

## Supplementary Information

### **First principles study on the mechanical and thermodynamic properties of aluminium doped magnesium alloys**

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The mechanical stability criteria of *hcp* Mg, *fcc* Al and  $Mg_xAl_y$  alloys are listed below.

**Hexagonal phase ( $C_{11}$ ,  $C_{33}$ ,  $C_{44}$ ,  $C_{12}$  and  $C_{13}$ )**

$$C_{44} > 0, C_{11} > |C_{12}|, (C_{11} + 2C_{12})C_{33} > 2C_{13}^2$$

**Cubic phase( $C_{11}$ ,  $C_{44}$  and  $C_{12}$ )**

$$C_{11} > 0, C_{44} > 0, C_{11} > |C_{12}|, (C_{11} + 2C_{12}) > 0$$

The formulas of elastic moduli of *hcp* Mg, *fcc* Al and  $Mg_xAl_y$  alloys are listed below.

**Hexagonal phase ( $C_{11}$ ,  $C_{33}$ ,  $C_{44}$ ,  $C_{12}$  and  $C_{13}$ )**

$$B_V = (1/9)[2(C_{11} + C_{12}) + 4C_{13} + C_{33}]$$

$$G_V = (1/30)(M + 12C_{44} + 12C_{66})$$

$$B_R = C^2/M$$

$$G_R = (5/2)[C^2C_{44}C_{66}]/[3B_VC_{44}C_{66} + C^2(C_{44} + C_{66})]$$

$$M = C_{11} + C_{12} + 2C_{33} - 4C_{13}$$

$$C^2 = (C_{11} + C_{12})C_{33} - 2C_{13}^2$$

**Cubic phase( $C_{11}$ ,  $C_{44}$  and  $C_{12}$ )**

$$B_V = B_R = (C_{11} + 2C_{12})/3$$

$$G_V = (C_{11} - C_{12} + 3C_{44})/5$$

$$G_R = 5(C_{11} - C_{12})C_{44}/[4C_{44} + 3(C_{11} - C_{12})]$$

It is worth noting that the structures listed in Tables S1, S2 and S3 are the metastable  $Mg_xAl_y$ , except for  $Mg_{11}Al_5$ ,  $Mg_5Al_3$  and  $Mg_9Al_7$ . Listing the data of these structures in Supplementary Information was just for reference purposes if necessary.  $Mg_{11}Al_5$ ,  $Mg_5Al_3$  and  $Mg_9Al_7$  are the most stable structures determined based on the formation energies and convex hull diagram, and their elastic properties, electronic properties and thermodynamic properties were discussed in the main text.

**Table S1. The elastic constants (Gpa) of Mg<sub>x</sub>Al<sub>y</sub> structures.**

<b>Structure</b>	<b><math>C_{11}</math></b>	<b><math>C_{12}</math></b>	<b><math>C_{13}</math></b>	<b><math>C_{33}</math></b>	<b><math>C_{44}</math></b>
<b>Mg<sub>15</sub>Al<sub>1</sub></b>	<b>64.6</b>	<b>24.3</b>	<b>22.8</b>	<b>71.8</b>	<b>18.7</b>
<b>Mg<sub>7</sub>Al</b>	<b>66.9</b>	<b>29.1</b>	<b>24.6</b>	<b>73.4</b>	<b>21.1</b>
<b>Mg<sub>13</sub>Al<sub>3</sub></b>	<b>62.6</b>	<b>32.2</b>	<b>25.0</b>	<b>66.2</b>	<b>15.1</b>
<b>Mg<sub>3</sub>Al</b>	<b>69.6</b>	<b>31.7</b>	<b>30.5</b>	<b>84.7</b>	<b>14.5</b>
<b>MgAl</b>	<b>74.3</b>	<b>43.9</b>	<b>35.7</b>	<b>89.9</b>	<b>20.6</b>
<b>Mg<sub>7</sub>Al<sub>9</sub></b>	<b>71.8</b>	<b>54.4</b>	<b>40.1</b>	<b>91.2</b>	<b>23.1</b>
<b>Mg<sub>3</sub>Al<sub>5</sub></b>	<b>71.9</b>	<b>42.7</b>	<b>39.1</b>	<b>100.8</b>	<b>19.3</b>
<b>Mg<sub>5</sub>Al<sub>11</sub></b>	<b>75.0</b>	<b>44.7</b>	<b>38.0</b>	<b>108.2</b>	<b>21.1</b>
<b>MgAl<sub>3</sub></b>	<b>81.4</b>	<b>45.4</b>	<b>43.5</b>	<b>114.1</b>	<b>19.9</b>
<b>Mg<sub>3</sub>Al<sub>13</sub></b>	<b>67.3</b>	<b>47.5</b>	<b>54.2</b>	<b>104.0</b>	<b>23.4</b>
<b>MgAl<sub>7</sub></b>	<b>75.2</b>	<b>49.9</b>	<b>50.7</b>	<b>112.9</b>	<b>17.8</b>
<b>MgAl<sub>15</sub></b>	<b>82.2</b>	<b>48.3</b>	<b>58.3</b>	<b>106.1</b>	<b>17.2</b>

**Table S2.** The elastic moduli with the unit of GPa ( $B_R$  and  $B_V$  represent the bulk moduli with the Reuss and Voigt averaging scheme, respectively;  $G_R$  and  $G_V$  represent the shear moduli with the Reuss and Voigt averaging scheme, respectively;  $E$ ,  $B$  and  $G$  represent Young's modulus, bulk modulus and shear modulus with the Voigt-Reuss-Hill averaging scheme, respectively), Poisson's ratio ( $\nu$ ), anisotropy ( $A^U$ ) and hardness ( $H_v$ ) of  $Mg_xAl_y$ .

Structure	$B_R$	$B_V$	$G_R$	$G_V$	$E$	$B$	$G$	$\nu$	$B/G$	$A^U$	$H_v$
$Mg_{15}Al_1$	37.82	37.87	20.10	20.25	51.4	37.8	20.2	0.27	1.88	0.04	3.8
$Mg_7Al$	40.42	40.42	20.65	20.81	53.1	40.4	20.7	0.28	1.95	0.04	3.7
$Mg_{13}Al_3$	39.51	39.53	16.07	16.36	42.8	39.5	16.2	0.32	2.44	0.09	2.4
$Mg_3Al$	45.19	45.48	17.60	18.34	47.6	45.3	18.0	0.32	2.52	0.22	2.5
$MgAl$	52.04	52.12	18.70	19.49	51.1	52.1	19.1	0.34	2.73	0.21	2.4
$Mg_7Al_9$	55.96	56.00	14.01	17.66	43.4	56.0	15.8	0.37	3.54	1.30	1.5
$Mg_3Al_5$	53.15	54.04	17.93	18.89	49.6	53.6	18.4	0.35	2.91	0.28	2.1
$Mg_5Al_{11}$	54.66	55.51	19.24	20.64	53.4	55.1	19.9	0.34	2.76	0.38	2.4
$MgAl_3$	59.02	60.19	20.37	21.19	55.9	59.6	20.8	0.34	2.87	0.22	2.4
$Mg_3Al_{13}$	57.21	61.16	14.37	16.85	43.1	59.2	15.6	0.38	3.79	0.93	1.4
$MgAl_7$	60.65	62.88	16.00	17.12	45.6	61.8	16.6	0.38	3.73	0.39	1.5
$MgAl_{15}$	64.37	66.70	17.18	17.31	47.6	65.5	17.2	0.38	3.80	0.07	1.5

**Table S3. Theoretically calculated thermal properties of  $\text{Mg}_x\text{Al}_y$  alloys including longitudinal sound velocity ( $v_l$ ), shear sound velocity ( $v_t$ ) and average sound velocity ( $v_m$ ) and Debye temperature ( $\Theta_D$ ).**

Structure	$v_l(\text{m}\cdot\text{s}^{-1})$	$v_t(\text{m}\cdot\text{s}^{-1})$	$v_m(\text{m}\cdot\text{s}^{-1})$	$\Theta_D$ (K)
$\text{Mg}_{15}\text{Al}$	6011.21	3355.94	3736.24	394.39
$\text{Mg}_7\text{Al}$	6077.94	3354.36	3737.87	397.21
$\text{Mg}_{13}\text{Al}_3$	5692.69	2931.37	3282.52	350.69
$\text{Mg}_3\text{Al}$	5928.49	3019.14	3383.16	365.86
$\text{MgAl}$	5945.14	2950.51	3311.36	367.42
$\text{Mg}_7\text{Al}_9$	5844.30	2648.40	2986.05	333.64
$\text{Mg}_3\text{Al}_5$	5839.48	2834.30	3184.89	356.82
$\text{Mg}_5\text{Al}_{11}$	5907.32	2918.80	3276.60	368.79
$\text{MgAl}_3$	6031.89	2942.58	3305.65	374.28
$\text{Mg}_3\text{Al}_{13}$	5705.09	2520.21	2844.63	323.88
$\text{MgAl}_7$	5773.07	2565.58	2895.11	331.43
$\text{MgAl}_{15}$	5852.68	2582.92	2915.52	335.97