Supplementary

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## Dominant higher-order vortex gyromodes in circular magnetic nanodots

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## Numerical experiment interpretation

Regarding the spectral representation we propose a novel approach, we will explain the advantages of using comparisons with time-domain evolution. The previous works focus either on the Fourier image  $\tilde{m}_z$  of the magnetization z component and ignoring all the rest, or study the collective  $\tilde{m}_i \tilde{m}^{i*}$  distribution (repeating indexes denote summation). In regards to tying the distributions to vortex motion the latter is much more difficult than the former, because for the former the z component circular maxima are directly showing the trace of the vortex core, and because people have a much better picture of the simpler  $m_z$  time domain motion in their mind.

However, counter-intuitively the  $\tilde{m}_z$  component makes up a very small portion of the total mode intensity, for the  $G_2$  in 200 nm nanodots it is only  $\eta \{\tilde{m}_z\} = \int dV |\tilde{m}_z|^2 / \int dV \tilde{m}_i \tilde{m}^{i*} = 0.137$ , for the  $H_{\text{amp.}} = 5 \text{ Oe}$  strong microwave field. The ratio  $\eta \{\tilde{m}_z\}$  is only decreased when the field  $H_{\text{amp.}}$  is made smaller. Even worse, the  $\tilde{m}_z$  magnetization component is quickly vanishing locally near points where the core approaches the rotation axis. The  $\tilde{m}_z$  component thus has a much worse signal-to-noise, making identifying low-intensity modes hard.

The biggest portion of the mode intensity is in the planar degrees of freedom  $\eta \{\tilde{m}_x\} = \eta \{\tilde{m}_y\} = 0.432$ , and we find further that a linear combination gives  $\eta \{\tilde{m}_{CCW} = (\tilde{m}_x + i\tilde{m}_y)/\sqrt{2}\} = 0.537$ (roughly 2/3 of the combined  $\tilde{m}_x$  and  $\tilde{m}_y$  intensity), and is the easiest to analyse because it possesses a phase while it is similar in amplitude to the total distribution by the virtue of being its largest contributor. Compared to the  $m_z$  component, this combination scales much better at low  $H_{\rm amp}$  fields and low mode amplitudes. It also has much gentler variation, slowly decaying from the axis of the nanodot to somewhere beyond the vertex core rotation radius, which is important for data on finite cell computational grids like the ones in Mumax3.

Giving an interpretation to the  $\tilde{m}_{CCW}$  distribution, however, is not as easy, primarily because the motion of planar magnetization components is often overlooked. To gain an understanding it is important to consider the three extremes first. Assuming that a stationary trajectory is simply rotating around the nanodot axis, like most modes presented in the paper, we can immediately say that we have a maximum at the axis of the nanodot. We can understand this maximum by considering the spins situated on the axis of the nanodot, these, because of the magnetization rotating as a whole, themselves simply rotate around the axis. The range of motion of the spin at the centre is quickly maximized when the core is far away and the spin stays completely in plane, and is smaller the closer the core is precessing to the centre.

A spin lying on a circular trajectory of the vortex core is another easy reference point, when the vortex core cross such a point, a planar moment is abruptly flipped as it reappears after the core  $m_z = 1$  crosses over. The planar  $(m_x, m_y)$  vector draws a closed curve in only half of a plane crossing **0** exactly once (the line separating the half plane is dependent on which point the observation is taken in). As a result of this, the Fourier intensity is necessarily smaller than that of the point at the axis.

The spins lying at the edge of the nanodot in contrast experiences very little motion to minimize the stray fields generated by the moving vortex. It fast converges to zero as the nanodot diameter is increased. These three combined for a typical monotonous decrease  $m_z$  core gives a single peak in intensity at the particle axis OZ r = 0.

Briefly looking at the  $\tilde{m}_{CCW}$  phase, it has an azimuthal number m = 0 axial symmetry as all the necessary phase shifts needed to get a gyrational motion to turn out to be already baked into the stationary component  $\tilde{m}(f = 0 \text{ Hz})$ . And any phase variation over the nanodot thickness contributes to the relative rotational shifts of vortex core positions between layers.

Armed with the above, we can now see how in Fig. ?? the frequency domain representation describes the time domain core motion. The axial maxima of the dynamic magnetization distribution coincide well with the most distant portions of the core line, and minima, denoted by A, B, and C, correspond to the closest points. We observe that core lines get very close to the nanodot axis of the nanodot however they avoid crossing it. For the mode presented closest is C at just 1.6 nm and the furthest point is at the nanodot base: 13.6 nm.

In the Fourier intensity distribution a similar ratio between the maximal and minimal intensities is observed. To better understand the behaviour at low intensities in the frequency domain it is necessary to now check the phase distributions. Because of the axial symmetry at r = 0 phase discontinuities of  $\pi$  are still allowing the core line to be differentiable, representing a core line crossing the nanodot axis, however, contrary to early predictions, no such effect is observed and the distribution is completely congruent with observed time-domain behaviour.

We can reverse our analysis to also speculate how the distribution of other idealistic hypothetical solutions would look like in the frequency domain. For example, if the core would be completely in a single rotating plane, then the phase would have abrupt jumps at the nodes. And if it would be a proper equidistant helix, then we would observe no minimums or maximums and a phase would vary smoothly across the depth of the nanodot. Instead, an unequal mix of the two is observed in practice Fig. ??, indicating that the real core helix is squished.