | 0.084  |
| 02        | H <sub>68</sub> C <sub>38</sub> | <i>P</i> 12 <sub>1</sub> ̄1̄                          | 7.74             | 7.79             | 11.04            | 10.67            | 11.38            | 11.84            | 8.862            | 8.994            | -0.133 |
| 03        | H <sub>32</sub> C <sub>20</sub> | <i>P</i> 4̄2 <sub>1</sub> <i>c</i>                    | 6.87             | 7.30             | 6.87             | 7.30             | 9.34             | 9.76             | 8.477            | 9.992            | -1.515 |
| 04        | H <sub>02</sub> C <sub>02</sub> | <i>P</i> 3̄ <i>m</i> 1                                | 2.54             | 1.78             | 2.54             | 1.78             | 5.49             | 5.36             | 7.670            | 1.684            | 5.986  |
| 05        | H <sub>24</sub> C <sub>48</sub> | <i>P</i> 12 <sub>1</sub> /c1                          | 5.10             | 5.00             | 10.35            | 10.48            | 16.53            | 15.99            | 11.289           | 10.990           | 0.299  |
| 06        | H <sub>8</sub> C <sub>12</sub>  | <i>C</i> 12/c1  | 7.41             | 4.22             | 7.41             | 7.76             | 4.15             | 7.76             | 10.489           | 11.121           | -0.632 |
| 07        | H <sub>02</sub> C <sub>02</sub> | <i>C</i> mcm  | 4.12             | 4.21             | 4.12             | 4.21             | 2.47             | 2.41             | 10.357           | 10.608           | -0.251 |
| 08        | H <sub>04</sub> C <sub>04</sub> | <i>I</i> 2 <sub>1</sub> ̄3̄                           | 3.74             | 3.61             | 3.74             | 3.61             | 3.74             | 3.61             | 5.027            | 4.544            | 0.483  |
| group I   |                                 |   |                  |                  |                  |                  |                  |                  |                  |                  |        |
| 09        | C <sub>120</sub>                | <i>P</i> nnm  | 9.12             | 8.86             | 9.61             | 9.47             | 12.98            | 13.32            | 9.666            | 9.073            | 0.593  |
| 10        | C <sub>48</sub>                 | <i>P</i> 2 <sub>1</sub> /c                            | 4.98             | 4.63             | 10.03            | 10.03            | 15.65            | 15.70            | 15.705           | 14.568           | 1.137  |
| 11        | C <sub>80</sub>                 | <i>P</i> 1  | 7.27             | 7.37             | 7.92             | 9.52             | 22.53            | 23.55            | 16.041           | 19.165           | -3.123 |
| 12        | C <sub>80</sub>                 | <i>P</i> 2 <sub>1</sub> 2 <sub>1</sub> 2 <sub>1</sub> | 8.19             | 9.35             | 12.14            | 11.33            | 14.31            | 14.52            | 16.540           | 18.949           | -2.409 |
| 13        | C <sub>28</sub>                 | <i>P</i> 2 <sub>1</sub>                               | 6.09             | 6.19             | 8.65             | 8.54             | 9.42             | 9.46             | 17.611           | 17.583           | 0.028  |
| 14        | C <sub>29</sub>                 | <i>I</i> 4̄3 <i>m</i>                                 | 7.97             | 7.73             | 7.97             | 7.74             | 7.74             | 7.76             | 13.457           | 12.319           | 1.139  |
| group II  |                                 |   |                  |                  |                  |                  |                  |                  |                  |                  |        |
| 15        | C <sub>140</sub>                | <i>C</i> mcm  | 10.52            | 9.87             | 10.52            | 9.87             | 17.62            | 18.00            | 12.627           | 10.700           | 1.927  |
| 16        | C <sub>240</sub>                | <i>P</i> ā3̄   | 14.88            | 14.02            | 14.88            | 14.02            | 14.02            | 14.30            | 13.718           | 11.479           | 2.240  |
| 17        | C <sub>60</sub>                 | <i>F</i> m̄3̄   | 10.46            | 9.91             | 10.46            | 9.91             | 9.915            | 9.92             | 13.500           | 11.504           | 1.995  |
| 18        | C <sub>120</sub>                | <i>P</i> nnm  | 9.11             | 8.97             | 10.74            | 10.00            | 14.41            | 14.65            | 12.697           | 10.772           | 1.924  |
| 19        | C <sub>60</sub>                 | <i>I</i> mm <sub>m</sub>                              | 10.41            | 8.92             | 10.41            | 8.97             | 9.81             | 9.97             | 11.307           | 10.013           | 1.294  |
| 20        | C <sub>60</sub>                 | <i>R</i> ̄3 <i>m</i>                                  | 10.20            | 9.02             | 10.20            | 9.02             | 9.68             | 9.80             | 10.660           | 9.594            | 1.066  |
| 21        | C <sub>120</sub>                | <i>P</i> nnm  | 8.31             | 8.19             | 9.60             | 9.34             | 13.56            | 14.08            | 9.365            | 8.644            | 0.722  |
| group III |                                 |   |                  |                  |                  |                  |                  |                  |                  |                  |        |
| 22        | C <sub>4</sub>                  | <i>P</i> 6 <sub>3</sub> /mmc                          | 2.47             | 2.39             | 2.47             | 2.39             | 8.69             | 8.69             | 11.451           | 10.731           | 0.720  |
| 23        | C <sub>2</sub>                  | <i>R</i> ̄3 <i>m</i>                                  | 4.27             | 2.39             | 4.27             | 2.39             | 4.25             | 4.27             | 10.607           | 9.935            | 0.672  |
| 24        | C <sub>2</sub>                  | <i>F</i> mmm  | 4.53             | 4.16             | 4.18             | 4.16             | 2.39             | 2.47             | 10.524           | 9.860            | 0.665  |
| 25        | C <sub>4</sub>                  | <i>C</i> mme  | 2.47             | 2.39             | 2.47             | 2.39             | 8.03             | 8.03             | 10.590           | 9.919            | 0.671  |
| 26        | C <sub>4</sub>                  | <i>C</i> mme  | 2.47             | 2.39             | 2.47             | 2.39             | 8.15             | 8.15             | 10.752           | 10.070           | 0.682  |
| 27        | C <sub>4</sub>                  | <i>P</i> 3̄ <i>m</i> 1                                | 2.47             | 2.39             | 2.47             | 2.39             | 25.83            | 25.83            | 34.267           | 31.913           | 2.354  |
| 28        | C <sub>12</sub>                 | <i>P</i> 6 <sub>3</sub> /mmc                          | 2.47             | 2.39             | 2.47             | 2.39             | 31.98            | 31.98            | 14.080           | 13.172           | 0.908  |
| 29        | C <sub>2</sub>                  | <i>P</i> 6/mmm  | 2.47             | 2.39             | 2.47             | 2.39             | 3.83             | 3.83             | 10.238           | 9.464            | 0.773  |
| 30        | C <sub>2</sub>                  | <i>C</i> 2/m  | 2.47             | 2.39             | 2.47             | 2.39             | 3.74             | 3.74             | 9.843            | 9.215            | 0.628  |
| 31        | C <sub>8</sub>                  | <i>C</i> mmm  | 3.68             | 3.57             | 3.68             | 3.57             | 6.30             | 6.27             | 10.572           | 9.853            | 0.719  |
| 32        | C <sub>2</sub>                  | <i>C</i> mcm  | 1.62             | 1.55             | 1.62             | 1.55             | 9.88             | 9.88             | 12.900           | 11.925           | 0.975  |
| 33        | C <sub>2</sub>                  | <i>C</i> mcm  | 1.62             | 1.55             | 1.62             | 1.55             | 6.40             | 6.40             | 8.362            | 7.723            | 0.639  |
| group IV  |                                 |   |                  |                  |                  |                  |                  |                  |                  |                  |        |

| ID | system    | spg          | $a_{DFT}$ | $a_{ANN}$ | $b_{DFT}$ | $b_{ANN}$ | $c_{DFT}$ | $c_{ANN}$ | $V_{DFT}$ | $V_{ANN}$ | $\Delta V$ |
|----|-----------|--------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|------------|
| 34 | $C_8$     | $Fd\bar{3}m$ | 3.57      | 3.51      | 3.57      | 3.51      | 3.51      | 3.56      | 5.705     | 5.401     | 0.304      |
| 35 | $C_{16}$  | $P6_3/mmc$   | 2.52      | 2.47      | 2.52      | 2.47      | 16.35     | 16.56     | 5.706     | 5.416     | 0.290      |
| 36 | $C_{12}$  | $P6_3/mmc$   | 2.52      | 2.47      | 2.52      | 2.47      | 12.30     | 12.43     | 5.707     | 5.420     | 0.287      |
| 37 | $C_8$     | $P6_3/mmc$   | 2.52      | 2.47      | 2.52      | 2.47      | 8.24      | 8.31      | 5.711     | 5.428     | 0.283      |
| 38 | $C_4$     | $P6_3/mmc$   | 2.51      | 2.46      | 2.51      | 2.46      | 4.15      | 4.18      | 5.718     | 5.442     | 0.276      |
| 39 | $C_8$     | $Cmmm$       | 4.87      | 4.19      | 4.87      | 4.76      | 2.50      | 2.51      | 5.866     | 5.586     | 0.280      |
| 40 | $C_{16}$  | $Pnma$       | 2.53      | 2.41      | 4.15      | 4.18      | 9.95      | 9.07      | 5.961     | 5.645     | 0.316      |
| 41 | $C_8$     | $C2/m$       | 4.77      | 2.41      | 4.77      | 4.17      | 4.68      | 4.15      | 5.975     | 5.655     | 0.320      |
| 42 | $C_4$     | $I4/mmm$     | 3.34      | 2.52      | 3.34      | 3.23      | 3.28      | 3.34      | 6.024     | 5.770     | 0.254      |
| 43 | $C_4$     | $Cmmm$       | 4.14      | 4.12      | 4.14      | 4.12      | 2.43      | 2.51      | 6.633     | 6.427     | 0.206      |
| 44 | $C_8$     | $Im\bar{3}m$ | 4.22      | 4.06      | 4.22      | 4.06      | 4.06      | 4.22      | 7.250     | 6.431     | 0.818      |
| 45 | $C_{20}$  | $Pm\bar{3}m$ | 5.22      | 5.17      | 5.22      | 5.17      | 5.17      | 5.22      | 7.096     | 6.916     | 0.180      |
| 46 | $C_8$     | $Ia\bar{3}$  | 3.88      | 3.80      | 3.88      | 3.80      | 3.80      | 3.88      | 5.606     | 5.269     | 0.338      |
| 47 | $C_{100}$ | $P1$         | 10.18     | 9.99      | 11.00     | 10.66     | 9.79      | 10.12     | 11.268    | 10.387    | 0.881      |
| 48 | $C_{100}$ | $P1$         | 10.65     | 10.31     | 10.73     | 10.50     | 10.27     | 10.43     | 11.843    | 11.070    | 0.773      |
| 49 | $C_{12}$  | $I4/mmm$     | 5.77      | 5.67      | 5.77      | 5.67      | 5.67      | 5.80      | 12.289    | 11.674    | 0.616      |
| 50 | $C_{100}$ | $P1$         | 10.19     | 9.77      | 10.42     | 10.10     | 10.32     | 19.54     | 11.166    | 10.174    | 0.992      |
| 51 | $C_1$     | $P2/m$       | 3.34      | 3.34      | 1.29      | 1.24      | 3.43      | 3.43      | 14.622    | 14.067    | 0.555      |
| 52 | $C_{20}$  | $I4/mmm$     | 8.96      | 6.69      | 8.96      | 8.79      | 8.80      | 8.96      | 23.364    | 22.131    | 1.233      |
| 53 | $C_4$     | $I4_132$     | 3.57      | 3.53      | 3.57      | 3.53      | 3.53      | 3.57      | 8.783     | 8.467     | 0.317      |

Table S5: Mean percentage error (MPE) (in Å) of lattice constants  $a$ ,  $b$ , and  $c$  in the 3D systems. The values for C-H\* are after excluding the layered system.

|         | C-H   | C-H*   | C-I    | C-II   | C-III  | C-IV   |
|---------|-------|--------|--------|--------|--------|--------|
| $MPE_a$ | 2.751 | -1.130 | -0.720 | 6.606  | 7.131  | 7.359  |
| $MPE_b$ | 2.820 | -1.052 | -1.325 | 7.462  | 6.490  | 2.503  |
| $MPE_c$ | 0.163 | -0.152 | -1.597 | -1.773 | -0.256 | -4.907 |

## 6 Structural and mechanical properties of the novel carbon polymorph

The structure of the discovered carbon polymorph is shown in Figure S32. Details of the structural parameters and atomic positions are summarized in Table S6.

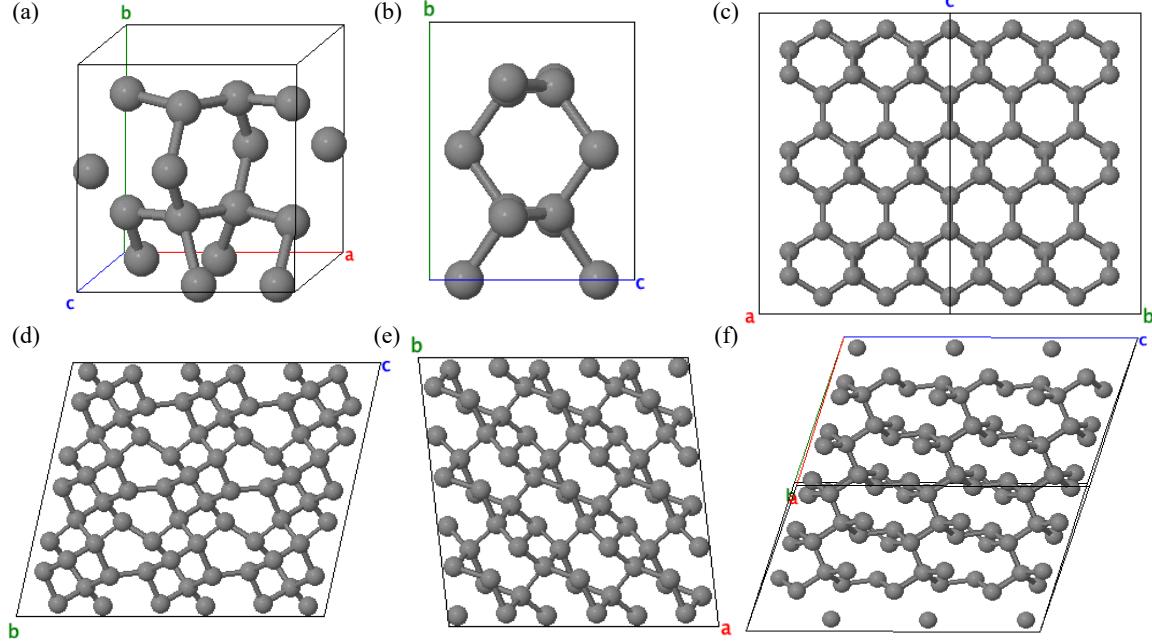


Figure S32: Different views of the unit cell (panels (a) and (b)) and  $3 \times 3 \times 3$  supercell (panels (c)-(f)) of the new carbon polymorph.

Table S6: Lattice vectors  $a$ ,  $b$ , and  $c$  along with the reduced atomic positions of the  $C2/m$  carbon polymorph.

|        | x        | y        | z       |
|--------|----------|----------|---------|
| a ( Å) | 2.36357  | -2.52698 | 0.00000 |
| b ( Å) | 2.36357  | 2.52698  | 0.00000 |
| c ( Å) | -1.21528 | 0.00000  | 3.99645 |
| C1     | 0.52701  | 0.52701  | 0.82729 |
| C2     | 0.11464  | 0.11464  | 0.16998 |
| C3     | 0.33627  | 0.85558  | 0.38327 |
| C4     | 0.14442  | 0.66373  | 0.61673 |
| C5     | 0.85558  | 0.33627  | 0.38327 |
| C6     | 0.66373  | 0.14442  | 0.61673 |
| C7     | 0.88536  | 0.88536  | 0.83002 |
| C8     | 0.47299  | 0.47299  | 0.17271 |

The 13 independent elastic constants are listed in Table S7. As shown in the table, the  $C_{11}$  and  $C_{22}$  are smaller, but comparable with  $C_{11}$  in diamond; however,  $C_{33}$  is close to that one for diamond.

Therefore, the new structure displays strong resistance to compression deformation along the  $x$ -,  $y$ -, and  $z$ -axis. The bulk modulus (B), Young's modulus (E) and shear modulus (G) are also provided in the table. Based on the values, B/G ratio for  $C2/m$  is 0.89 comparable to diamond which is 0.84, smaller than 1.75, indicating the brittleness of  $C2/m$  phase. Based on harmonic approximation,

Table S7: Calculated elastic constants  $C_{ij}$ , bulk modulus B, Young's modulus E, and shear modulus G for the  $C2/m$  carbon polymorph along with other experimental values for diamond. All values are in GPa.

|          | $C2/m$ | Diamond |                      |
|----------|--------|---------|----------------------|
|          |        | Present | Experiments          |
| $C_{11}$ | 1034.3 | 1050.0  | 1074.8 [1], 1079 [2] |
| $C_{12}$ | 27.6   | 127.4   | 125.6 [1], 124 [2]   |
| $C_{13}$ | 86.0   |         |                      |
| $C_{15}$ | -3.0   |         |                      |
| $C_{22}$ | 1023.3 |         |                      |
| $C_{23}$ | 131.8  |         |                      |
| $C_{25}$ | -17.3  |         |                      |
| $C_{33}$ | 1074.9 |         |                      |
| $C_{35}$ | -3.5   |         |                      |
| $C_{44}$ | 505.1  | 559.3   | 720.6 [1], 578 [2]   |
| $C_{46}$ | -15.7  |         |                      |
| $C_{55}$ | 440.8  |         |                      |
| $C_{66}$ | 372.2  |         |                      |
| B        | 401.8  | 434.9   | 442 [1, 2]           |
| E        | 988.1  | 1112.1  | 1048.5 [1]           |
| G        | 453.2  | 517.8   | 609.5 [1], 535 [2]   |

its molar heat capacity ( $C_v$ ) at room temperature (300 K) is determined to be 6.94 J/mol/K. This value is comparable to the heat capacities of diamond (6.40 J/mol/K) and mp-47 (6.43 J/mol/K), as shown in Figure S33. As the last considered properties, we calculated the electronic structure of this polymorph. It was found that the structure is insulator with an indirect band gap of 5.06 eV. The calculated electronic band structure is shown in Figure S32.

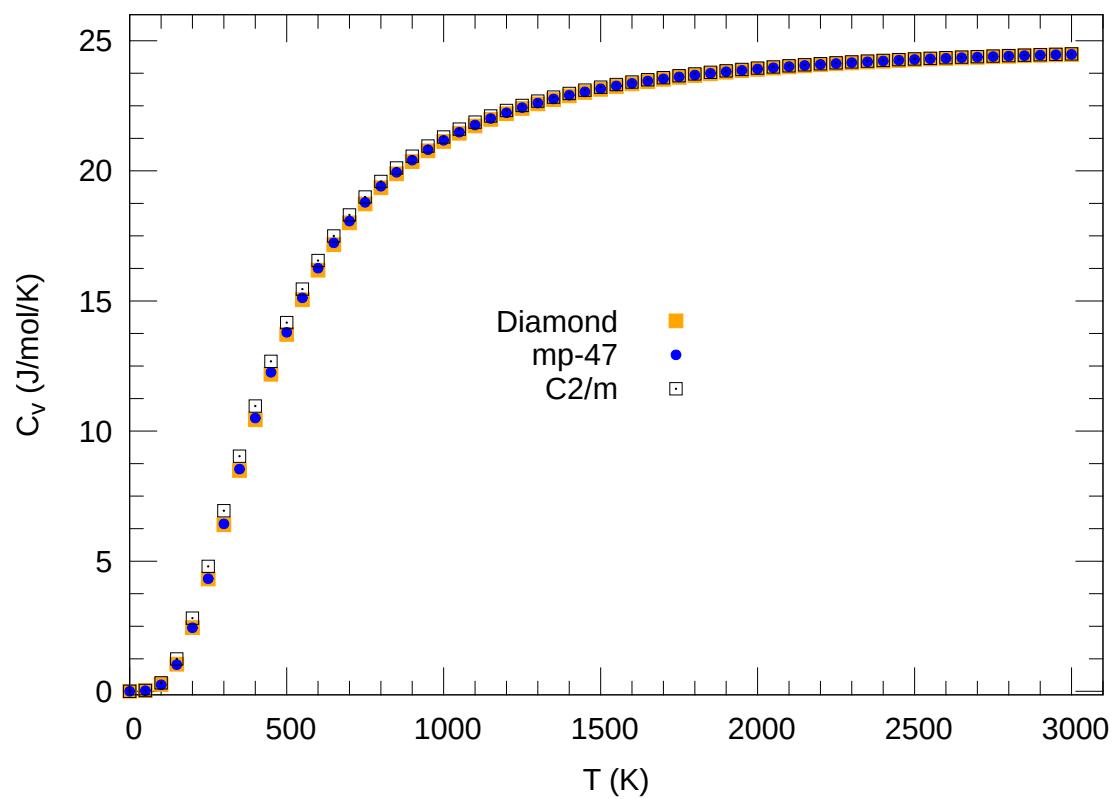


Figure S33: The molar heat capacity of the new carbon polymorph along with diamond and two other reported structures.

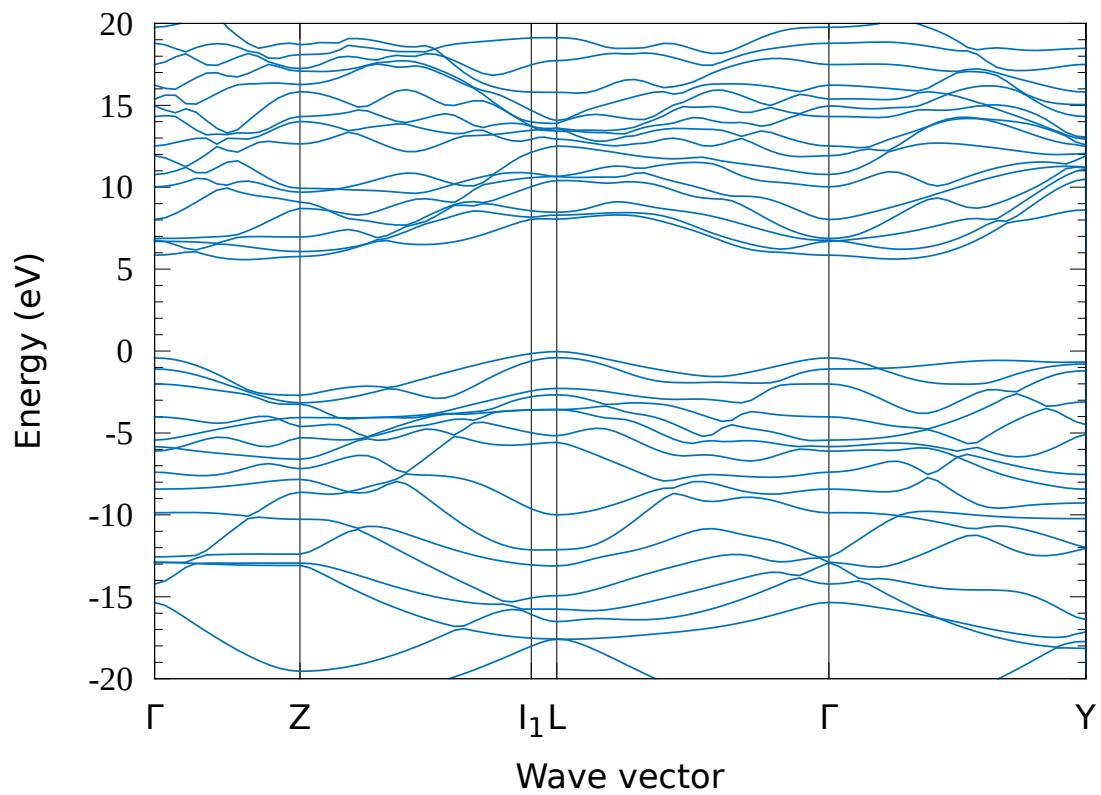


Figure S34: The electronic band structure of the new carbon polymorph with symmetry  $C2/m$ . The Fermi energy is set to zero.

## References

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