

ESI: Critical Role of Lattice Vacancies in Pressure-Induced Phase Transitions of Baroplastic Diblock Copolymers

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1 Compressible Self-Consistent Field Theory for Diblock Copolymer Systems

Let us consider a compressible system of diblock copolymers *A-block-B* (*d*) under constant temperature T and isotropic pressure P . Let n_d be the number of diblock copolymers, and let N_A and N_B be the number of segments of each block, respectively. The total number of segments is $N_d = N_A + N_B$, and the block ratio of the A-component is $f = N_A/N_d$. Based on the lattice vacancy concept¹⁻³, we consider the free volume of the system, described as voids (*v*) below. This model treats voids as another component of n_v conceptual solvent molecules composed of a single segment ($N_v = 1$), enabling the straightforward conversion of the classical incompressible lattice model to a compressible one. In this model, changes in the number of voids express changes in the system volume V . The average segment number density is written as $\rho_0 = (n_d N_d + n_v)/V$. For simplicity, we assume that all segments occupy equal volume, $v_0 = 1/\rho_0$, and polymer segments have equal Kuhn length, $a = a_A = a_B$, as in the usual SCF theories⁴.

The compressible SCF theory with voids can be formulated in the straightforward way of the incompressible SCF theory⁵. The detailed procedure for deriving SCF equations and a free energy functional of an incompressible system is explained in references^{4,6-8}. Consequently, we here outline the formulation of the compressible SCF theory.

The Hamiltonian \mathcal{H} of the system is divided into the ideal chain part \mathcal{H}_0 and the non-bonded interaction part \mathcal{W} . The ideal chain part is given by $\mathcal{H}_0 = (3/2N_d a^2) \int_0^1 ds (d\mathbf{r}_\alpha(s)/ds)^2$, where s is the counter variable from 0 to 1 scaled by the chain length, and $\mathbf{r}_\alpha(s)$ specifies the space curve of the α -th chain of the block copolymer. We give the dimensionless density operators at position \mathbf{r} , $\hat{\phi}_A(\mathbf{r}) = (N_d/\rho_0) \sum_{\alpha=1}^{n_d} \int_0^f ds \delta(\mathbf{r} - \mathbf{r}_\alpha(s))$, $\hat{\phi}_B(\mathbf{r}) = (N_d/\rho_0) \sum_{\alpha=1}^{n_d} \int_f^1 ds \delta(\mathbf{r} - \mathbf{r}_\alpha(s))$, and $\hat{\phi}_v(\mathbf{r}) = (1/\rho_0) \sum_{\gamma=1}^{n_v} \delta(\mathbf{r} - \mathbf{r}_\gamma)$ where \mathbf{r}_γ expresses a local position of the γ -th void. Then, the partition function \mathcal{Z} can be written as

$$\mathcal{Z} = \int \prod_{\alpha=1}^{n_d} \tilde{\mathcal{D}}\mathbf{r}_\alpha \prod_{\gamma=1}^{n_v} d\mathbf{r}_\gamma \delta(1 - \hat{\phi}_A - \hat{\phi}_B - \hat{\phi}_v) \exp(-\beta\mathcal{W}) \quad (1)$$

where $\beta \equiv 1/k_B T$, and the first functional integral over all configurations is weighted with the ideal chain part, $\tilde{\mathcal{D}}\mathbf{r}_\alpha \equiv \mathcal{D}\mathbf{r}_\alpha \exp(-\beta\mathcal{H}_0)$. The delta functional enforces the incompressibility constraint of $\hat{\phi}_A(\mathbf{r}) + \hat{\phi}_B(\mathbf{r}) + \hat{\phi}_v(\mathbf{r}) = 1$. The non-bonded interaction part of the Hamiltonian, \mathcal{W} , can be written with

the Flory interaction parameters, χ_{ij} , as $\beta\mathcal{W} = (\rho_0/2) \int d\mathbf{r} \sum_{i,j} \chi_{ij} \hat{\phi}_i \hat{\phi}_j$ where $i, j = A, B, v$. The Flory interaction parameters are defined as $\chi_{ij} = \chi_{ji} \equiv \beta(\epsilon_{ii}^* + \epsilon_{jj}^* - 2\epsilon_{ij}^*)$ where $\epsilon_{ij}^* (\geq 0)$ is the characteristic self-interaction energy between the i -th and the j -th components having a unit of [J]. In the lattice vacancy model, the interactions between the i -th components and voids are assigned zero energy, $\epsilon_{Av}^* = \epsilon_{Bv}^* = \epsilon_{vv}^* = 0$ [J]¹⁻³.

After the replacement of operators by field valuables and the saddle-point approximation, the partition function can be rewritten as

$$\mathcal{Z} = \int \prod_i (\mathcal{D}\phi_i \mathcal{D}\omega_i) \mathcal{D}\xi \prod_k \left(\frac{\mathcal{Q}_k^{n_k}}{n_k!} \right) \exp \left\{ -\frac{\rho_0}{N_d} \int d\mathbf{r} \left[\frac{1}{2} \sum_{i,j} \chi_{ij} N_d \phi_i \phi_j - \sum_i \omega_i \phi_i - \xi \left(1 - \sum_i \phi_i \right) \right] \right\} \quad (2)$$

where $k = d, v$, and \mathcal{Q}_d and \mathcal{Q}_v are the partition functions for a single molecule of the diblock copolymer and the void, respectively. The field valuables, ϕ_i, ω_i , and ξ , denote the density field of the i -th component, the self-consistent field of the i -th component, and the Lagrange multiplier field. The partition function gives free energy of

$$\frac{N_d F}{\rho_0 k_B T V} = - \sum_k \left[\frac{N_d}{N_k} \bar{\phi}_k \ln \left(\frac{N_k \mathcal{Q}_k}{N_d V \bar{\phi}_k} \right) \right] + \frac{1}{V} \int d\mathbf{r} \left[\frac{1}{2} \sum_{i,j} \chi_{ij} N_d \phi_i \phi_j - \sum_i \omega_i \phi_i - \xi \left(1 - \sum_i \phi_i \right) \right] \quad (3)$$

where $\bar{\phi}_k$ is the average volume fraction of the k -th component. The SCF equations are also derived from the partition function as follows:

$$\omega_i(\mathbf{r}) = \sum_j \chi_{ij} N_d \phi_j(\mathbf{r}) + \xi(\mathbf{r}) \quad (4)$$

$$\phi_A(\mathbf{r}) + \phi_B(\mathbf{r}) + \phi_v(\mathbf{r}) = 1 \quad (5)$$

$$\phi_A(\mathbf{r}) = \frac{\bar{\phi}_d V}{\mathcal{Q}_d} \int_0^f ds q(\mathbf{r}, s) q^\dagger(\mathbf{r}, s) \quad (6)$$

$$\phi_B(\mathbf{r}) = \frac{\bar{\phi}_d V}{\mathcal{Q}_d} \int_f^1 ds q(\mathbf{r}, s) q^\dagger(\mathbf{r}, s) \quad (7)$$

$$\phi_v(\mathbf{r}) = \frac{\bar{\phi}_v V}{\mathcal{Q}_v} \exp \left[-\frac{1}{N_d} \omega_v(\mathbf{r}) \right] \quad (8)$$

The end-segment distribution function of the diblock copolymers, $q(\mathbf{r}, s)$, is the solution to the modified diffusion equation of $\partial q / \partial s = (1/6) N_d a^2 \nabla^2 q - \omega_d q$ where $\omega_d(\mathbf{r}, s) = \omega_A(\mathbf{r})$ for $s \in [0, f]$ and $\omega_d(\mathbf{r}, s) = \omega_B(\mathbf{r})$ for $s \in [f, 1]$. The initial condition is $q(\mathbf{r}, 0) = 1$. A second end-segment distribution function for the opposite chain end $q^\dagger(\mathbf{r}, s)$ satisfies the above-modified diffusion equation with a minus sign on the right-hand side and the initial condition $q^\dagger(\mathbf{r}, 1) = 1$. The single-molecule partition functions can be written as $\mathcal{Q}_d = \int d\mathbf{r} q(\mathbf{r}, 1)$ and $\mathcal{Q}_v = \int d\mathbf{r} \exp[-\omega_v(\mathbf{r})/N_d]$. In this study, the modified diffusion equation was solved by the pseudo-spectral method^{6,9}. One-dimensional calculation cell dimensions were optimized by the valuable cell shape method^{6,10}. We continued the iterative calculation until the differences of the self-consistent fields were less than 10^{-6} . Note that applications of this compressible SCF theory to copolymer systems of more complicated structures, such as linear multiblock copolymers and starpolymers, are straightforward^{6,7,11-13}.

The number of voids can be determined using the Sanchez-Lacombe equation of state (SL-EOS), derived based on the lattice vacancy model¹⁻³. The segment volume and the segment number are written as $v_i^* = k_B T_i^* / P_i^*$ and $N_i = M_i / \rho_i^* v_i^*$, where $P_i^* (= \epsilon_{ii}^* / v_i^*)$, $T_i^* (= \epsilon_{ii}^* / k_B)$, and ρ_i^* are the i -th pure-component EOS parameters, and M_i is the molecular weight of the i -th component. The EOS parameters for mixtures, P^* , T^* , and ρ^* can be obtained by combining pure-component parameters. The combining rules of diblock copolymers (binary case) are given by^{2,3}

$$v_0 = \frac{f}{f + (v_A^*/v_B^*)(1-f)} v_A^* + \frac{1-f}{(1-f) + (v_B^*/v_A^*)f} v_B^* \quad (9)$$

$$P^* = f^2 P_A^* + (1-f)^2 P_B^* + 2f(1-f)P_{AB}^* \quad (10)$$

$$T^* = P^* v_0 / k_B \quad (11)$$

$$\rho^* = \frac{1}{w_A / \rho_A^* + w_B / \rho_B^*} \quad (12)$$

where w_A and w_B are the weight fractions of A and B. The cross term of characteristic pressure, P_{ij}^* , is written with a dimensionless adjustable parameter ξ_{ij} as $P_{ij}^* = \xi_{ij} \sqrt{P_i^* P_j^*}$. The parameter ξ_{ij} is determined to reproduce the phase behavior of the system. The dimensionless interaction parameter, χ_{ij} , is related as $\chi_{ij} = \beta \Delta P_{ij}^* v_0$ where $\Delta P_{ij}^* \equiv P_i^* + P_j^* - 2P_{ij}^*$ ^{2,3}. The conventional form of the SL-EOS is $\tilde{\rho}^2 + \tilde{P} + \tilde{T}[\ln(1 - \tilde{\rho}) + \tilde{\rho}] = 0$, where $\tilde{\rho}$ ($\equiv \rho / \rho^*$), \tilde{P} ($\equiv P / P^*$), and \tilde{T} ($\equiv T / T^*$) are the dimensionless mass density, pressure, and temperature, respectively. This EOS can be rewritten by using the valuables in the SCF theory as follows:

$$P = \frac{k_B T}{v_0} \left\{ -\ln \bar{\phi}_v - (1 - \bar{\phi}_v) + [\chi_{AB} f(1-f) - \chi_{Av} f - \chi_{Bv}(1-f)](1 - \bar{\phi}_v)^2 \right\} \quad (13)$$

At given T and P , $\bar{\phi}_v$ can be calculated by Equation (13).

2 Extension of the Compressible SCF Theory to Multi-Component Systems

The compressible SCF theory based on the lattice vacancy model can straightforwardly introduce additional polymer and solvent species. For example, let us consider an additional component of a gas pressure medium (g) absorbed into the polymer phase. Let n_g be the number of gas solvents, and let N_g be the number of segments of each solvent. In this case, the average segment number density is written as $\rho_0 = (n_d N_d + n_g N_g + n_v) / V$ where V is the system volume. The partition function \mathcal{Z} is modified as

$$\mathcal{Z} = \int \prod_i (\mathcal{D}\phi_i \mathcal{D}\omega_i) \mathcal{D}\xi \prod_k \left(\frac{\mathcal{Q}_k^{n_k}}{n_k!} \right) \exp \left\{ -\frac{\rho_0}{N_d} \int d\mathbf{r} \left[\frac{1}{2} \sum_{i,j} \chi_{ij} N_d \phi_i \phi_j - \sum_i \omega_i \phi_i - \xi \left(1 - \sum_i \phi_i \right) \right] \right\} \quad (14)$$

where $i, j = A, B, g, v$, $k = d, g, v$, and \mathcal{Q}_g is the partition functions for a single molecule of the gas. This partition function \mathcal{Z} yields the following free energy and SCF equations:

$$\frac{N_d F}{\rho_0 k_B T V} = - \sum_k \left[\frac{N_d}{N_k} \bar{\phi}_k \ln \left(\frac{N_k \mathcal{Q}_k}{N_d \bar{\phi}_k V} \right) \right] + \frac{1}{V} \int d\mathbf{r} \left[\frac{1}{2} \sum_{i,j} \chi_{ij} N_d \phi_i \phi_j - \sum_i \omega_i \phi_i - \xi \left(1 - \sum_i \phi_i \right) \right] \quad (15)$$

$$\omega_i(\mathbf{r}) = \sum_j \chi_{ij} N_d \phi_j(\mathbf{r}) + \xi(\mathbf{r}) \quad (16)$$

$$\sum_i \phi_i(\mathbf{r}) = 1 \quad (17)$$

$$\phi_A(\mathbf{r}) = \frac{\bar{\phi}_d V}{\mathcal{Q}_d} \int_0^f ds q(\mathbf{r}, s) q^\dagger(\mathbf{r}, s) \quad (18)$$

$$\phi_B(\mathbf{r}) = \frac{\bar{\phi}_d V}{\mathcal{Q}_d} \int_f^1 ds q(\mathbf{r}, s) q^\dagger(\mathbf{r}, s) \quad (19)$$

$$\phi_g(\mathbf{r}) = \frac{\bar{\phi}_g V}{\mathcal{Q}_g} \exp \left[-\frac{N_g}{N_d} \omega_g(\mathbf{r}) \right] \quad (20)$$

$$\phi_v(\mathbf{r}) = \frac{\bar{\phi}_v V}{\mathcal{Q}_v} \exp \left[-\frac{1}{N_d} \omega_v(\mathbf{r}) \right] \quad (21)$$

where

$$\mathcal{Q}_g = \int d\mathbf{r} \exp\left[-\frac{N_g}{N_d}\omega_g(\mathbf{r})\right] \quad (22)$$

When little gas is absorbed into the polymer phase, we can approximate the combining rules for SL-EOS by those of pure diblock copolymers. Henry's law determines the absorption amount of gases with applied pressure: $w_g = HP$ where w_g is the weight fraction of the gas absorbed by the polymer and H is the Henry's law constant. According to the SL-EOS theory, the Henry's law constant of diblock copolymer solutions is given by¹⁴

$$H = \frac{M_g}{RT\rho^*} \exp\left\{(1-\bar{\phi}_v)\frac{M_g}{\rho_g^*k_B T}[P_g^* - f\Delta P_{Ag}^* - (1-f)\Delta P_{Bg}^* + f(1-f)\Delta P_{AB}^*] - N_g\left[1 + \frac{\bar{\phi}_v}{1-\bar{\phi}_v} \ln \bar{\phi}_v\right]\right\} \quad (23)$$

where N_g is the number of segments of the gas molecules. From obtained w_g , we can calculate the average gas volume fraction as^{2,3}

$$\bar{\phi}_g = \frac{w_g/\rho_g^*}{w_g/\rho_g^* + (1-w_g)/\rho^*}(1-\bar{\phi}_v) \quad (24)$$

3 Determination of Equation-of-State Parameters

Table S1: SL-EOS Parameters of Polymers and Gases

Parameters	PS	PnPMA	PMMA	N ₂	He
P_i^* [MPa] ^{a)}	366.9	324.8	366.6	617.3	31.8
T_i^* [K] ^{a)}	773.0	684.4	772.4	162.2	7.8
ρ_i^* [g/cm ³] ^{a)}	1.0928	1.0560	1.2373	1.8943	0.5300
$\xi_{PS,i}$		1.00154 ^{b)}	0.99735 ^{c)}	0.75 ^{b)}	0.80 ^{b)}
$\xi_{PnPMA,i}$	1.00154 ^{b)}			1.05 ^{b)}	1.25 ^{b)}

^{a)} Literature values for PS¹⁵, N₂¹⁶, and He¹⁶, and determined values by fitting literature *PVT* data for PnPMA¹⁶ and PMMA¹⁷. ^{b)} Determined to reproduce the literature data^{16,18}. ^{c)} Determined to reproduce the literature data¹⁹.

We determined the equation-of-state (EOS) parameters from the *PVT* data and the phase behavior in references as shown in Table S1. We referred to the literature values of pure-component parameters of PS¹⁵, N₂¹⁶, and He¹⁶. We determined the parameters of PnPMA and PMMA by fitting literature value at 468 K¹⁶ and at 398 K¹⁷, assuming the fixed segment volume identical to the segment volume of PS (17.52 cm³/mol) as shown in Figure S1 and S3, respectively. In this paper, we investigate PS-*b*-PnPMA of $N_d \simeq 2600$ and $f = 0.5$ at $T = 468$ K, which corresponds to the original experimental data for PS-*b*-PnPMA with a molecular weight of $M_w = 48700$ ¹⁶. We adjusted the parameter $\xi_{PS,PnPMA}$ to reproduce the experimental phase behavior at 468 K (P_{ODT} : 3–4 MPa)^{16,18}. We also investigate PS-*b*-PMMA of $N_d \simeq 1100$ ($M_n = 22160$) and $f = 0.5$ at $T = 393$ K and adjusted $\xi_{PS,PMMA}$ to reproduce the phase behavior at 393 K ($P_{ODT} = 0.1$ MPa)¹⁹. The parameters for PS-*b*-PnPMA/He, $\xi_{PS,He}$ and $\xi_{PnPMA,He}$, are adjusted to reproduce $P_{ODT} \simeq 4$ MPa at 468 K¹⁶. The parameters for PS-*b*-PnPMA/N₂, ξ_{PS,N_2} and ξ_{PnPMA,N_2} are chosen to reproduce the non-baroplastic (barotropic) phase behavior at 468 K¹⁶. In this work, we have considered baroplastic BCPs in a narrow temperature-pressure window. The equation-of-state parameters may have different values in the other temperature-pressure window. A complex temperature and pressure dependence of interaction parameters in a large temperature-pressure window is crucial for future research endeavors.

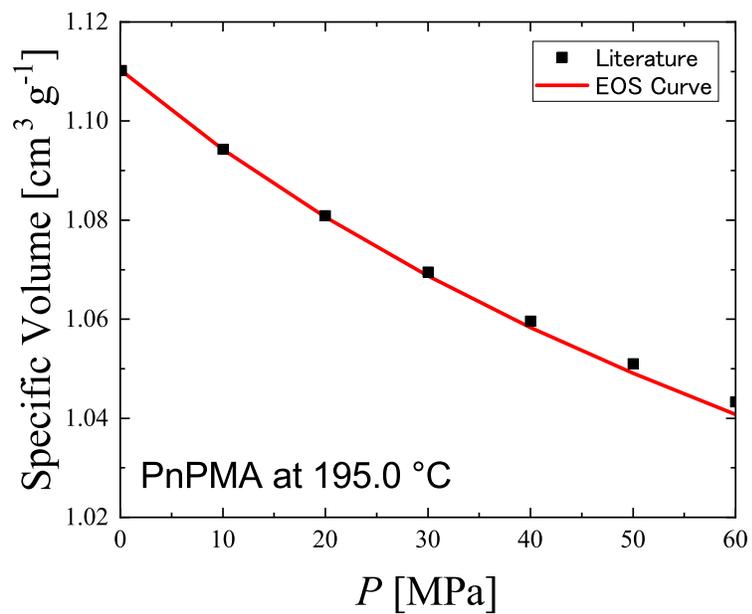


Figure S1: SL-EOS curve fitting to the literature data¹⁶ of PnPMA at 195.0 °C (468 K).

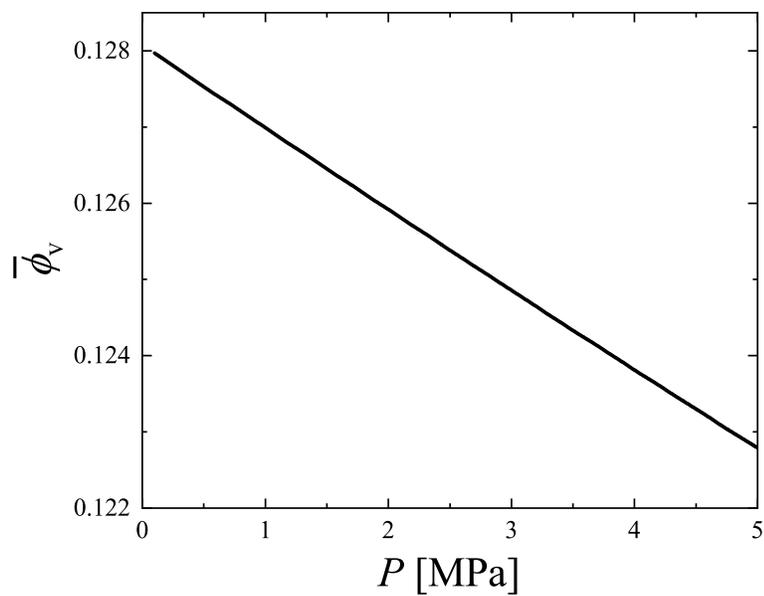


Figure S2: SL-EOS curve of PS-*b*-PnPMA at 468 K ($N_d \simeq 2600$, $f = 0.5$).

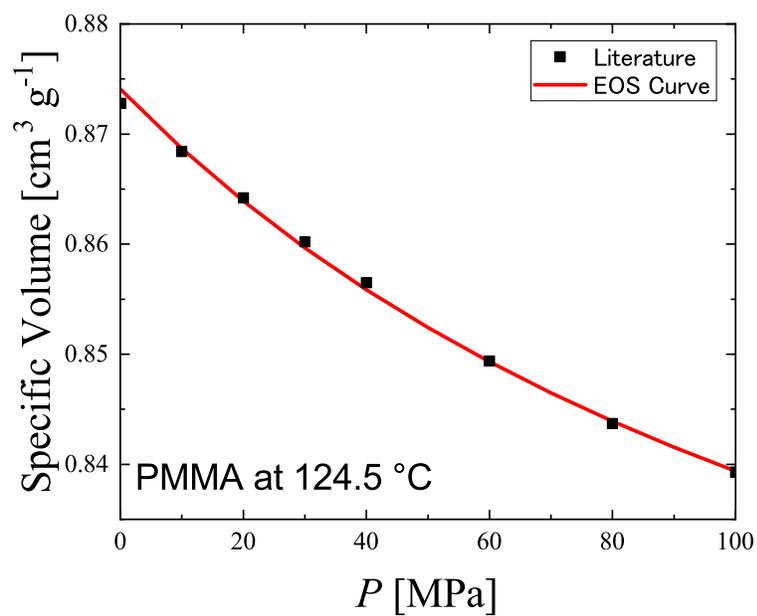


Figure S3: SL-EOS curve fitting to the literature data¹⁷ of PMMA at 124.5 °C (398 K).

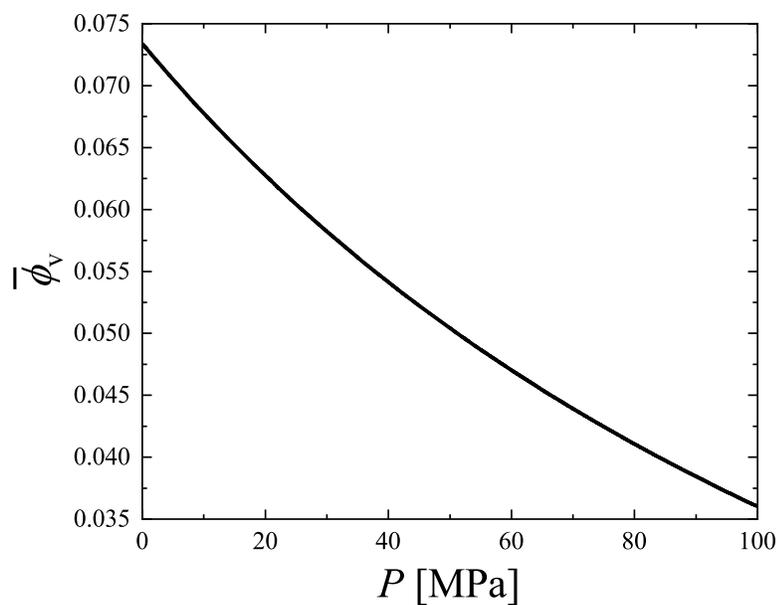


Figure S4: SL-EOS curve of PS-*b*-PMMA at 393 K ($N_d \simeq 1100$, $f = 0.5$).

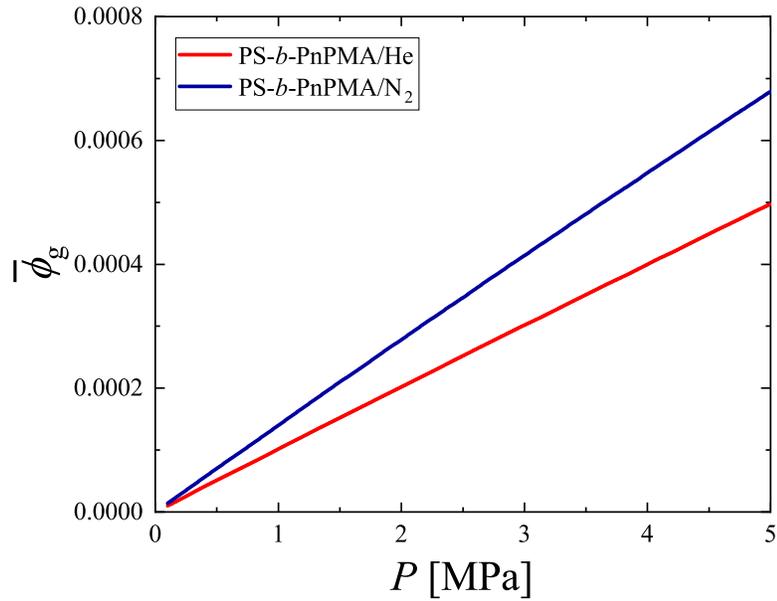


Figure S5: The volume fraction of gases absorbed into PS-*b*-PnPMA as a function of pressure ($N_d \simeq 2600$, $f = 0.5$, $T = 468$ K).

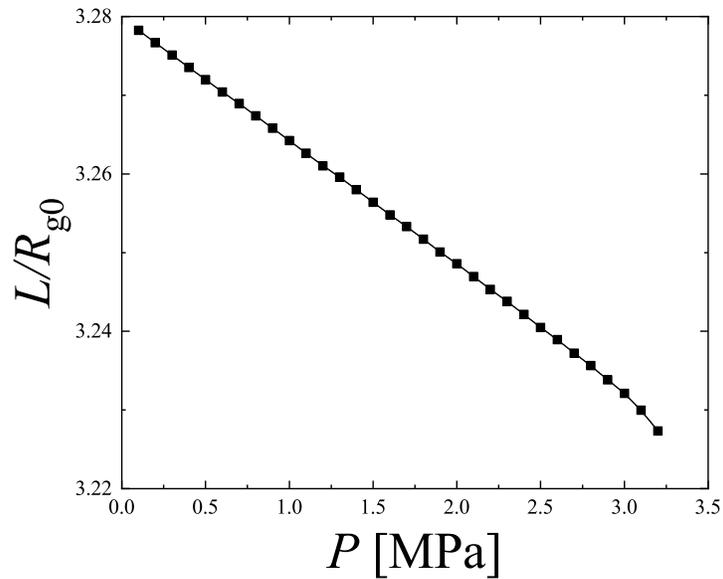


Figure S6: PS-*b*-PnPMA lamellar period as a function of pressure at 468 K ($N_d \simeq 2600$, $f = 0.5$). The length is scaled by the radius of gyration of an ideal chain, R_{g0} .

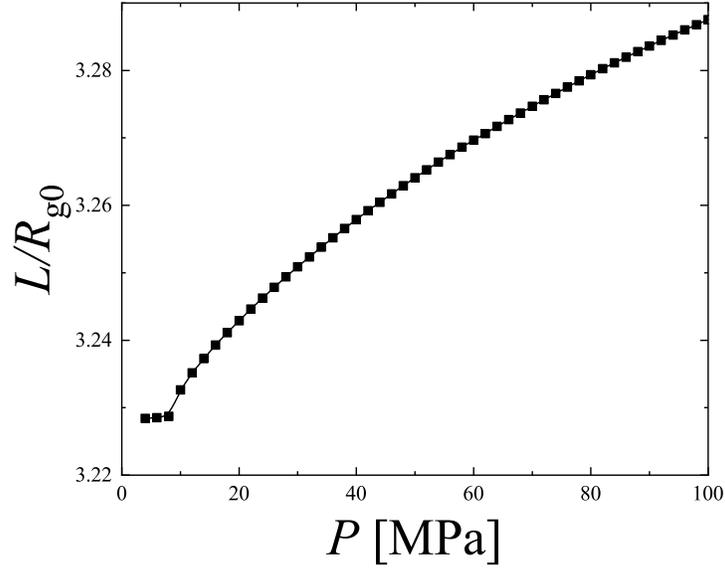


Figure S7: PS-*b*-PMMA lamellar period as a function of pressure at 393 K ($N_d \simeq 1100$, $f = 0.5$). The length is scaled by the radius of gyration of an ideal chain, R_{g0} .

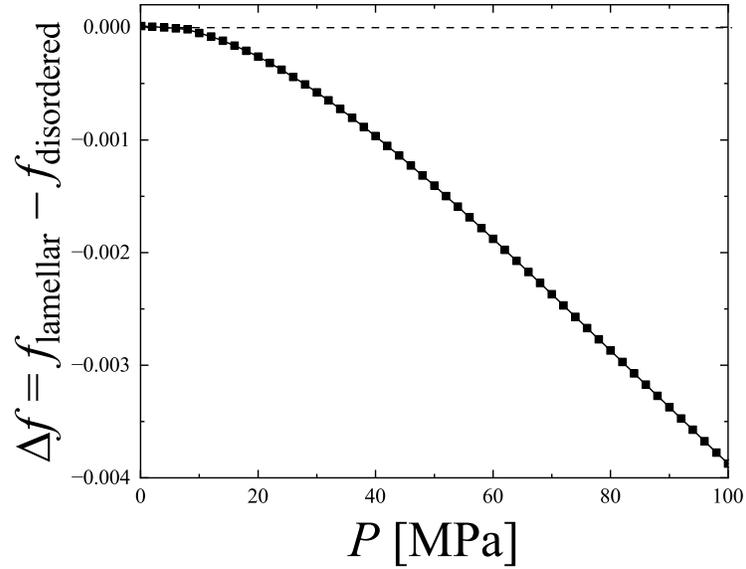


Figure S8: Pressure dependence of Δf ($= f_{\text{lamellar}} - f_{\text{disordered}}$) of PS-*b*-PMMA ($N_d \simeq 1100$, $f = 0.5$, $T = 393$ K).

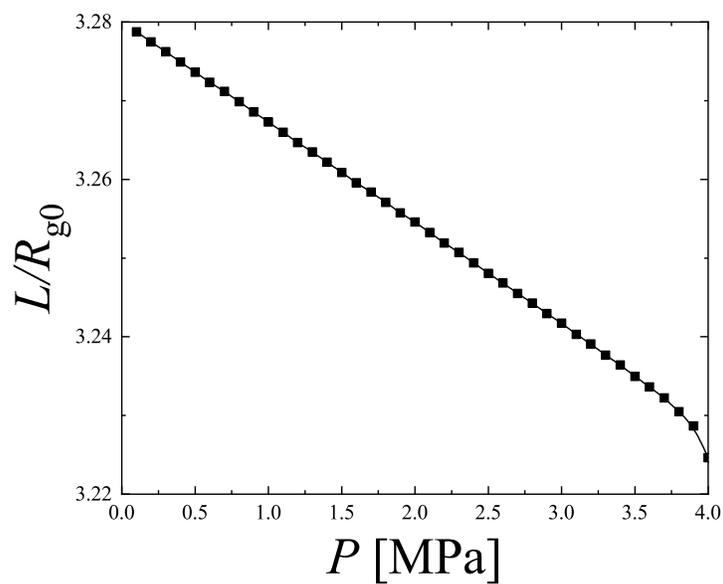


Figure S9: PS-*b*-PnPMA/He lamellar period as a function of pressure at 468 K ($N_d \simeq 2600$, $f = 0.5$). The length is scaled by the radius of gyration of an ideal chain, R_{g0} .

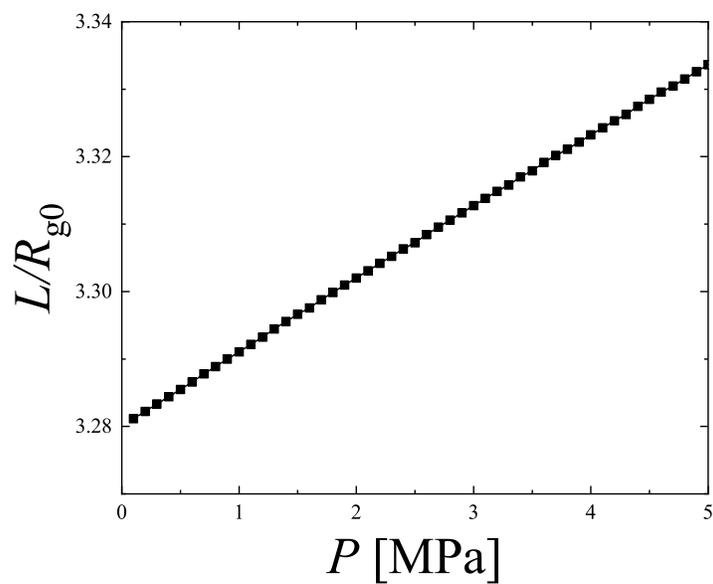


Figure S10: PS-*b*-PnPMA/ N_2 lamellar period as a function of pressure at 468 K ($N_d \simeq 2600$, $f = 0.5$). The length is scaled by the radius of gyration of an ideal chain, R_{g0} .

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