Supplementary Information for

Prediction of high-temperature superconductors at ambient

pressure with diamond-like structures: $B_2CX(X = N, P)$

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I. SUPPLEMENTAL FORMULAS

The total EPC constant λ is obtained via isotropic Eliashberg function ¹⁻³

$$\alpha^{2}F(\omega) = \frac{1}{2\pi N(E_{F})} \sum_{qv} \delta(\omega - \omega_{qv}) \frac{\gamma_{qv}}{\omega_{qv}},$$
(S1)

$$\lambda = 2 \int \frac{\alpha^2 F(\omega)}{\omega} d\omega = \sum_{qv} \lambda_{qv}, \qquad (S2)$$

where $\alpha^2 F(\omega)$ is Eliashberg function and $N(E_F)$ is the DOS at the Fermi level, ω_{qv} is the phonon frequency of the vth phonon mode with wave vector q, and γ_{qv} is the phonon linewidth ¹⁻³. The γ_{qv} can be estimated by

$$\gamma_{qv} = \frac{2\pi\omega_{qv}}{\Omega_{\rm BZ}} \sum_{k,n,m} \left| g_{kn,k+qm}^{v} \right|^2 \delta(\varepsilon_{kn} - E_F) \delta(\varepsilon_{k+qm} - E_F), \tag{S3}$$

where Ω_{BZ} is the volume of the BZ, ε_{kn} and ε_{k+qm} indicate the Kohn-Sham energy, and $g_{kn,k+qm}^{v}$ represents the screened electron-phonon matrix element. λ_{qv} is the EPC constant for phonon mode qv, which is defined as

$$\lambda_{qv} = \frac{\gamma_{qv}}{\pi \hbar N(E_F)\omega_{qv}^2}.$$
(S4)

 T_c is estimated by McMillan-Allen-Dynes formula ³

$$T_{c} = \frac{\omega_{log}}{1.2} exp \left[\frac{-1.04(1+\lambda)}{\lambda - \mu^{*}(1+0.62\lambda)} \right].$$
 (S5)

The Coulomb repulsion μ^* in Eq.(S5) is set to 0.10 and the logarithmic average of the phonon frequencies ω_{log} is defined as

$$\omega_{log} = exp \left[\frac{2}{\lambda} \int \alpha^2 F(\omega) \frac{\ln \omega}{\omega} d\omega \right].$$
 (S6)

Furthermore, to estimate the thermodynamic properties such as critical magnetic field $H_c(0)$, superconducting gap $\Delta(0)$, specific heat jump $\Delta C(T_c)$ and isotope coefficient β , we used the following semiempirical formulas ⁴

$$\frac{\gamma T_c^2}{H_c^2(0)} = 0.168 \left[1 - 12.2 \left(\frac{T_c}{\omega_{log}} \right)^2 \ln \left(\frac{\omega_{log}}{3T_c} \right) \right],\tag{S7}$$

$$\frac{2\Delta(0)}{k_B T_c} = 3.53 \left[1 + 12.5 \left(\frac{T_c}{\omega_{log}} \right)^2 \ln \left(\frac{\omega_{log}}{2T_c} \right) \right],\tag{S8}$$

$$\frac{\Delta C(T_c)}{\gamma T_c} = 1.43 \left[1 + 53 \left(\frac{T_c}{\omega_{log}} \right)^2 \ln \left(\frac{\omega_{log}}{3T_c} \right) \right],\tag{S9}$$

$$\beta = \frac{1}{2} \left[1 - \frac{1.04(1+\lambda)(1+0.62\lambda)}{[\lambda - \mu^*(1+0.62\lambda)]^2} \mu^{*2} \right],$$
(S10)

here k_B is the Boltzmann constant and γ is the Sommerfeld constant, which can be obtained by

$$\gamma = \frac{2}{3}\pi^2 k_B^2 N(E_F)(1+\lambda).$$
 (S11)

II. SUPPLEMENTAL FIGURES



Fig. S1. Phonon spectra of (a) d-B₂CN, (b) d-B₂CP-I and (c) d-B₂CP-II.



Fig. S2. AIMD simulations of (a) d-B₂CN, (b) d-B₂CP-I and (c) d-B₂CP-II at 300 K.



Fig. S3. Orbital-projected band structures of (a) B, (b) C and (c) P for d-B₂CP-II along the high-symmetry line Γ -A-L- Γ -M-K-H. (d) Total DOS of d-B₂CP-II, B, C and P.



Fig. S4. (a) Phonon dispersion weighted by the magnitude of EPC λ_{qv} , (b) PhDOS, (c) Eliashberg spectral function $\alpha^2 F(\omega)$ and cumulative frequency dependence of EPC $\lambda(\omega)$ for d-B₂CP-II. (d)-(f) Its VMs for the prominent λ_{qv} I, II and III, respectively.



Fig. S5. Critical temperature T_c as a function of Coulomb pseudopotential μ^* for *d*-B₂CN, *d*-B₂CP-I and *d*-B₂CP-II. The vertical line marks the value $\mu^* = 0.10$ used in this work.

Notes and references

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