



Supporting Information for:

**Multimodal nanoparticle analysis enabled by a polymer electrolyte
nanopore combined with nanoimpact electrochemistry**

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Below is a COMSOL-generated 'Model Report' representing the measurements where the bath contains 20 mM KCl (no PEG) and the nanopipette 20 mM KCl with 25% 35K-PEG. The PEG / KCl interface is set at 8 μm above the pore opening, with 25% PEG + 20mM KCl and 20mM KCl conditions in the nanopipette and external bath respectively. This model was used to create the data in Figures 2 and 3 in the main text and Figures S6, S7, S8, and S10 in the supporting information. Other variants of this model, which were generated by changing the conditions in the nanopipette and bath domains, were used to generate data for other nanopore measurements. The information presented below included all the boundary and subdomain conditions, mesh settings, solver settings, etc., necessary to recreate this model.

Glass nanopore

| | |
|-------------|--------------------------|
| Report date | Jun 14, 2024, 3:41:57 PM |
|-------------|--------------------------|

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1 Global Definitions

| | |
|------|---------------------------|
| Date | Jun 14, 2024, 11:54:39 AM |
|------|---------------------------|

GLOBAL SETTINGS

| | |
|---------|---|
| Name | ModelReportPEG,KCl.mph |
| Path | /scrfs/storage/egyasiag/home/ModelReportPEG,KCl.mph |
| Version | COMSOL Multiphysics 6.0 (Build: 354) |

USED PRODUCTS

| |
|--------------------------------------|
| COMSOL Multiphysics |
| Chemical Reaction Engineering Module |

COMPUTER INFORMATION

| | |
|------------------|---|
| CPU | Intel(R) Xeon(R) Gold 6130 CPU @ 2.10GHz, 2 sockets |
| Operating system | Linux |

1.1 PARAMETERS

PARAMETERS

| Name | Expression | Value | Description |
|------------------|--|-----------------------|---|
| BathConductivity | 3.86[mS/cm] | 0.386 S/m | Experimentally measured conductivity of the bath medium |
| BeyondPoreRadius | PoreRadius*200*SizeFactor | 1.42E-4 m | Radius for domain representing bath solution |
| PoreConductivity | 1.19[mS/cm] | 0.119 S/m | Experimentally measured conductivity of the pipette fill solution |
| cPore | 20[mM] | 20 mol/m ³ | Concentration of the pipette fill solution |
| cbath | 20[mM] | 20 mol/m ³ | Bath concentration |
| ConeApex | - PoreHeight*PoreRadius/(Pore TopRadius - PoreRadius) | -8.3062E-7 m | z value where the apex of the inner cone (if continued) would lie |

| Name | Expression | Value | Description |
|-----------------|---|------------------------------|--|
| DCIPore | $0.55 * \text{PoreConductivity} * (R_{\text{const}} * T) / (F_{\text{const}}^2 * c_{\text{bath}})$ | 8.568E-10 m ² /s | Estimated diffusion coefficient of anions in pore (25% 35K PEG) |
| DCIBath | $0.51 * \text{BathConductivity} * (R_{\text{const}} * T) / (F_{\text{const}}^2 * c_{\text{Pore}})$ | 2.5771E-9 m ² /s | CRC Handbook, 97th ed. Haynes, W. M., Ed CRC Press, 2016 |
| DebyeLengthPore | $\text{sqrt}((\text{EpsilonRPore} * \text{epsilon0_const} * R_{\text{const}} * T) / (2 * c_{\text{Pore}} * F_{\text{const}}^2))$ | 1.736E-9 m | Debye Length (pore) |
| DebyeLengthBath | $\text{sqrt}((\text{EpsilonRBath} * \text{epsilon0_const} * R_{\text{const}} * T) / (2 * c_{\text{bath}} * F_{\text{const}}^2))$ | 2.1532E-9 m | Debye Length (bath) |
| Delta | PoreRadius/50 | 1.42E-9 m | Used for selecting geometric boundaries |
| DKPore | $0.45 * \text{PoreConductivity} * (R_{\text{const}} * T) / (F_{\text{const}}^2 * c_{\text{bath}})$ | 7.0102E-10 m ² /s | Estimated diffusion coefficient of anions in pore (25% PEG) |
| DKBath | $0.49 * \text{BathConductivity} * (R_{\text{const}} * T) / (F_{\text{const}}^2 * c_{\text{Pore}})$ | 2.476E-9 m ² /s | CRC Handbook, 97th ed. Haynes, W. M., Ed CRC Press, 2016 |
| EpsilonRPore | 52 | 52 | Koizum, N.; Hanai, T. Dielectric Properties of Lower-Membered Polyethylene Glycols at Low Frequencies. J. Phys. Chem. 1956 |
| EpsilonRBath | 80 | 80 | CRC Handbook, 97th ed. Haynes, W. M., Ed CRC Press, 2016 |
| gammaPore | $(\exp(1 * e_{\text{const}} * \text{GCSurfacePotentialPore} / (2 * k_{\text{B_const}} * T)) - 1) / (\exp(1 * e_{\text{const}} * \text{GCSurfacePotential} / (2 * k_{\text{B_const}} * T)) + 1)$ | -0.074217 | Helper for Gouy-Chapman (pore) |

| Name | Expression | Value | Description |
|------------------------|---|-------------------------|--|
| gammaBath | $(\exp(1 * e_{\text{const}} * GCSurfacePotentialBath / (2 * k_B_{\text{const}} * T)) - 1) / (\exp(1 * e_{\text{const}} * GCSurfacePotentialBath / (2 * k_B_{\text{const}} * T)) + 1)$ | -0.059951 | Helper for Gouy-Chapman (bath) |
| GCSurfacePotentialPore | $\text{asinh}(\text{PoreSurfaceCharge} / \sqrt{8 * c_{\text{Pore}} * N_A_{\text{const}} * \epsilon_{\text{R}} * \text{Pore} * \epsilon_{0_{\text{const}}} * k_B_{\text{const}} * T}) * (2 * k_B_{\text{const}} * T) / (1 * e_{\text{const}})$ | -0.0075132 V | Surface potential difference from Gouy-Chapman (pore) |
| GCSurfacePotentialBath | $\text{asinh}(\text{PoreSurfaceCharge} / \sqrt{8 * c_{\text{bath}} * N_A_{\text{const}} * \epsilon_{\text{R}} * \text{Bath} * \epsilon_{0_{\text{const}}} * k_B_{\text{const}} * T}) * (2 * k_B_{\text{const}} * T) / (1 * e_{\text{const}})$ | -0.0060651 V | Surface potential difference from Gouy-Chapman (bath) |
| PoreAngle | 4.885666895[deg] | 0.085271 rad | Inner truncated hollow cone half angle; |
| PoreHeight | 50[um]*SizeFactor | 5E-4 m | Truncated hollow cone's height in model; measured along the axis taking the aperture as flat |
| PoreRadius | 71[nm] | 7.1E-8 m | Inner cone aperture radius; measured from SEM imaging |
| PoreSurfaceCharge | -2[mC/m^2] | -0.002 C/m ² | Surface charge of quartz glass wall, measured by best fit to experiment |
| PoreTopRadius | $\text{PoreRadius} + \tan(\text{PoreAngle}) * \text{PoreHeight}$ | 4.281E-5 m | Estimated radius of the inner cone at z = PoreHeight |
| RAccessBottom | $1 / (4 * \text{BathConductivity} * \text{PoreRadius})$ | 9.1221E6 Ω | Analytical expression for access resistance (assuming infinite RG and uniform concentration) |
| RadiusRatio | $\text{PoreRadius} / \text{PoreTopRadius}$ | 0.0016585 | Ratio between the inner cone radii at the nanopipette |

| Name | Expression | Value | Description |
|---------------|--|-------------------|--|
| | | | opening and at z = PoreHeight |
| RPore | $1/(\text{PoreConductivity} \cdot \pi \cdot \text{PoreRadius} \cdot \tan(\text{PoreAngle}))$ | 4.4075E8 Ω | Analytical resistance inside the pore (assuming uniform conductivity PEG throughout) |
| RTot | RAccessBottom + RPore | 4.4987E8 Ω | Total Resistance |
| SizeFactor | 10 | 10 | Scaling factor for selected boundaries dimensions |
| T | 20[degC] | 293.15 K | Temperature |
| VApp | 0.5 [V] | 0.5 V | Applied voltage |
| VBottom | $V_{\text{App}} \cdot R_{\text{AccessBottom}} / R_{\text{Tot}}$ | 0.010139 V | Potential drop outside the pore |
| WallAngle | $\text{atan}((\text{WallTopRadius} - \text{WallRadius}) / \text{PoreHeight})$ | 0.11507 rad | Estimated half cone angle of the outer cone (glass wall) |
| WallRadius | PoreRadius + WallThickness | 9.6E-8 m | Outer cone (glass wall) radius at z = 0 |
| WallThickness | 25[nm] | 2.5E-8 m | Glass thickness at the nanopipette opening; from SEM |
| WallTopRadius | WallRadius/RadiusRatio | 5.7884E-5 m | Outer cone (glass wall) radius at z = PoreHeight |
| Z_int | 8 [um] | 8E-6 m | Interface vertical distance within the nanopipette |

2 Model 1

SETTINGS

| Description | Value |
|---|----------------------------|
| Unit system | Same as global system (SI) |
| Avoid inverted elements by curving interior domain elements | Off |

2.1 DEFINITIONS

2.1.1 Variables

Excess Charge

SELECTION

| | |
|------------------------|--|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domains 1–4 |

| Name | Expression | Unit | Description |
|--------------|-----------------------|------------------|-------------|
| ExcessCharge | $F_const*(cK - cCl)$ | C/m ³ | |

Distance from glass (internal electrode)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Name | Internal Electrode |
| Selection | Named sel3: Geometry geom1: Dimension 1: Boundary 6 |

| Name | Expression | Unit | Description |
|--------|--|------|-------------|
| dGlass | $(PoreTopRadius - r)*\sin(90[\text{deg}] - PoreAngle)$ | m | |

Distance from glass (external electrode, upper part)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: Boundaries 13–14 |

| Name | Expression | Unit | Description |
|--------|---|------|-------------|
| dGlass | $\cos(\text{WallAngle})*(r - \text{WallRadius} - z*\tan(\text{WallAngle}))$ | m | |

Distance from glass (external electrode, bottom part)

SELECTION

| | |
|------------------------|--|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: Boundary 12 |

| Name | Expression | Unit | Description |
|--------|--|------|-------------|
| dGlass | $(r < \text{PoreRadius}) * (\text{sqrt}((r - \text{PoreRadius})^2 + z^2)) + (-z) * (r \geq \text{PoreRadius}) * (r < \text{WallRadius}) + (r \geq \text{WallRadius}) * (\text{sqrt}((r - \text{WallRadius})^2 + z^2))$ | m | |

Analytical Potential, No Charge (Bath)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domains 1, 3-4 |

| Name | Expression | Unit | Description |
|-----------------------------|-------------------------------------|------|-------------|
| AnalyticalPotentialNoCharge | DiskElec(r, -z, PoreRadius)*VBottom | V | |

Analytical Potential, No Charge (Pore)

SELECTION

| | |
|------------------------|---------------------------------------|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domain 2 |

| Name | Expression | Unit | Description |
|-----------------------------|---|------|-------------|
| AnalyticalPotentialNoCharge | $V_{\text{Bottom}} + (V_{\text{App}} - V_{\text{Bottom}}) * 1 / (\text{PoreConductivity} * \pi) * z / (\text{PoreRadius} * (\text{PoreRadius} + z / \text{PoreHeight} * (\text{PoreTopRadius} - \text{PoreRadius}))) / R_{\text{Pore}}$ | V | |

Domain Expressions Gouy-Chapman

SELECTION

| | |
|------------------------|--------------|
| Geometric entity level | Entire model |
|------------------------|--------------|

| Name | Expression | Unit | Description |
|------------------|----------------------------|--------------------|-------------|
| cCl_GC | c_GC(WallDistance, -1) | mol/m ³ | |
| cK_GC | c_GC(WallDistance, 1) | mol/m ³ | |
| Potential_GC | E_GC(WallDistance) | V | |
| cCl_GCBath | c_GCBath(WallDistance, -1) | mol/m ³ | |
| cK_GCBath | c_GCBath(WallDistance, 1) | mol/m ³ | |
| Potential_GCBath | E_GCBath(WallDistance) | V | |

Wall Distance (Pore)

SELECTION

| | |
|------------------------|--|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domains 2–3 |

| Name | Expression | Unit | Description |
|--------------|--|------|--|
| lambda | $(r * \text{PoreRadius} - z * \text{ConeApex} + \text{ConeApex}^2) / (\text{PoreRadius}^2 + \text{ConeApex}^2)$ | | helper value for point of nearest wall |
| WallDistance | $(\text{lambda} \geq 1) * \sqrt{(r - \text{lambda} * \text{PoreRadius})^2 + (z - (\text{ConeApex} - \text{lambda} * \text{ConeApex}))^2} + (\text{lambda} < 1) * \sqrt{(r - \text{PoreRadius})^2 + z^2}$ | m | |

Wall Distance (Bath, bottom part)

SELECTION

| | |
|------------------------|---------------------------------------|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domain 1 |

| Name | Expression | Unit | Description |
|--------------|--|------|-------------|
| WallDistance | $(r < \text{PoreRadius}) * \sqrt{(r - \text{PoreRadius})^2 + z^2} + (-z) * (r \geq \text{PoreRadius}) * (r < \text{WallRadius}) + (r \geq \text{WallRadius}) * \sqrt{(r - \text{WallRadius})^2 + z^2}$ | m | |

Wall Distance (Bath, upper part)

SELECTION

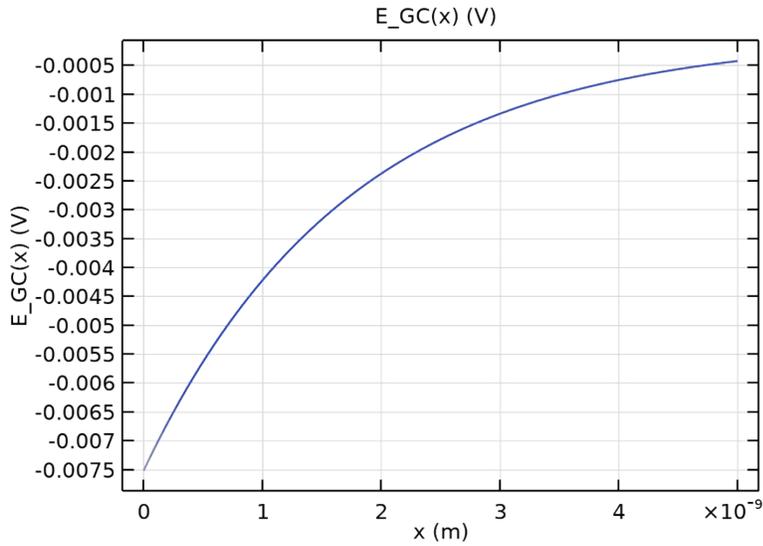
| | |
|------------------------|---------------------------------------|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domain 4 |

| Name | Expression | Unit | Description |
|--------------|---|------|-------------|
| WallDistance | $\cos(\text{WallAngle}) * (r - \text{WallRadius} - z * \tan(\text{WallAngle}))$ | m | |

2.1.2 Functions

Gouy-Chapman Potential (Pore)

| | |
|---------------|----------|
| Function name | E_GC |
| Function type | Analytic |



Gouy-Chapman Potential (Pore)

DEFINITION

| Description | Value |
|-------------|---|
| Expression | $(2 * k_B * const * T / e_{const}) * \log((1 + \gamma_{pore} * \exp(-x / DebyeLengthPore)) / (1 - \gamma_{pore} * \exp(-x / DebyeLengthPore)))$ |
| Arguments | x |

UNITS

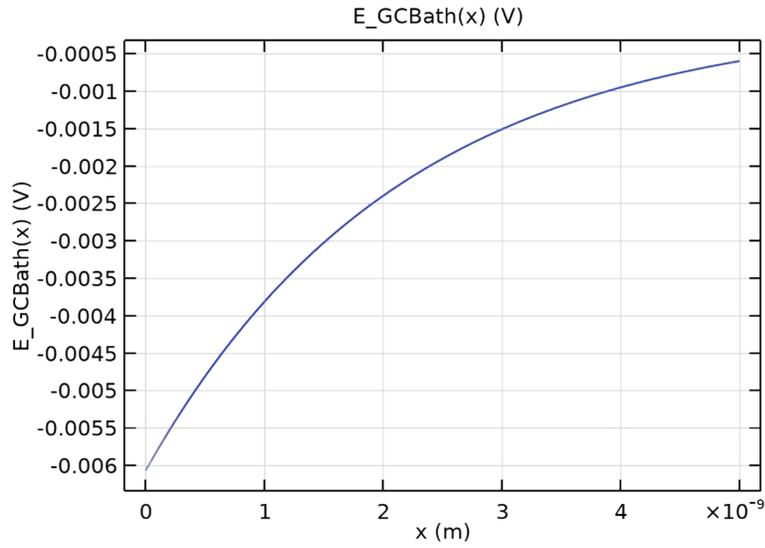
| Description | Value |
|-------------|-------|
| Function | V |

UNITS

| Argument | Unit |
|----------|------|
| x | m |

Gouy-Chapman Potential (Bath)

| | |
|---------------|----------|
| Function name | E_GCBath |
| Function type | Analytic |



Gouy-Chapman Potential (Bath)

DEFINITION

| Description | Value |
|-------------|---|
| Expression | $\frac{(2 * k_B * const * T / e_{const}) * \log(1 + \gamma_{Bath} * \exp(-x / DebyeLength_{Bath}))}{(1 - \gamma_{Bath} * \exp(-x / DebyeLength_{Bath}))}$ |
| Arguments | x |

UNITS

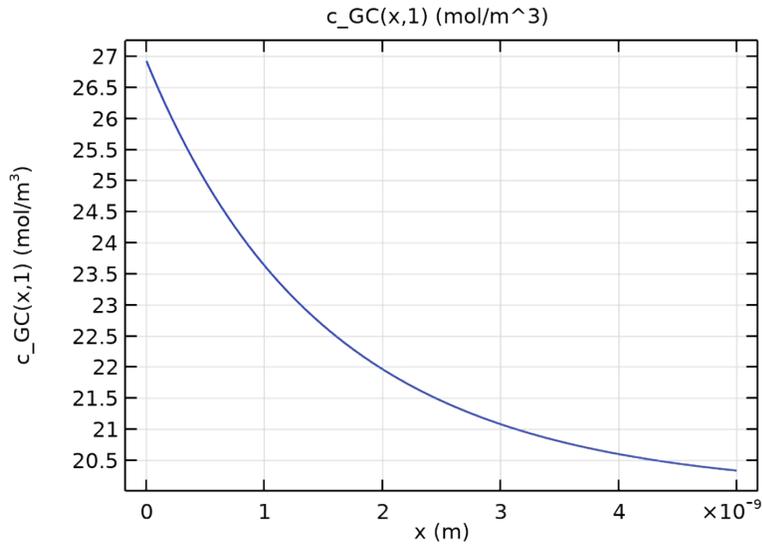
| Description | Value |
|-------------|-------|
| Function | V |

UNITS

| Argument | Unit |
|----------|------|
| x | m |

Gouy-Chapman Concentration (Pore)

| | |
|---------------|----------|
| Function name | c_GC |
| Function type | Analytic |



Gouy-Chapman Concentration (Pore)

DEFINITION

| Description | Value |
|-------------|--|
| Expression | $c_{\text{Pore}} \cdot \exp(-z \cdot e_{\text{const}} \cdot E_{\text{GC}}(x) / (k_{\text{B_const}} \cdot T))$ |
| Arguments | {x, z} |

UNITS

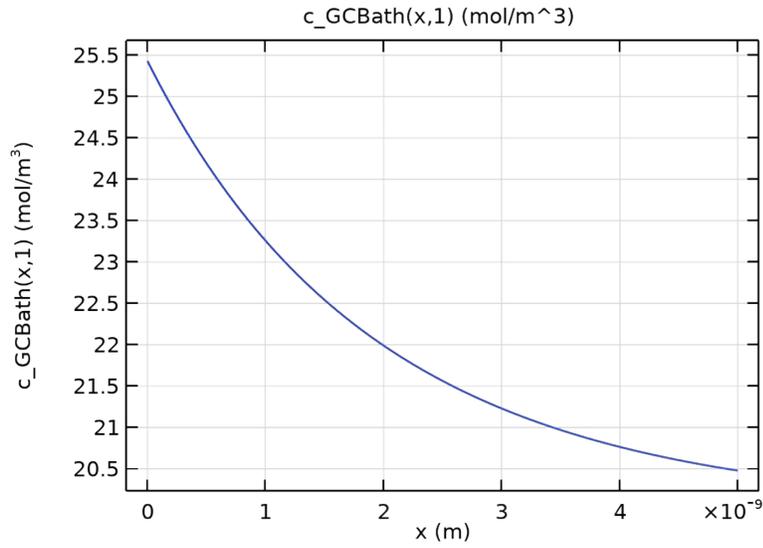
| Description | Value |
|-------------|--------------------|
| Function | mol/m ³ |

UNITS

| Argument | Unit |
|----------|------|
| x | m |
| z | 1 |

Gouy-Chapman Concentration (Bath)

| | |
|---------------|----------|
| Function name | c_GCBath |
| Function type | Analytic |



Gouy-Chapman Concentration (Bath)

DEFINITION

| Description | Value |
|-------------|--|
| Expression | $cbath \cdot \exp(-z \cdot e_{\text{const}} \cdot E_{\text{GCBath}}(x) / (k_{\text{B_const}} \cdot T))$ |
| Arguments | {x, z} |

UNITS

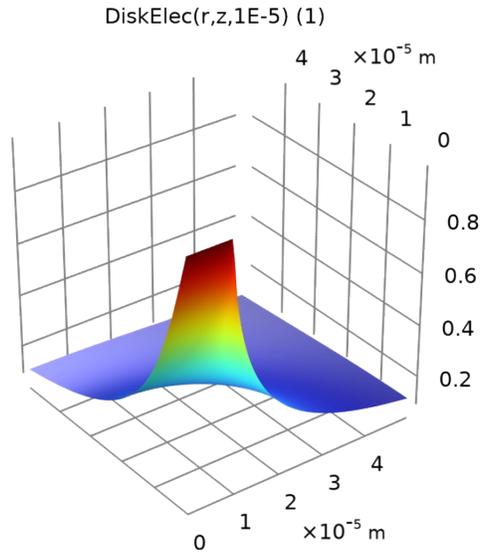
| Description | Value |
|-------------|--------------------|
| Function | mol/m ³ |

UNITS

| Argument | Unit |
|----------|------|
| x | m |
| z | 1 |

Disk Electrode (Normalized Distribution)

| | |
|---------------|----------|
| Function name | DiskElec |
| Function type | Analytic |



Disk Electrode (Normalized Distribution)

DEFINITION

| Description | Value |
|-------------|---|
| Expression | $2/\pi * \text{asin}(2*a/(\sqrt{(z + \text{eps})^2 + (a + \text{eps} + r)^2}) + \sqrt{(z + \text{eps})^2 + (a + \text{eps} - r)^2}))$ |
| Arguments | {r, z, a} |

UNITS

| Description | Value |
|-------------|-------|
| Function | 1 |

UNITS

| Argument | Unit |
|----------|------|
| r | m |
| z | m |
| a | m |

2.1.3 Selections

2.1.3.1 Symmetry Axis

| Selection type |
|----------------|
| Box |

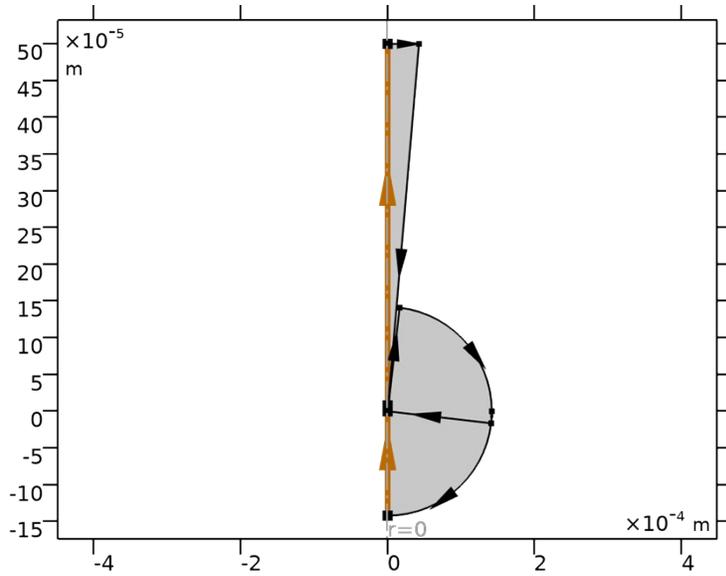
| Selection |
|-------------------|
| Boundaries 1–2, 4 |

GEOMETRIC ENTITY LEVEL

| Description | Value |
|-------------|----------|
| Level | Boundary |

OUTPUT ENTITIES

| Description | Value |
|-------------------|-------------------------|
| Include entity if | All vertices inside box |

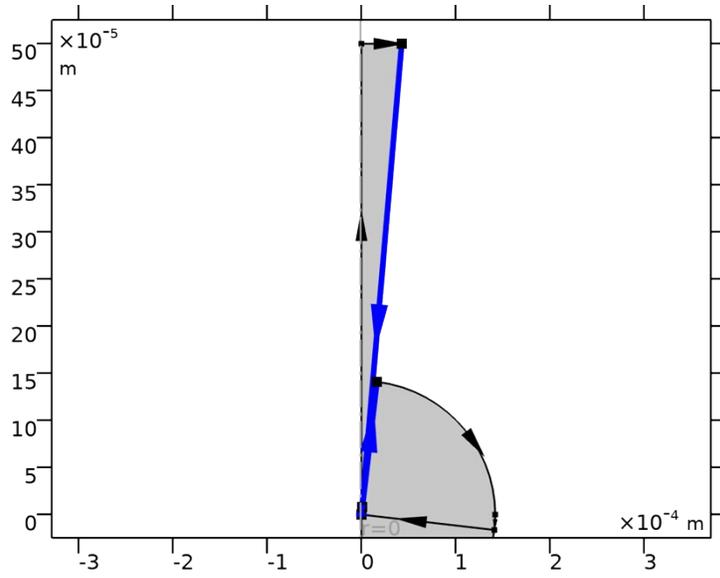


Symmetry Axis

2.1.3.2 Glass Wall

| Selection type |
|----------------|
| Explicit |

| Selection |
|--------------------|
| Boundaries 7–9, 11 |



Glass Wall

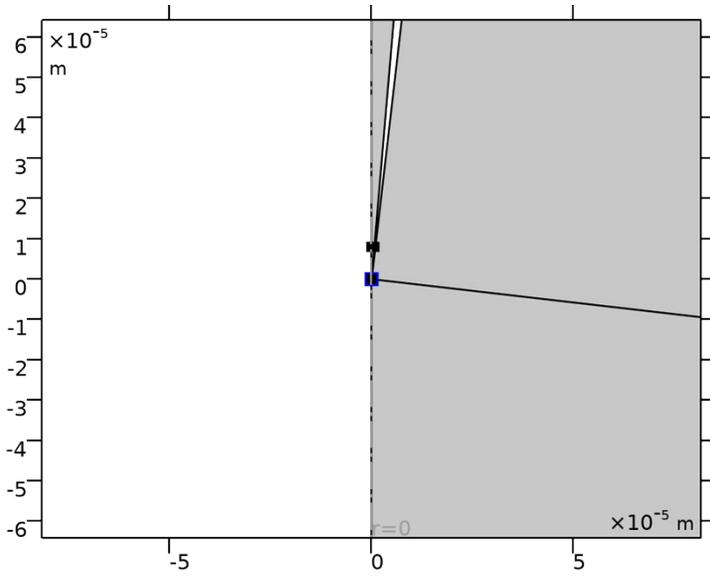
2.1.3.3 Aperture point

Selection type

Explicit

Selection

Point 5



Aperture point

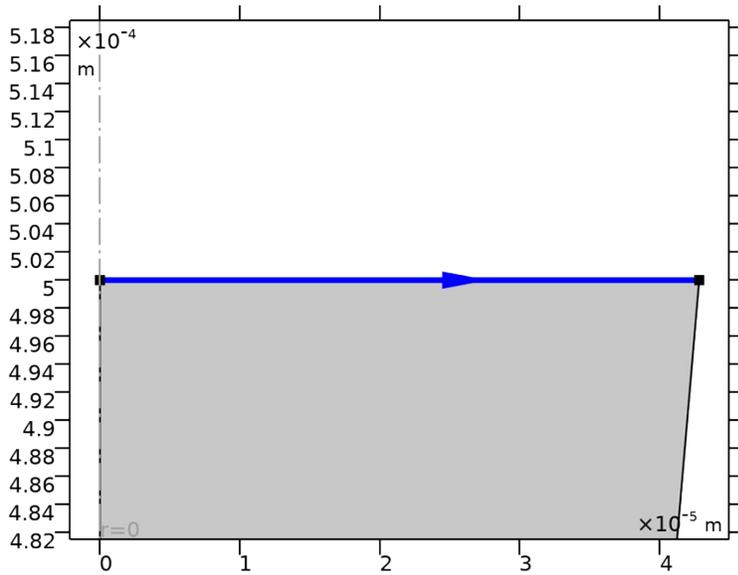
2.1.3.4 Internal Electrode

Selection type

Explicit

Selection

Boundary 6



Internal Electrode

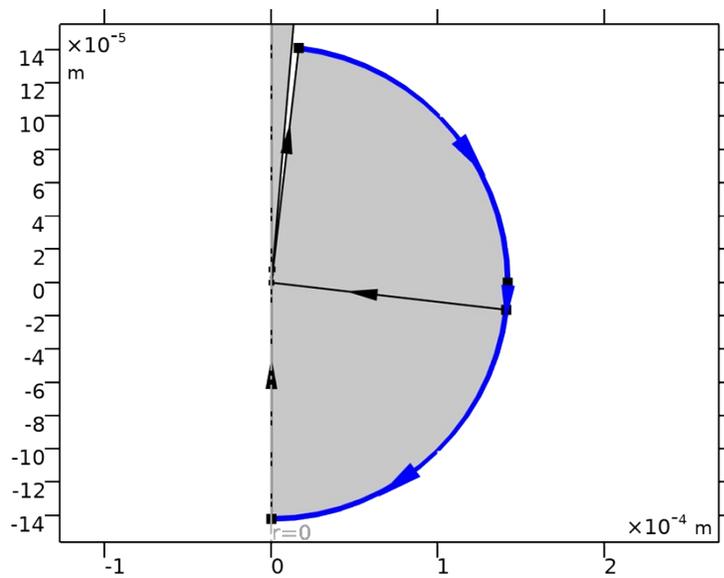
2.1.3.5 External Electrode

Selection type

Explicit

Selection

Boundaries 12–14



External Electrode

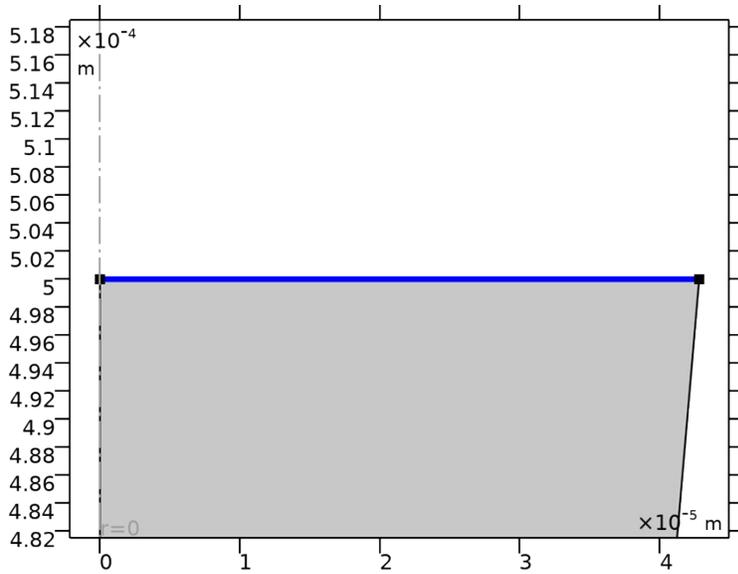
2.1.4 Probes

Current (internal electrode)

| | |
|------------|----------------|
| Probe type | Boundary probe |
|------------|----------------|

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: Boundary 6 |



Selection

PROBE TYPE

| Description | Value |
|-------------|----------|
| Type | Integral |

EXPRESSION

| Description | Value |
|---------------------|--|
| Expression | $-F_{\text{const}} * (\text{chds.bndFlux}_{\text{cK}} - \text{chds.bndFlux}_{\text{cCl}}) * 2 * \pi * r$ |
| Table and plot unit | nA |
| Description | Current (internal) |

TABLE AND WINDOW SETTINGS

| Description | Value |
|--------------|-------------------------------|
| Output table | Probe Table 1 |
| Plot window | Probe Plot 3 |

2.1.5 Coordinate Systems

Boundary System 1

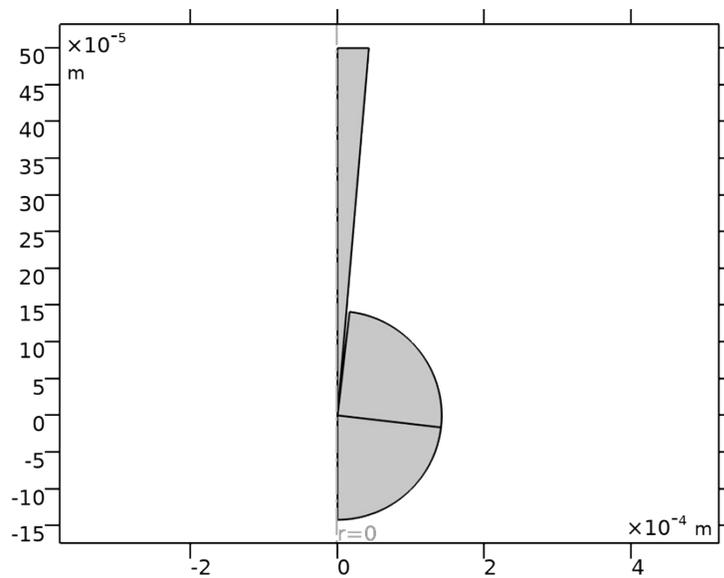
| | |
|------------------------|-----------------|
| Coordinate system type | Boundary system |
| Tag | sys1 |

COORDINATE NAMES

| First | Second | Third |
|-------|--------|-------|
| t1 | to | n |

2.2 NANOPIPETTE

Pore with glass of infinite width



Nanopipette

UNITS

| | |
|--------------|-----|
| Length unit | m |
| Angular unit | deg |

GEOMETRY STATISTICS

| Description | Value |
|----------------------|-------|
| Space dimension | 2 |
| Number of domains | 4 |
| Number of boundaries | 14 |
| Number of vertices | 11 |

2.2.1 Pore (b5)

POLYGON SEGMENTS

| Description | Value |
|----------------|---|
| Control points | {{0, 0, PoreTopRadius, PoreRadius, 0}, {0, PoreHeight, PoreHeight, 0, 0}} |
| Degree | {1, 1, 1, 1} |
| Weights | {1, 1, 1, 1, 1, 1, 1, 1} |
| Type | Solid |

2.2.2 Wall (pol1)

OBJECT TYPE

| Description | Value |
|-------------|------------|
| Type | Open curve |

COORDINATES

| Description | Value |
|-------------|-------|
| Data source | Table |

COORDINATES

| r (m) | z (m) |
|---------------|------------|
| PoreRadius | 0 |
| WallRadius | 0 |
| WallTopRadius | PoreHeight |

2.2.3 Bath (c1)

SELECTIONS OF RESULTING ENTITIES

| Description | Value |
|-----------------------------|-------|
| Resulting objects selection | On |

POSITION

| Description | Value |
|-------------|--------|
| Position | {0, 0} |

ROTATION ANGLE

| Description | Value |
|-------------|-------|
| Rotation | -90 |

SIZE AND SHAPE

| Description | Value |
|--------------|------------------|
| Radius | BeyondPoreRadius |
| Sector angle | 180 |

2.2.4 Difference Bath from Pipette & Wall (dif1)

SETTINGS

| Description | Value |
|--------------------------|-------|
| Keep objects to add | On |
| Keep objects to subtract | On |

2.2.5 Rectangle for dGlass (r1)

POSITION

| Description | Value |
|-------------|-----------------|
| Position | {WallRadius, 0} |

ROTATION ANGLE

| Description | Value |
|-------------|------------|
| Rotation | -WallAngle |

SIZE

| Description | Value |
|-------------|------------------|
| Width | BeyondPoreRadius |
| Height | BeyondPoreRadius |

2.2.6 Z_int Point 1 (pt1)

POINT

| Description | Value |
|------------------|-------------|
| Point coordinate | {0, 8.0E-6} |

2.2.7 Z_int Point 2 (pt2)

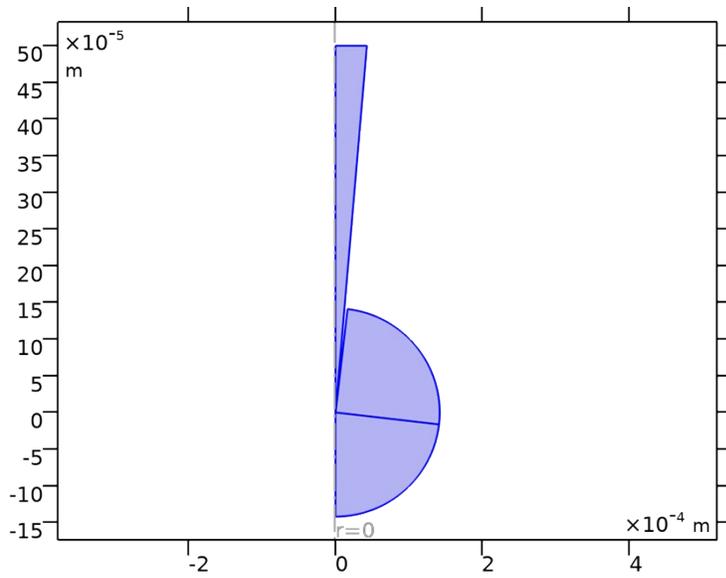
POINT

| Description | Value |
|------------------|--------------------------------|
| Point coordinate | {7.548259896776561E-7, 8.0E-6} |

2.3 TRANSPORT OF DILUTED SPECIES

USED PRODUCTS

| |
|--------------------------------------|
| COMSOL Multiphysics |
| Chemical Reaction Engineering Module |



Transport of Diluted Species

SELECTION

| | |
|------------------------|--|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: All domains |

EQUATIONS

$$\nabla \cdot \mathbf{J}_i = R_i$$

$$\mathbf{J}_i = -D_i \nabla C_i - z_i u_{m,j} F C_i \nabla V$$

2.3.1 Interface Settings

Discretization

SETTINGS

| Description | Value |
|---------------|-----------|
| Concentration | Quadratic |

SETTINGS

| Description | Value |
|---------------|------------------|
| Equation form | Study controlled |

Advanced Settings

SETTINGS

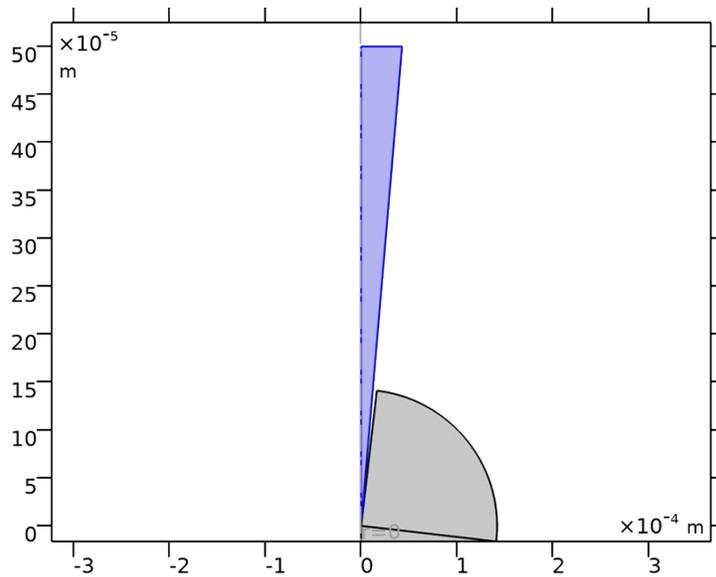
| Description | Value |
|-----------------|-------------------|
| Convective term | Conservative form |

Transport Mechanisms

SETTINGS

| Description | Value |
|-------------------------------|-------|
| Convection | Off |
| Migration in electric field | On |
| Mass transfer in porous media | Off |

2.3.2 Diffusion and Migration (Pore)



Diffusion and Migration (Pore)

SELECTION

| | |
|------------------------|--|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: All domains |

EQUATIONS

$$\nabla \cdot \mathbf{J}_i = R_i$$

$$\mathbf{J}_i = -D_i \nabla C_i - z_i u_{mj} F C_i \nabla V$$

Diffusion

SETTINGS

| Description | Value |
|-----------------------|--------------|
| Source | Material |
| Material | None |
| Diffusion coefficient | User defined |

| Description | Value |
|-----------------------|---|
| Diffusion coefficient | {{DKPore, 0, 0}, {0, DKPore, 0}, {0, 0, DKPore}} |
| Diffusion coefficient | User defined |
| Diffusion coefficient | {{DCIPore, 0, 0}, {0, DCIPore, 0}, {0, 0, DCIPore}} |

Migration in Electric Field

SETTINGS

| Description | Value |
|--------------------|----------------------------|
| Electric potential | Electric potential (es) |
| Mobility | Nernst - Einstein relation |
| Charge number | {1, -1} |

Coordinate System Selection

SETTINGS

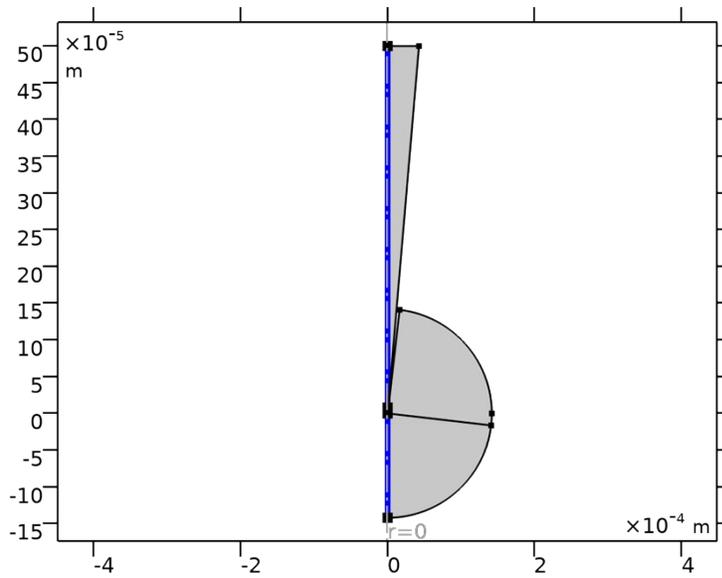
| Description | Value |
|-------------------|--------------------------|
| Coordinate system | Global coordinate system |

Model Input

SETTINGS

| Description | Value |
|-------------|--------------|
| Temperature | User defined |
| Temperature | T |

2.3.3 Axial Symmetry

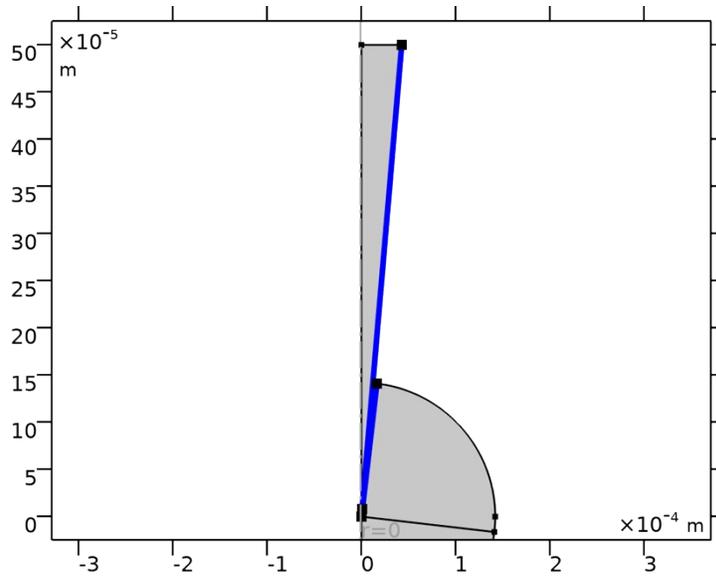


Axial Symmetry

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: All boundaries |

2.3.4 No Flux (Glass Wall)



No Flux (Glass Wall)

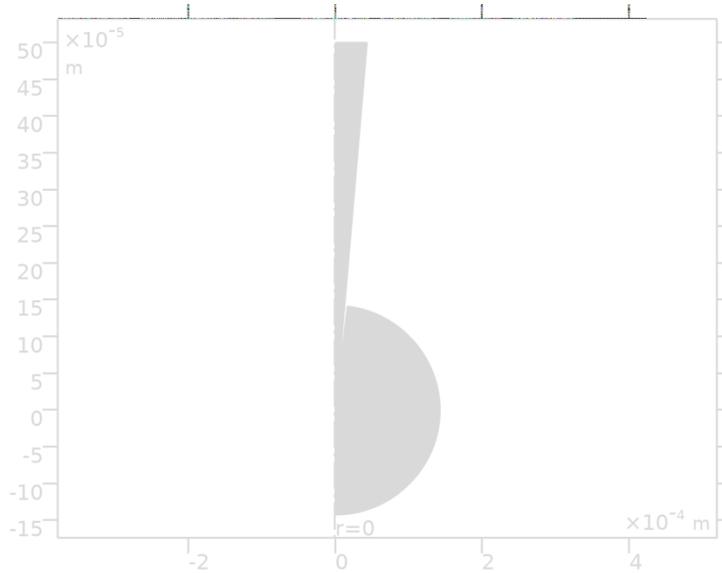
SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: All boundaries |

EQUATIONS

$$-\mathbf{n} \cdot \mathbf{J}_i = 0$$

2.3.5 Initial Values



Initial Values

SELECTION

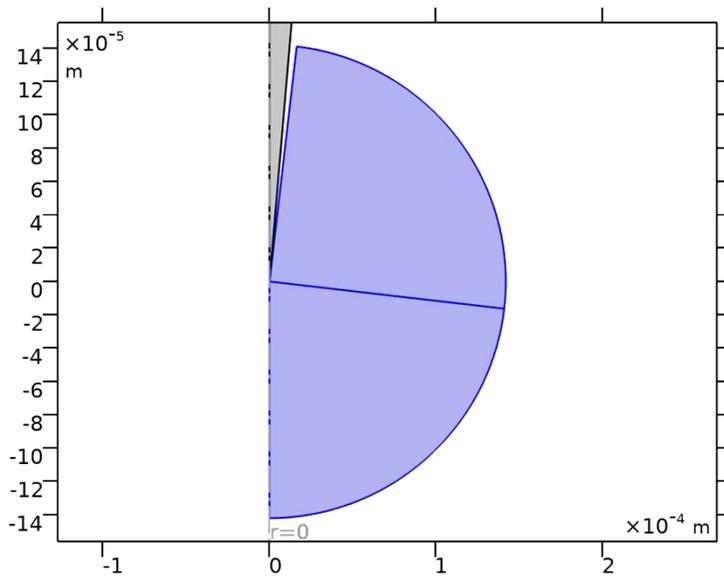
| | |
|------------------------|--|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: All domains |

Initial Values

SETTINGS

| Description | Value |
|---------------|----------------|
| Concentration | {cPore, cPore} |

2.3.6 Diffusion and Migration (Bath)



Diffusion and Migration (Bath)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domains 1–2, 4 |

EQUATIONS

$$\nabla \cdot \mathbf{J}_j = R_j$$

$$\mathbf{J}_j = -D_j \nabla C_j - z_j u_{mj} F C_j \nabla V$$

Diffusion

SETTINGS

| Description | Value |
|-----------------------|---|
| Source | Material |
| Material | None |
| Diffusion coefficient | User defined |
| Diffusion coefficient | {{DKBath, 0, 0}, {0, DKBath, 0}, {0, 0, DKBath}} |
| Diffusion coefficient | User defined |
| Diffusion coefficient | {{DCIBath, 0, 0}, {0, DCIBath, 0}, {0, 0, DCIBath}} |

Migration in Electric Field

SETTINGS

| Description | Value |
|--------------------|-------------------------|
| Electric potential | Electric potential (es) |

| Description | Value |
|---------------|----------------------------|
| Mobility | Nernst - Einstein relation |
| Charge number | {1, -1} |

Coordinate System Selection

SETTINGS

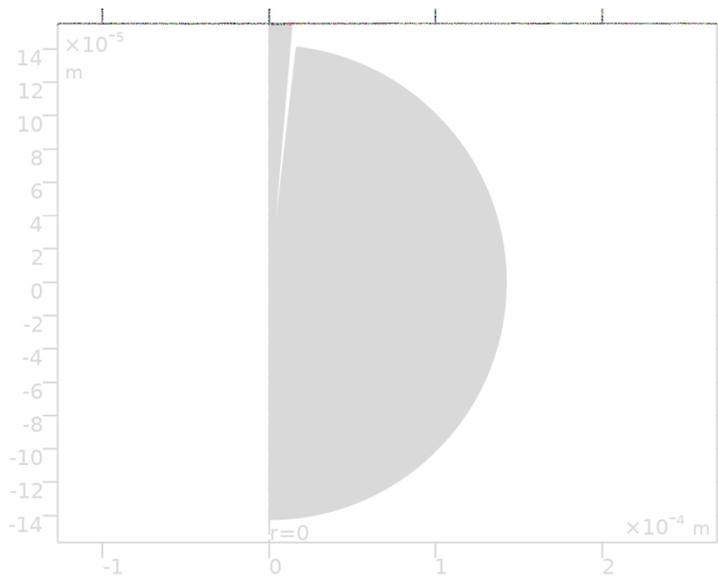
| Description | Value |
|-------------------|--------------------------|
| Coordinate system | Global coordinate system |

Model Input

SETTINGS

| Description | Value |
|-------------|--------------|
| Temperature | User defined |
| Temperature | T |

2.3.7 Initial Values (Gouy-Chapman, Bath)



Initial Values (Gouy-Chapman, Bath)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domains 1-2, 4 |

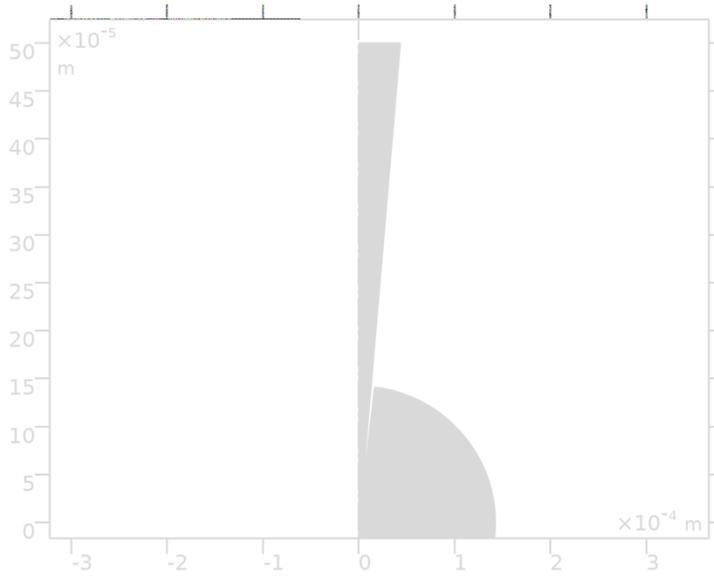
Initial Values

SETTINGS

| Description | Value |
|-------------|-------|
|-------------|-------|

| Description | Value |
|---------------|-------------------------|
| Concentration | {cK_GCBath, cCl_GCBath} |

2.3.8 Initial Values (Gouy-Chapman, Pore)



Initial Values (Gouy-Chapman, Pore)

SELECTION

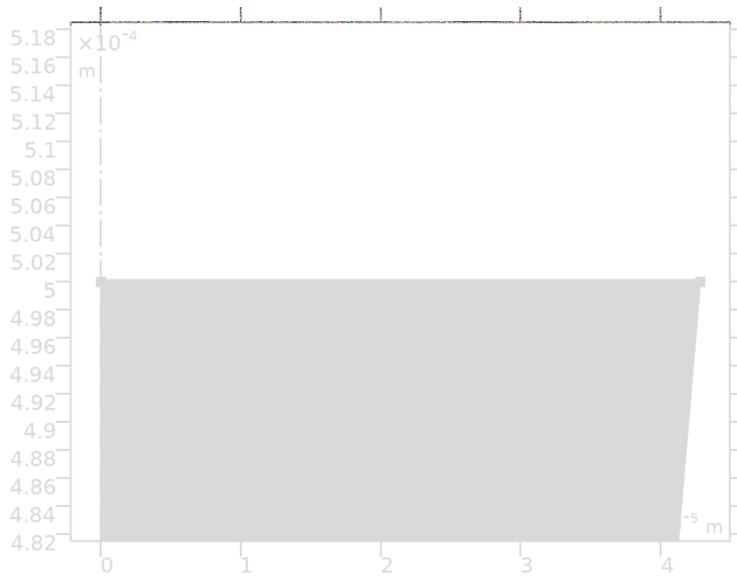
| | |
|------------------------|---------------------------------------|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domain 3 |

Initial Values

SETTINGS

| Description | Value |
|---------------|-----------------|
| Concentration | {cK_GC, cCl_GC} |

2.3.9 Concentration (Gouy-Chapman, internal electrode)



Concentration (Gouy-Chapman, internal electrode)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: Boundary 6 |

EQUATIONS

$$c_i = c_{0j}$$

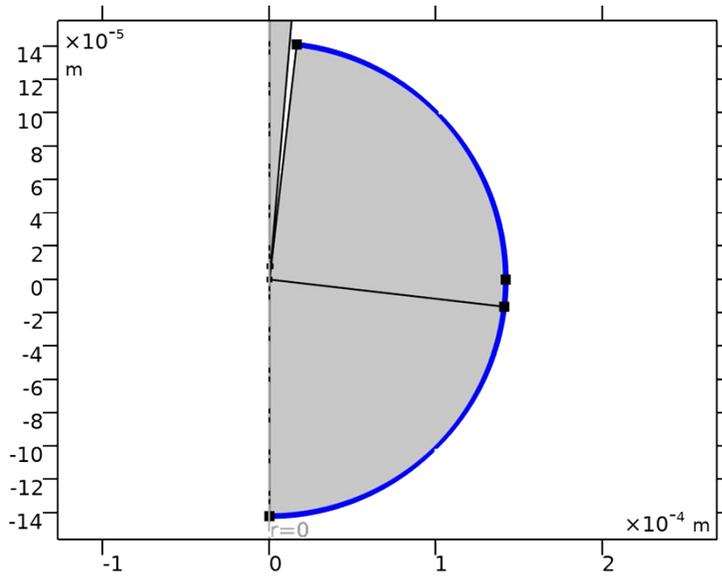
.....

Concentration

SETTINGS

| Description | Value |
|---------------|-------------------------------------|
| Species cK | On |
| Species cCl | On |
| Concentration | {c_GC(dGlass, 1), c_GC(dGlass, -1)} |

2.3.10 Concentration (Gouy-Chapman, external electrode)



Concentration (Gouy-Chapman, external electrode)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: Boundaries 12–14 |

EQUATIONS

$$c_j = c_{0j}$$

Concentration

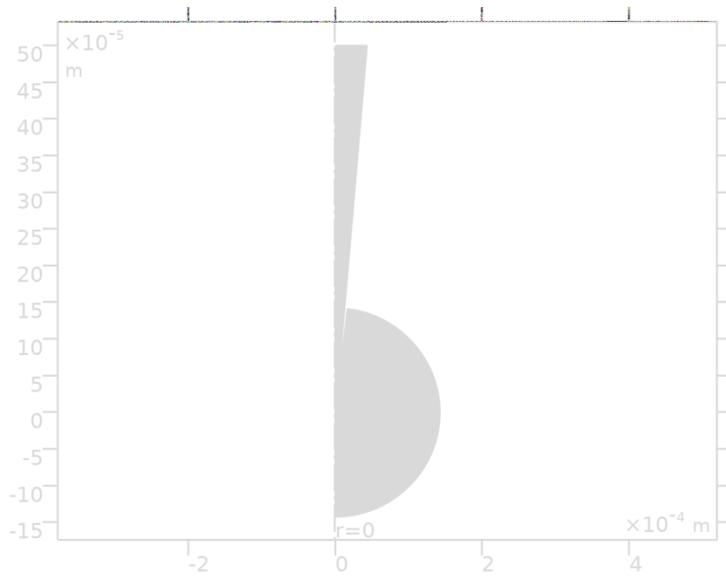
SETTINGS

| Description | Value |
|---------------|---|
| Species cK | On |
| Species cCl | On |
| Concentration | {c_GC Bath(dGlass, 1), c_GC Bath(dGlass, -1)} |

2.4 ELECTROSTATICS

USED PRODUCTS

| |
|---------------------|
| COMSOL Multiphysics |
|---------------------|



Electrostatics

SELECTION

| | |
|------------------------|--|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: All domains |

EQUATIONS

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\mathbf{E} = -\nabla V$$

2.4.1 Interface Settings

Discretization

SETTINGS

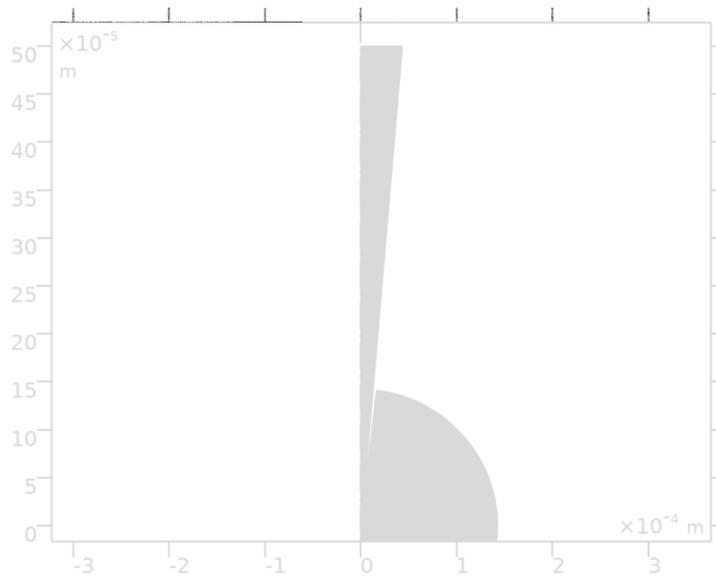
| Description | Value |
|--------------------|-----------|
| Electric potential | Quadratic |

Manual Terminal Sweep Settings

SETTINGS

| Description | Value |
|---------------------------|---------|
| Use manual terminal sweep | Off |
| Reference impedance | 50[ohm] |

2.4.2 Charge Conservation (Pore)



Charge Conservation (Pore)

SELECTION

| | |
|------------------------|--|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: All domains |

EQUATIONS

$$\mathbf{E} = -\nabla V$$

$$\nabla \cdot (\epsilon_0 \epsilon_r \mathbf{E}) = \rho_v$$

.....

Constitutive Relation D-E

SETTINGS

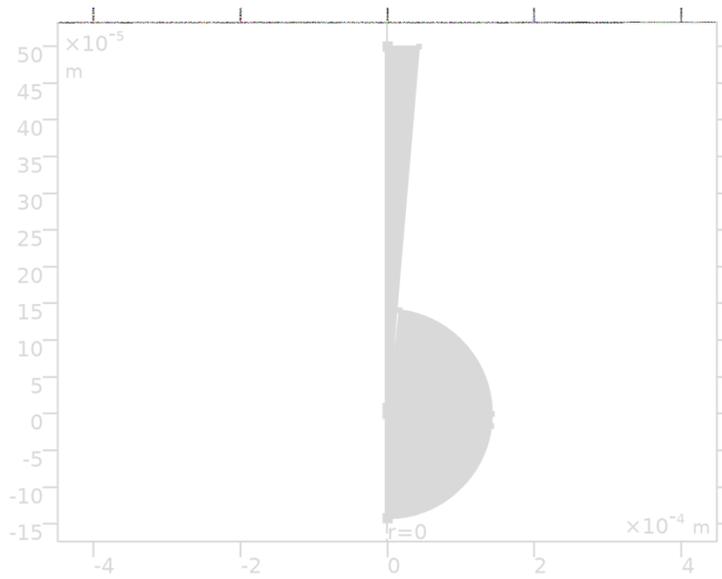
| Description | Value |
|-----------------------|--|
| Dielectric model | Relative permittivity |
| Relative permittivity | User defined |
| Relative permittivity | {{EpsilonRPore, 0, 0}, {0, EpsilonRPore, 0}, {0, 0, EpsilonRPore}} |

Coordinate System Selection

SETTINGS

| Description | Value |
|-------------------|--------------------------|
| Coordinate system | Global coordinate system |

Axial Symmetry

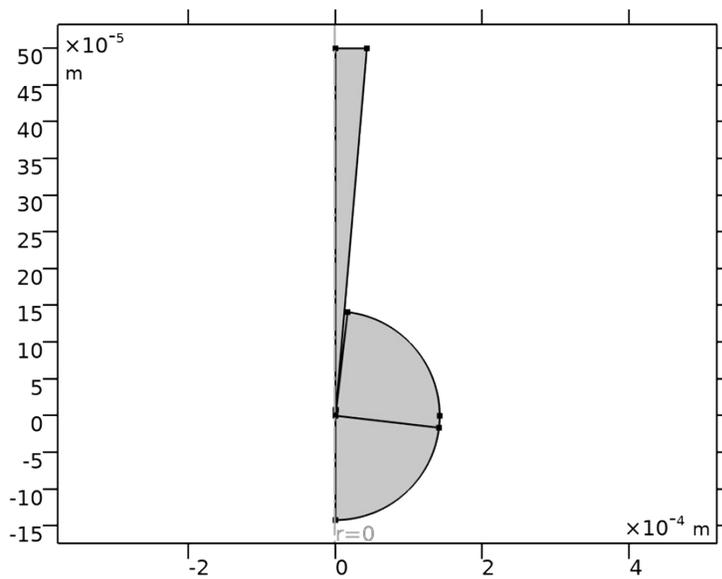


Axial Symmetry

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: All boundaries |

2.4.3 Zero Charge



Zero Charge

SELECTION

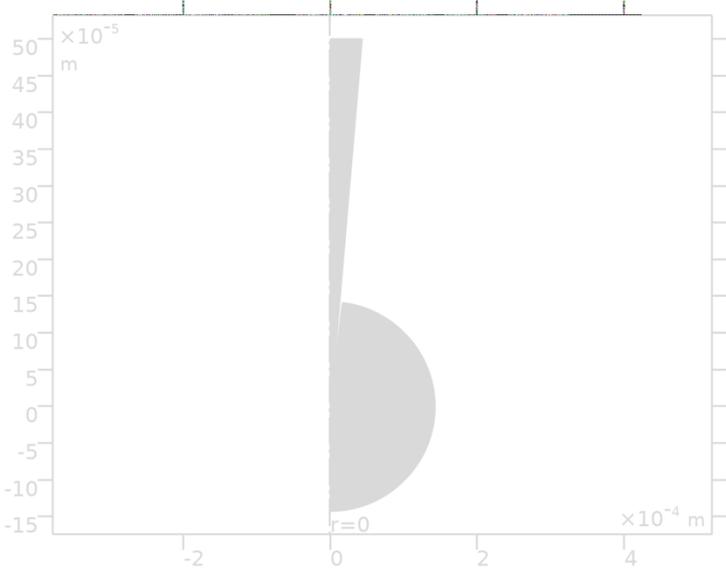
| | |
|------------------------|----------|
| Geometric entity level | Boundary |
|------------------------|----------|

| | |
|-----------|---|
| Selection | Geometry geom1: Dimension 1: All boundaries |
|-----------|---|

EQUATIONS

$\mathbf{n} \cdot \mathbf{D} = 0$

2.4.4 Initial Values



Initial Values

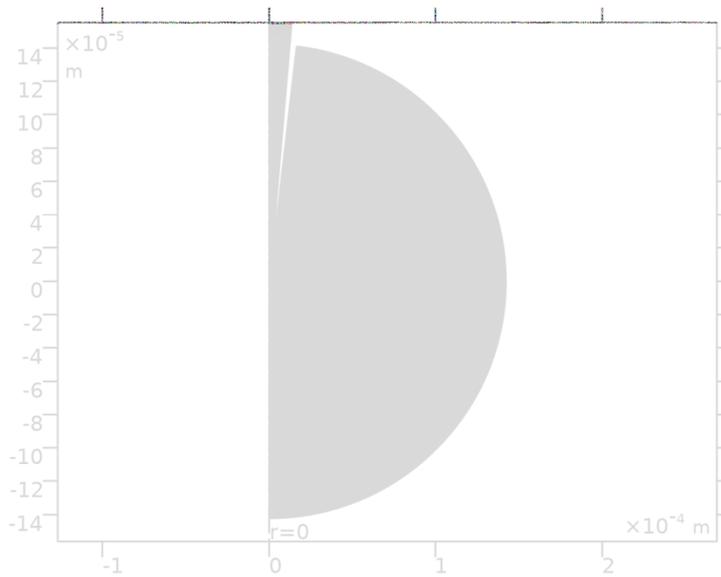
SELECTION

| | |
|------------------------|--|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: All domains |

SETTINGS

| Description | Value |
|--------------------|-------|
| Electric potential | 0 |

2.4.5 Charge Conservation (Bath)



Charge Conservation (Bath)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domains 1–2, 4 |

EQUATIONS

$$\mathbf{E} = -\nabla V$$

$$\nabla \cdot (\epsilon_0 \epsilon_r \mathbf{E}) = \rho_v$$

Constitutive Relation D-E

SETTINGS

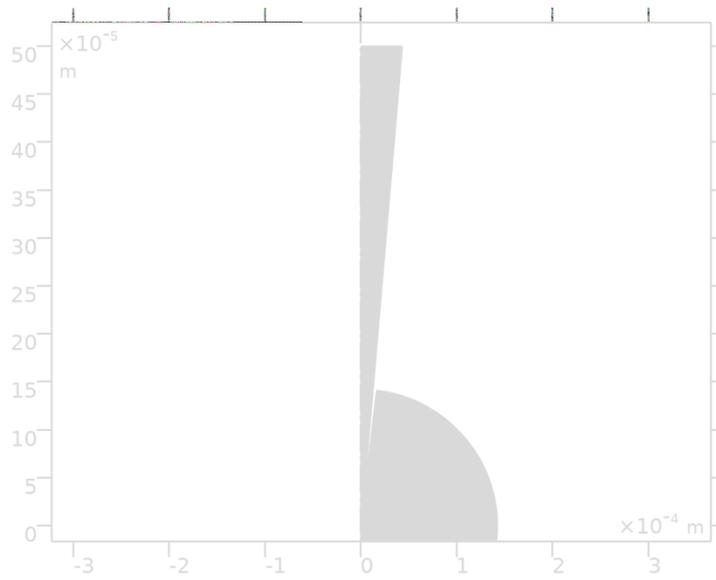
| Description | Value |
|-----------------------|--|
| Dielectric model | Relative permittivity |
| Relative permittivity | User defined |
| Relative permittivity | {{EpsilonRBath, 0, 0}, {0, EpsilonRBath, 0}, {0, 0, EpsilonRBath}} |

Coordinate System Selection

SETTINGS

| Description | Value |
|-------------------|--------------------------|
| Coordinate system | Global coordinate system |

2.4.6 Initial Values (Gouy-Chapman in Pore)



Initial Values (Gouy-Chapman in Pore)

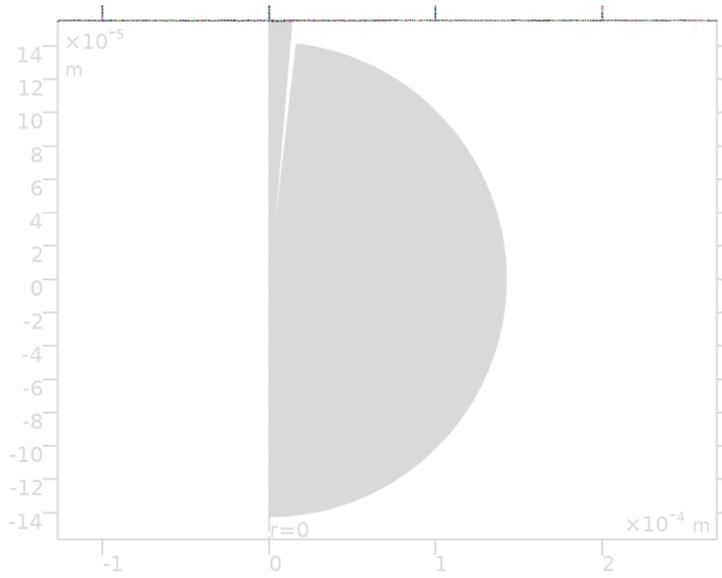
SELECTION

| | |
|------------------------|---------------------------------------|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domain 3 |

SETTINGS

| Description | Value |
|--------------------|--|
| Electric potential | Potential_GC + AnalyticalPotentialNoCharge |

2.4.7 Initial Values (Gouy-Chapman in Bath)



Initial Values (Gouy-Chapman in Bath)

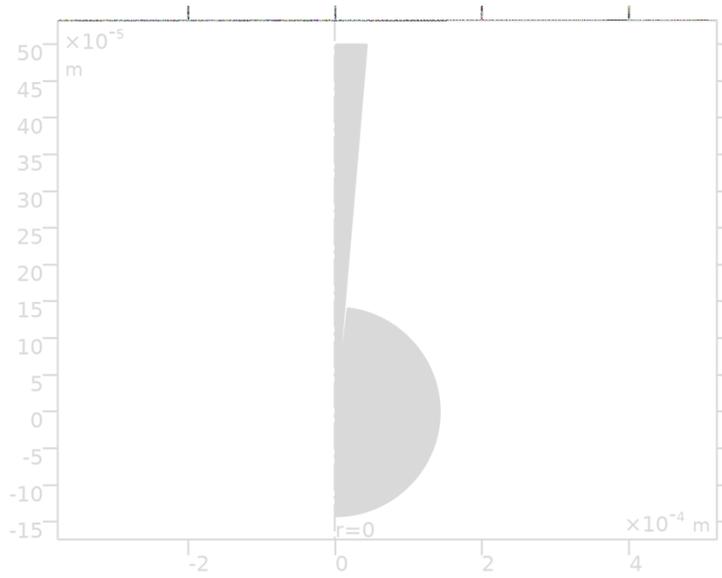
SELECTION

| | |
|------------------------|---|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domains 1–2, 4 |

SETTINGS

| Description | Value |
|--------------------|---|
| Electric potential | Potential_GC Bath + AnalyticalPotentialNoCharge |

2.4.8 Space Charge Density



Space Charge Density

SELECTION

| | |
|------------------------|--|
| Geometric entity level | Domain |
| Selection | Geometry geom1: Dimension 2: Domains 1-4 |

EQUATIONS

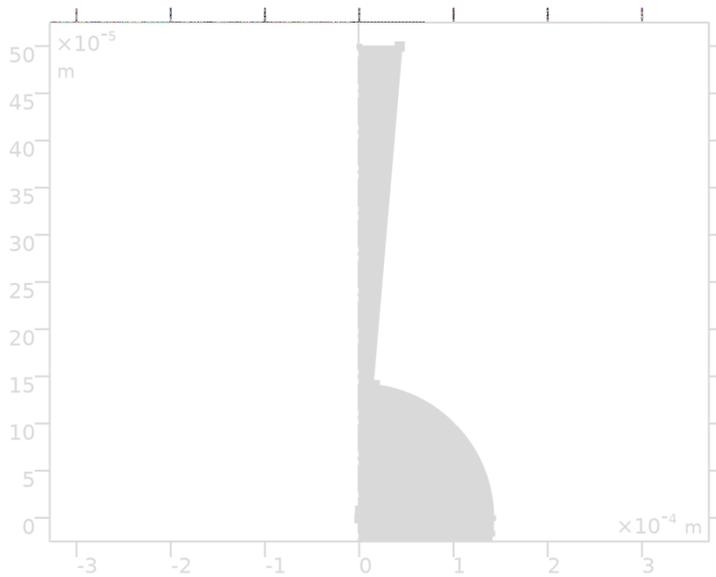
$$\nabla \cdot \mathbf{D} = \rho_v$$

Coordinate System Selection

SETTINGS

| Description | Value |
|-------------------|--------------------------|
| Coordinate system | Global coordinate system |

2.4.9 Surface Charge (Glass Wall)



Surface Charge (Glass Wall)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Name | Glass Wall |
| Selection | Named sel1: Geometry geom1: Dimension 1: Boundaries 7–9, 11 |

EQUATIONS

$$\mathbf{n} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$$

Surface Charge Density

SETTINGS

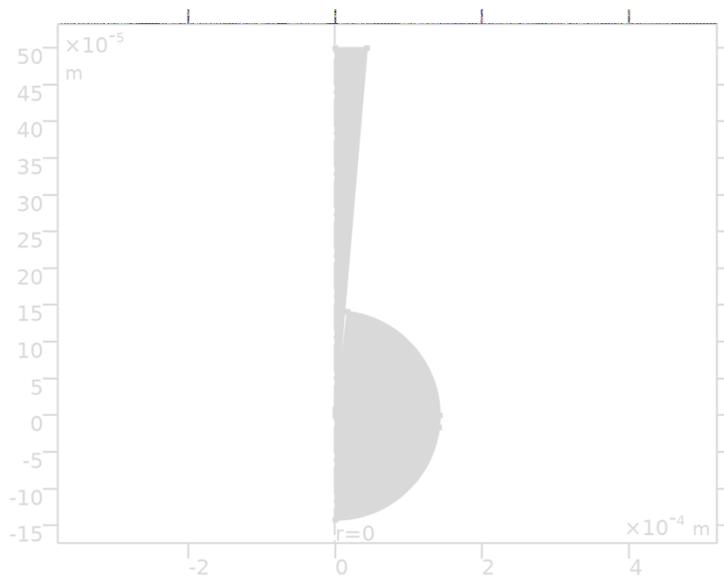
| Description | Value |
|------------------------|-------------------|
| Surface charge density | PoreSurfaceCharge |

Coordinate System Selection

SETTINGS

| Description | Value |
|-------------------|--------------------------|
| Coordinate system | Global coordinate system |

2.4.10 Surface Charge (Au nanoparticle)



Surface Charge (Au nanoparticle)

SELECTION

| | |
|------------------------|--|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: No boundaries |

EQUATIONS

$$\mathbf{n} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$$

Surface Charge Density

SETTINGS

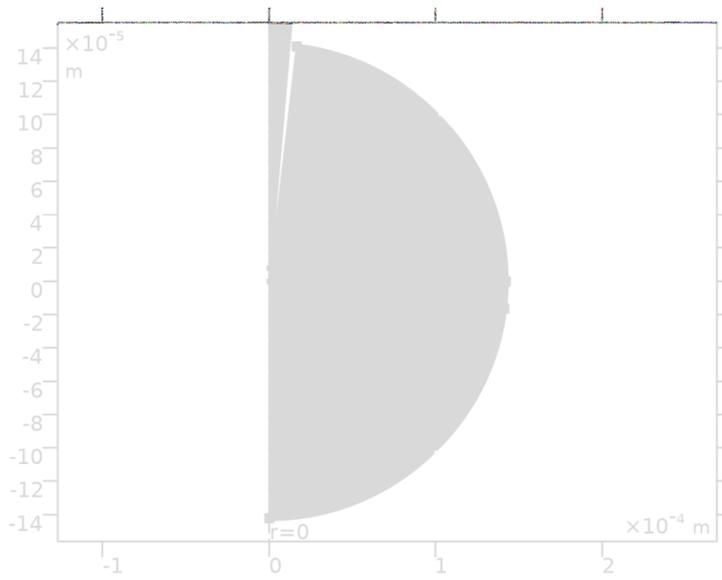
| Description | Value |
|------------------------|-----------------------|
| Surface charge density | ParticleSurfaceCharge |

Coordinate System Selection

SETTINGS

| Description | Value |
|-------------------|--------------------------|
| Coordinate system | Global coordinate system |

2.4.11 Electric Potential (Gouy-Chapman, external electrode)



Electric Potential (Gouy-Chapman, external electrode)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Name | External Electrode |
| Selection | Named sel4: Geometry geom1: Dimension 1: Boundaries 12–14 |

EQUATIONS

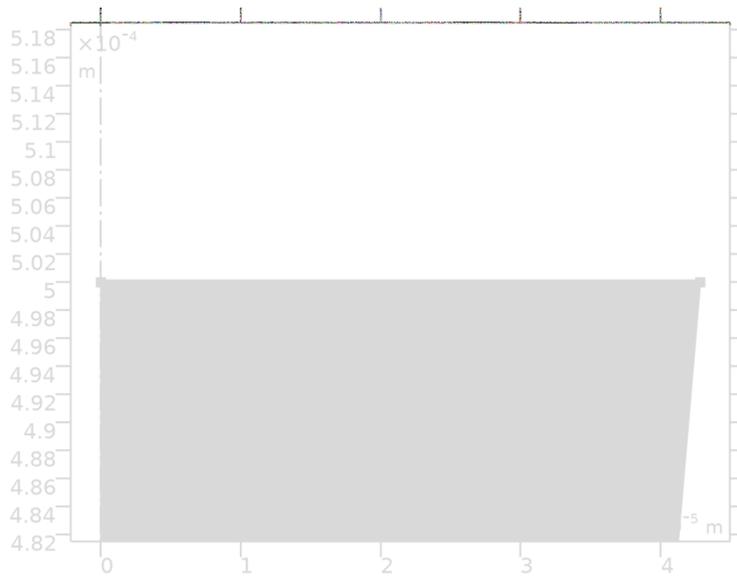
$$V = V_0$$

Electric Potential

SETTINGS

| Description | Value |
|--------------------|-------------------|
| Electric potential | E_GC Bath(dGlass) |

2.4.12 Electric Potential (Gouy-Chapman, internal electrode)



Electric Potential (Gouy-Chapman, internal electrode)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Name | Internal Electrode |
| Selection | Named sel3: Geometry geom1: Dimension 1: Boundary 6 |

EQUATIONS

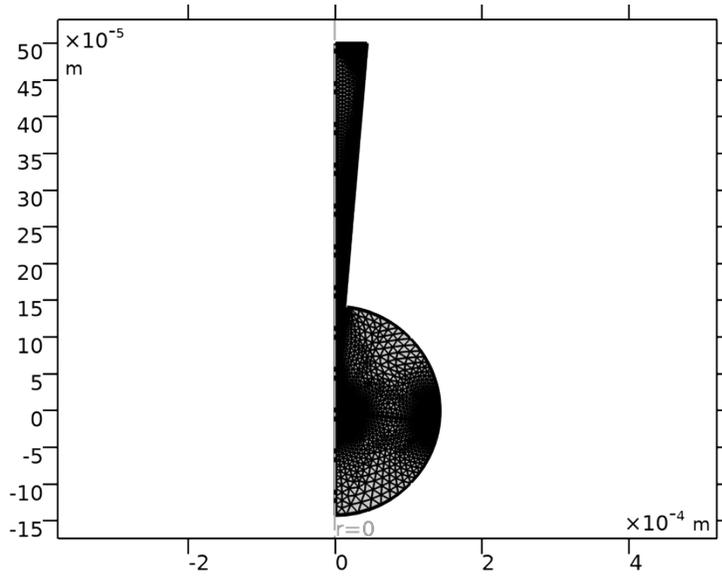
$$V = V_0$$

Electric Potential

SETTINGS

| Description | Value |
|--------------------|---------------------|
| Electric potential | VApp + E_GC(dGlass) |

2.5 FINAL MESH



Final Mesh

2.5.1 Size Overall (size)

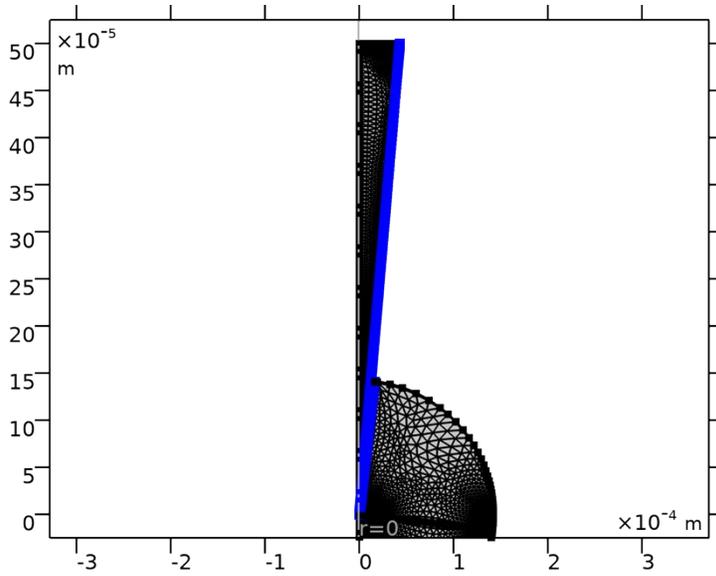
SETTINGS

| Description | Value |
|------------------------------|----------------|
| Maximum element size | 200E-7 |
| Minimum element size | 5E-10 |
| Curvature factor | 0.2 |
| Resolution of narrow regions | 10 |
| Predefined size | Extremely fine |
| Custom element size | Custom |

2.5.2 Size for Glass Wall (size2)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Name | Glass Wall |
| Selection | Named sel1: Geometry geom1: Dimension 1: Boundaries 7-9, 11 |



Size for Glass Wall

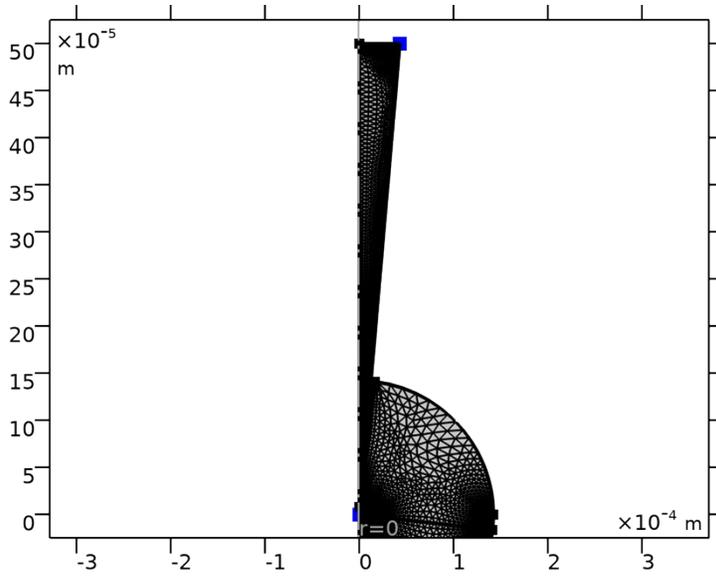
SETTINGS

| Description | Value |
|----------------------|----------------|
| Maximum element size | 6.42E-6 |
| Minimum element size | 1.28E-8 |
| Curvature factor | 0.2 |
| Predefined size | Extremely fine |

2.5.3 Size Aperture Points (size1)

SELECTION

| | |
|------------------------|--|
| Geometric entity level | Point |
| Selection | Geometry geom1: Dimension 0: Points 5–6, 9 |



Size Aperture Points

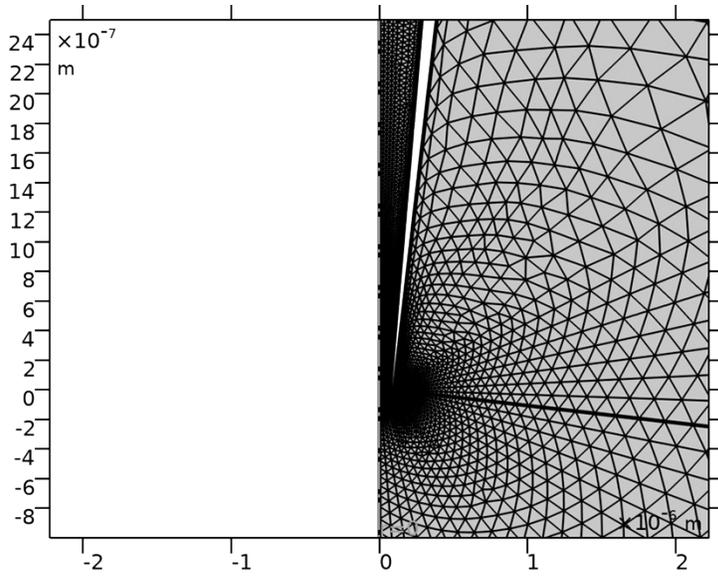
SETTINGS

| Description | Value |
|------------------------------|----------------|
| Maximum element size | PoreRadius/100 |
| Minimum element size | 6.45E-10 |
| Minimum element size | Off |
| Curvature factor | 0.3 |
| Curvature factor | Off |
| Resolution of narrow regions | Off |
| Maximum element growth rate | 1.3 |
| Maximum element growth rate | Off |
| Custom element size | Custom |

2.5.4 Boundary Layers (bl1)

SELECTION

| | |
|------------------------|----------------|
| Geometric entity level | Domain |
| Selection | Geometry geom1 |



Boundary Layers

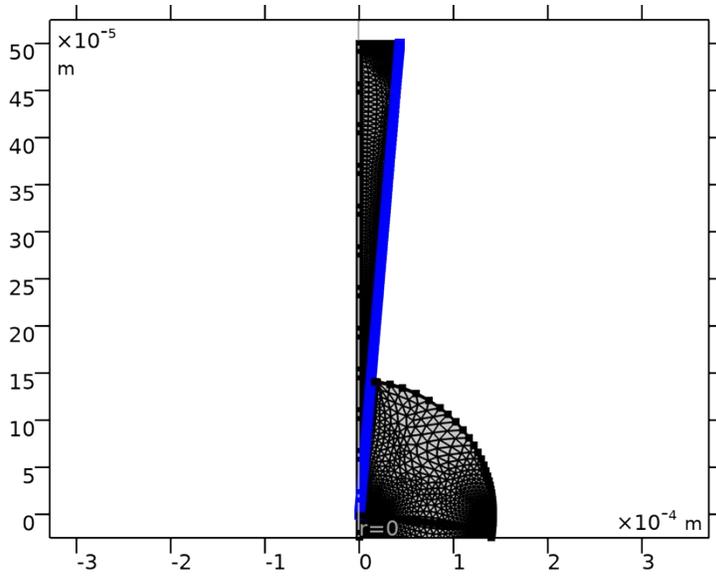
SETTINGS

| Description | Value |
|----------------------------------|---|
| Number of iterations | 30 |
| Maximum element depth to process | 30 |
| Last build time | 7 |
| Built with | COMSOL 6.0.0.354 (glnxa64) 2024 - 06 - 06T16:50:16.266986 |

Boundary Layer Properties (Internal) (blp)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: Boundaries 8, 11 |



Boundary Layer Properties (Internal)

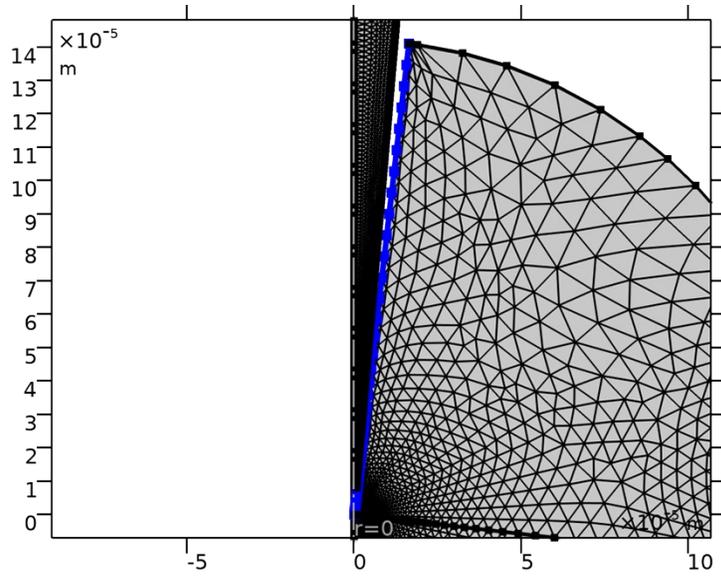
SETTINGS

| Description | Value |
|-------------------------|--------------------|
| Number of layers | 12 |
| Thickness specification | First layer |
| Thickness | DebyeLengthPore/10 |

Boundary Layer Properties (External) (blp1)

SELECTION

| | |
|------------------------|--|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: Boundaries 7, 9 |



Boundary Layer Properties (External)

SETTINGS

| Description | Value |
|-------------------------|--------------------|
| Number of layers | 12 |
| Thickness specification | First layer |
| Thickness | DebyeLengthBath/10 |

3 Steady State

COMPUTATION INFORMATION

| | |
|------------------|------|
| Computation time | 42 s |
|------------------|------|

3.1 STATIONARY

STUDY SETTINGS

| Description | Value |
|--------------------------------|-------|
| Include geometric nonlinearity | Off |

PHYSICS AND VARIABLES SELECTION

| Physics interface | Discretization |
|-------------------------------------|----------------|
| Transport of Diluted Species (chds) | physics |
| Electrostatics (es) | physics |

MESH SELECTION

| Geometry | Mesh |
|---------------------|-------|
| Nanopipette (geom1) | mesh2 |

3.2 SOLVER CONFIGURATIONS

3.2.1 Solution 1

Compile Equations: Stationary (st1)

STUDY AND STEP

| Description | Value |
|----------------|------------------------------|
| Use study | Steady State |
| Use study step | Stationary |

Dependent Variables 1 (v1)

GENERAL

| Description | Value |
|-----------------------|----------------------------|
| Defined by study step | Stationary |

Concentration (mod1.cCl) (mod1_cCl)

GENERAL

| Description | Value |
|--------------------|----------------------------------|
| Field components | mod1.cCl |
| Internal variables | {mod1.uflux.cCl, mod1.dflux.cCl} |

Concentration (mod1.cK) (mod1_cK)

GENERAL

| Description | Value |
|--------------------|--------------------------------|
| Field components | mod1.cK |
| Internal variables | {mod1.uflux.cK, mod1.dflux.cK} |

Electric potential (mod1.V) (mod1_V)

GENERAL

| Description | Value |
|------------------|--------|
| Field components | mod1.V |

Stationary Solver 1 (s1)

GENERAL

| Description | Value |
|-----------------------|----------------------------|
| Defined by study step | Stationary |

RESULTS WHILE SOLVING

| Description | Value |
|-------------|------------------------------------|
| Plot | On |
| Plot group | Probe Plot Group 1 |

Advanced (aDef)

ASSEMBLY SETTINGS

| Description | Value |
|------------------------|-------|
| Reuse sparsity pattern | On |

Fully Coupled 1 (fc1)

GENERAL

| Description | Value |
|---------------|--|
| Linear solver | Direct, concentrations (chds) (merged) |

METHOD AND TERMINATION

| Description | Value |
|------------------------------|--------|
| Initial damping factor | 0.01 |
| Minimum damping factor | 1.0E-6 |
| Maximum number of iterations | 50 |

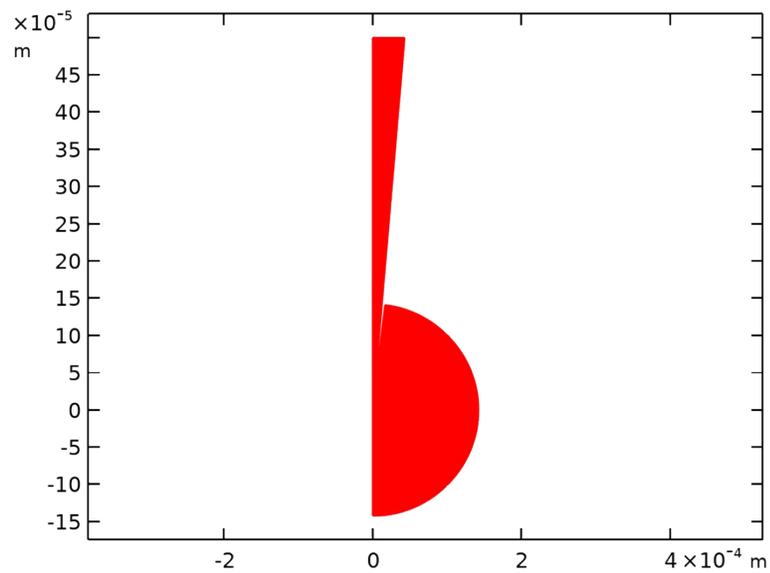
4 Results

4.1 DATA SETS

4.1.1 Steady State/Solution 1

SOLUTION

| Description | Value |
|-------------|----------------------------|
| Solution | Solution 1 |
| Component | Model 1 (mod1) |

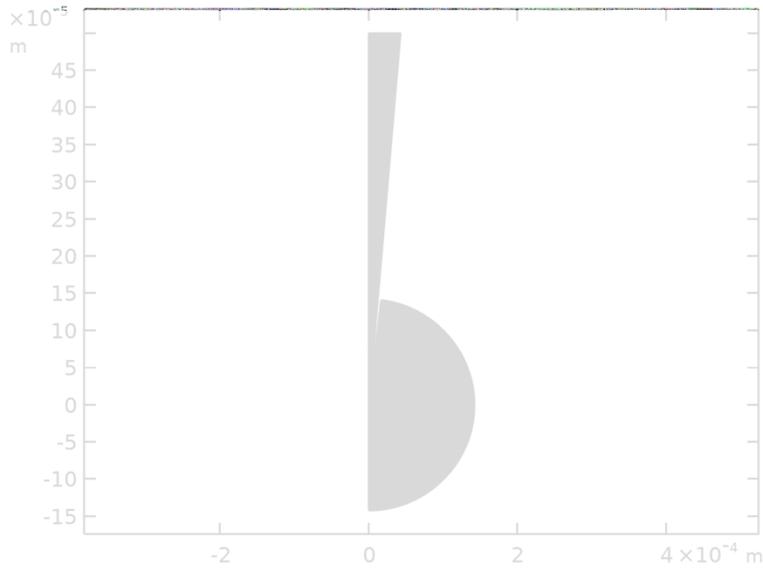


Dataset: Steady State/Solution 1

4.1.2 Probe Solution 2

SOLUTION

| Description | Value |
|-------------|----------------------------|
| Solution | Solution 1 |
| Component | Model 1 (mod1) |



Dataset: Probe Solution 2

4.1.3 Current (internal electrode)

SELECTION

| | |
|------------------------|---|
| Geometric entity level | Boundary |
| Selection | Geometry geom1: Dimension 1: Boundary 6 |

DATA

| Description | Value |
|-------------|----------------------------------|
| Dataset | Probe Solution 2 |

SETTINGS

| Description | Value |
|-------------------|-------------|
| Method | Integration |
| Integration order | 4 |
| Integration order | On |

4.2 DERIVED VALUES

4.2.1 Current (internal electrode)

OUTPUT

| | |
|--------------|-------------------------------|
| Evaluated in | Probe Table 1 |
|--------------|-------------------------------|

DATA

| Description | Value |
|-------------|--|
| Dataset | Current (internal electrode) |

EXPRESSIONS

| Expression | Unit | Description |
|--|------|--------------------|
| $-F_{\text{const}} \cdot (\text{chds.bndFlux}_{\text{cK}} - \text{chds.bndFlux}_{\text{cCl}}) \cdot 2 \cdot \pi \cdot r$ | nA | Current (internal) |