## Supplemental Material:

Effects of the surface energy and surface stress on the phase stability of spin crossover nano-objects: a thermodynamic approach

Shiteng Mi, Karl Ridier, Gábor Molnár, William Nicolazzi*, Azzedine Bousseksou**

LCC, CNRS \& Université de Toulouse, 205 route de Narbonne, 31077 Toulouse, France

## I. The exact stationary solution of thin film model

$$
\begin{equation*}
\frac{B_{H S} V_{H S}}{2}\left(\gamma_{o}-1\right)\left[\left(\frac{V_{\min }}{V_{H S}}-1\right)^{2}-\frac{m}{\beta}\left(\frac{V_{\min }}{V_{H S}}-\frac{1}{m}\right)^{2}\right]+(\Delta H-T \Delta S)+k T \ln \left(\frac{x^{b}}{1-x^{b}}\right)=0 \tag{S1}
\end{equation*}
$$

$$
\begin{gather*}
\frac{B_{H S} V_{H S}}{2}\left(\gamma_{o}-1\right)\left[\left(\frac{V_{\min }}{V_{H S}}-1\right)^{2}-\frac{m}{\beta}\left(\frac{V_{\min }}{V_{H S}}-\frac{1}{m}\right)^{2}\right]+(\Delta H-T \Delta S)+k T \ln \left(\frac{x^{s}}{1-x^{s}}\right)+\frac{B_{H S} V_{H S} R_{\gamma}}{2 a_{H S}^{0}}\left(1-\frac{a_{H S}^{0} \Gamma_{\gamma}}{a_{\downarrow}^{0} m}\right)+\frac{1}{2} B_{H S} \\
R_{\sigma} V_{\min }^{2 / 3}[1-\alpha]-\frac{B_{H S} R_{\sigma} V_{H S}}{2 a_{H S}^{0}}\left[1-\frac{a_{H S}^{0} \alpha}{a_{L S}^{0} m}\right]=0 \tag{S2}
\end{gather*}
$$

$$
\begin{align*}
& \left(L^{\text {tot }}-2 L^{s}\right) B_{H S}\left(\gamma_{o}-1\right)\left[x^{b}\left(\frac{V_{\min }}{V_{H S}}-1\right)+\left(1-x^{b}\right) \frac{m}{\beta}\left(\frac{V_{\min }}{V_{H S}}-\frac{1}{m}\right)\right]+2 L^{s} B_{H S}\left(\gamma_{o}-1\right) \\
& {\left[x^{s}\left(\frac{V_{\min }}{V_{H S}}-1\right)+\left(1-x^{s}\right) \frac{m}{\beta}\left(\frac{V_{\min }}{V_{H S}}-\frac{1}{m}\right)\right]+\frac{2}{3} L^{s} B_{H S} R_{\sigma} V_{\min }^{-1 / 3}\left[x^{s}+\left(1-x^{s}\right) \alpha\right]=0,} \tag{S3}
\end{align*}
$$

## II. Spherical core-shell model

Assuming that the nanoparticle is in the HS state and that the shell is made up of a monolayer, the core volume is: $V_{b}=N_{H S}^{b} V_{H S}=\frac{4 \pi R_{1}^{3}}{3}$, where $R_{1}$ is the core radius and $V_{H S}$ the equilibrium volume of a molecule in the HS state. Then, the shell volume can be deduced: $V_{S}=N_{H S}^{s} V_{H S}=N V_{H S}-N_{H S}^{b} V_{H S}=\frac{4 \pi R^{3}}{3}-\frac{4 \pi R_{1}^{3}}{3}$, where $R$ is the radius of the nanoparticle. Finally, the number of molecules in the core $\left(N^{b}\right)$ and in the shell $\left(N^{s}\right)$ can be expressed as:

$$
\begin{gather*}
N^{b}=N\left(\frac{R_{1}}{R}\right)^{3},  \tag{S4}\\
N^{s}=N\left(1-\left(\frac{R_{1}}{R}\right)^{3}\right) . \tag{S5}
\end{gather*}
$$

Besides, considering that all the molecules at the surface are in the HS state, the relation between the area of a single molecule and the total surface area of the nanoparticle is:

$$
\begin{equation*}
N^{a} \frac{V_{H S}}{a_{o}^{H S}}=4 \pi R^{2}, \tag{S6}
\end{equation*}
$$

where $N^{a}$ stands for the total number of molecules at the surface and $a_{0}^{H S}$ is HS lattice parameter. On the other hand, the relation between the volume of a single molecule and the total volume of the particle is:

$$
\begin{equation*}
N V_{H S}=\frac{4 \pi R^{3}}{3} . \tag{S7}
\end{equation*}
$$

From eqn (S6) and eqn (S7), the number of molecules at the surface $N^{a}$ can be deduced:

$$
\begin{equation*}
N^{a}=N \frac{3 a_{o}^{H S}}{R} . \tag{S8}
\end{equation*}
$$

The $N_{H S}^{a}$ and $N_{L S}^{a}$ in eqn (18) of the main text can be calculated as follow:

$$
\begin{gather*}
N_{H S}^{a}=x^{s} N^{a}=x^{s} N \frac{3 a_{o}^{H S}}{R},  \tag{S9}\\
N_{L S}^{a}=\left(1-x^{s}\right) N^{a}=\left(1-x^{s}\right) N \frac{3 a_{o}^{H S}}{R}, \tag{S10}
\end{gather*}
$$

Considering the shell is made up of a monolayer of molecules ( $N^{s}=N^{a}$ in this case), the total energy of the spherical system is written as:
$G_{\text {total }}\left(V_{\text {min }}, x^{b}, x^{s}\right)=N\left(\frac{R_{1}}{R}\right)^{3} \frac{B_{H S} V_{H S}}{2}\left(\gamma_{o}-1\right)\left[x^{b}\left(\frac{V_{\text {min }}}{V_{H S}}-1\right)^{2}+\left(1-x^{b}\right) \frac{m}{\beta}\left(\frac{V_{\text {min }}}{V_{H S}}-\frac{1}{m}\right)^{2}\right]_{+}$
$N\left(1-\left(\frac{R_{1}}{R}\right)^{3}\right) \frac{B_{H S} V_{H S}}{2}\left(\gamma_{o}-1\right)\left[x^{S}\left(\frac{V_{\min }}{V_{H S}}-1\right)^{2}+\left(1-x^{S}\right) \frac{m}{\beta}\left(\frac{V_{\min }}{V_{H S}}-\frac{1}{m}\right)^{2}\right]_{+}$
$N\left[x^{S}\left(1-\left(\frac{R_{1}}{R}\right)^{3}\right)+x^{b}\left(\frac{R_{1}}{R}\right)^{3}\right](\Delta H-T \Delta S)+N\left(H_{L S}-T S_{L S}\right)+$

$$
N k T\left(\frac{R_{1}}{R}\right)^{3}\left[x^{b} \ln \left(x^{b}\right)+\left(1-x^{b}\right) \ln \left(1-x^{b}\right)\right]_{+}
$$

$$
N k T\left(1-\left(\frac{R_{1}}{R}\right)^{3}\right)\left[x^{s} \ln \left(x^{s}\right)+\left(1-x^{s}\right) \ln \left(1-x^{s}\right)\right]_{+}
$$

$$
N \frac{3 B_{H S} V_{H S} R_{\gamma}}{2 R}\left[x^{s}+\left(1-x^{s}\right) \frac{a_{H S}^{0} \Gamma_{\gamma}}{a_{L S}^{0} m}\right]_{+}
$$

$$
\begin{equation*}
N \frac{{ }_{N H S}^{B_{H S} V_{H S} R_{\sigma}}}{R}\left[x^{s}\left(\frac{V_{\text {min }}}{V_{H S}}-1\right)+\alpha \frac{a_{H S}^{0}}{a_{L S}^{0}}\left(1-x^{s}\right)\left(\frac{V_{\text {min }}}{V_{H S}}-\frac{1}{m}\right)\right], \tag{S11}
\end{equation*}
$$

The exact stationary solution of the spherical model is found using eqn (24) à the main text, which brings out the equation set:

$$
\begin{equation*}
\frac{B_{H S} V_{H S}}{2}\left(\gamma_{o}-1\right)\left[\left(\frac{V_{\min }}{V_{H S}}-1\right)^{2}-\frac{m}{\beta}\left(\frac{V_{\min }}{V_{H S}}-\frac{1}{m}\right)^{2}\right]+(\Delta H-T \Delta S)+k T \ln \left(\frac{x^{b}}{1-x^{b}}\right)=0 \tag{S12}
\end{equation*}
$$

$$
\begin{gather*}
\left(1-\left(\frac{R_{1}}{R}\right)^{3}\right)\left[\frac{B_{H S} V_{H S}}{2}\left(\gamma_{o}-1\right)\left[\left(\frac{V_{\min }}{V_{H S}}-1\right)^{2}-\frac{m}{\beta}\left(\frac{V_{\min }}{V_{H S}}-\frac{1}{m}\right)^{2}\right]+(\Delta H-T \Delta S)+k \ln \left(\frac{x^{s}}{1-x^{s}}\right)\right]+\frac{3 B_{H S} V_{H S} R_{\gamma}}{2 R} \\
\left(1-\frac{a_{H S}^{0} \Gamma_{\gamma}}{a_{L S}^{0} m}\right)+\frac{B_{H S} V_{H S} R_{\sigma}}{R}\left[\frac{V_{\min }}{V_{H S}}-1-\alpha \frac{a_{H S}^{0}}{a_{L S}^{0}}\left(\frac{V_{\min }}{V_{H S}}-\frac{1}{m}\right)\right]=0 \tag{S13}
\end{gather*}
$$

$$
B_{H S}\left(\frac{R_{1}}{R}\right)^{3}\left(\gamma_{o}-1\right)\left[x^{b}\left(\frac{V_{\min }}{V_{H S}}-1\right)+\left(1-x^{b}\right) \frac{m}{\beta}\left(\frac{V_{\min }}{V_{H S}}-\frac{1}{m}\right)\right]+B_{H S}\left(\gamma_{o}-1\right)\left(1-\left(\frac{R_{1}}{R}\right)^{3}\right)
$$

$$
\begin{equation*}
\left[x^{s}\left(\frac{V_{\min }}{V_{H S}}-1\right)+\left(1-x^{s}\right) \frac{m}{\beta}\left(\frac{V_{\min }}{V_{H S}}-\frac{1}{m}\right)\right]+\frac{B_{H S} R_{\sigma}}{R}\left[x^{s}+\alpha \frac{a_{H S}^{0}}{a_{L S}^{0}}\left(1-x^{s}\right)\right]=0 \tag{S14}
\end{equation*}
$$



Fig. S1. Calculated temperature dependence of the HS fraction (heating mode) in a 5 nm spherical particle of $[\mathrm{Fe}($ pyrazine $)]\left[\mathrm{Ni}(\mathrm{CN})_{4}\right]$ in the case of (a) $\Gamma_{\gamma}=\frac{\gamma_{L S}}{\gamma_{H S}}=1.4$ and (b) $\Gamma_{\gamma}=\frac{\gamma_{L S}}{\gamma_{H S}}=2.4$ for different values of $\gamma_{H S}$ varying from 10 to $190 \mathrm{~mJ} / \mathrm{m}^{2}$ with an increment of $10 \mathrm{~mJ} / \mathrm{m}^{2}$. The red curve represents the bulk material. Green and red arrows are guides for the eye to follow the evolution the residual HS fraction and the equilibrium temperature, respectively, with increasing of $\gamma_{H S}$.


Fig. S2: Calculated temperature dependence of the HS fraction (heating mode) in a 5 nm spherical particle of $[\mathrm{Fe}($ pyrazine $)]\left[\mathrm{Ni}(\mathrm{CN})_{4}\right]$ in the case of $\alpha=\frac{\sigma_{L S}}{\sigma_{H S}}=1.4$ for different values of $\sigma_{H S}$ ranging from (a) +10 to $+410 \mathrm{~mJ} / \mathrm{m}^{2}$ and (b) -410 to $-10 \mathrm{~mJ} / \mathrm{m}^{2}$ with an increment of $20 \mathrm{~mJ} / \mathrm{m}^{2}$. The red curve represents the bulk material. Green and red arrows are guides for the eye to follow the evolution the residual LS fraction and the equilibrium temperature, respectively, with increasing of $\sigma_{H S}$.


Fig. S3. Calculated temperature dependence of the total HS fraction (heating mode) for different particle radii of the compound $[\mathrm{Fe}($ pyrazine $)]\left[\mathrm{Ni}(\mathrm{CN})_{4}\right]$.

## III. Molecular dynamics calculations

Since no surface energy data in the literature for SCO materials have been found, we employed molecular dynamics (MD) simulations to calculate the surface energy of $[\mathrm{Fe}($ pyrazine $)]\left[\mathrm{Ni}(\mathrm{CN})_{4}\right]$ in the HS and LS spin states. The surface energy is related to the excess of potential energy per unit area in a system having free surfaces in comparison with its bulk counterpart. Accordingly, the slab method is used in the present MD simulation, ${ }^{1}$ where the potential energies of a bulk material, i.e. without surface and a finite-size system with free surfaces (or surface-vacuum interface) are needed to be calculated separately.

First, the $[\mathrm{Fe}($ pyrazine $)]\left[\mathrm{Ni}(\mathrm{CN})_{4}\right]$ structure with $20 \times 20 \times 20$ unit-cells is built in the LS state and the HS state, which corresponds to simulation box sizes of $140.26 \times 140.26 \times 135.52$ $\AA^{3}$ and $145.15 \times 145.15 \times 145.15 \AA^{3}$, respectively, containing 128000 atoms. Periodic boundary conditions are applied along the $x, y$ and $z$ directions to simulate the bulk material.

Then, a thin film having two solid-vacuum interfaces constituted with equivalent pyrazineterminated surfaces at the top and at the bottom is created in the HS and LS states (see Fig. S4). The model system with free surfaces, containing 130400 atoms, is composed with $20 \times 20$ unitcells in the $x$ and $y$ directions and 21 layers in the $z$ direction. Periodic boundary conditions are applied along the $x$ and $y$ directions, while open boundary conditions are applied along the $z$ direction to avoid self-interactions between the two surfaces. The surface energy ( $E_{\text {sur }}$ ) is simply estimated as follow: ${ }^{1}$

$$
\begin{equation*}
E_{\text {sur }}=\frac{E_{\text {tot }}-\sum_{i} n_{i} E_{\text {bulk-i }}}{2 S}, \tag{S15}
\end{equation*}
$$

where $S$ is the surface area after mechanical relaxation, $E_{\text {tot }}$ is the total energy of the model system with free surfaces. $n_{i}$ corresponds to the total number of atoms $i(i=\mathrm{Ni}, \mathrm{Fe}, \mathrm{C}$ and N$)$ at the surface and $E_{b u l k-i}$ stands for the energy of a single $i$ atom in the bulk material.

The present calculations are performed through the large-scale atomic/molecular massively parallel simulator package (LAMMPS) ${ }^{2}$ with a recently constructed $[\mathrm{Fe}($ pyrazine $)]\left[\mathrm{Ni}(\mathrm{CN})_{4}\right]$ force field. ${ }^{3}$ The temperature and pressure are controlled by the Nosé-Hoover method ${ }^{4,5}$ and a timestep of 1 fs is selected. Each modeled system has been fully relaxed at 300 K for 100000 MD steps to reach equilibrium state under the isothermal-isobaric (NPT) ensemble.


- Fe
- Ni
- C
- N
$y \leftarrow \int_{x}^{z}$
Fig. S4: Schematic representation of the $[\mathrm{Fe}($ pyrazine $)]\left[\mathrm{Ni}(\mathrm{CN})_{4}\right]$ film structure with periodic boundary condition along the $x$ and $y$-directions.

S1. A. Fahs, S. Mi, W. Nicolazzi, G. Molnár and A. Bousseksou, Adv. Phys. Res., 2023, 2, 2200055.

S2. S. Plimpton, J. Comput. Phys., 1995, 117, 1-19.
S3. S. Mi, A. Fahs, G. Molnár, W. Nicolazzi and A. Bousseksou, Chem. Phys. Lett., 2023, 811, 140232.
S4. S. Nosé, J. Chem. Phys., 1984, 81, 511-519.
S5. W. G. Hoover, Phys. Rev. A, 1985, 31, 1695.

