Electronic Supplementary Material (ESI) for Soft Matter.
This journal is © The Royal Society of Chemistry 2024

Electronic supplementary information

Inelastic effects in bulge formation of inflated polymer tubes

Fatemeh Rouhani[1], Jack Wurster Pazin[1], Brian A. Young[3], Qihan Liu[1], Sachin Velankar[1,2*]

[1]Department of Mechanical Engineering and Material Science, [2]Department of Chemical Engineering, University of Pittsburgh, Pittsburgh PA 15261 USA

[3]Plastics Engineering and Technology Department, Penn State University Behrend, Erie PA 16563 USA

Section S 1: Additional supplementary information

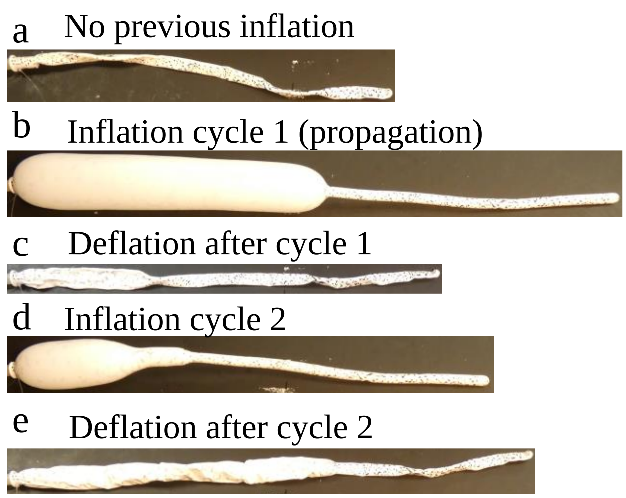

Figure S 1: A long rubber balloon during two cycles of inflation-deflation with air. After the first deflation (image c), the portion of the balloon that had bulged is permanently deformed. The second inflation (image d) shows three coexisting diameters, the smallest of which corresponds to the portion of the balloon that has never experienced large inflation, and the largest corresponds to the portion of the balloon that has bulged in both cycles. Black marks are ink marks to visualize the deformation.

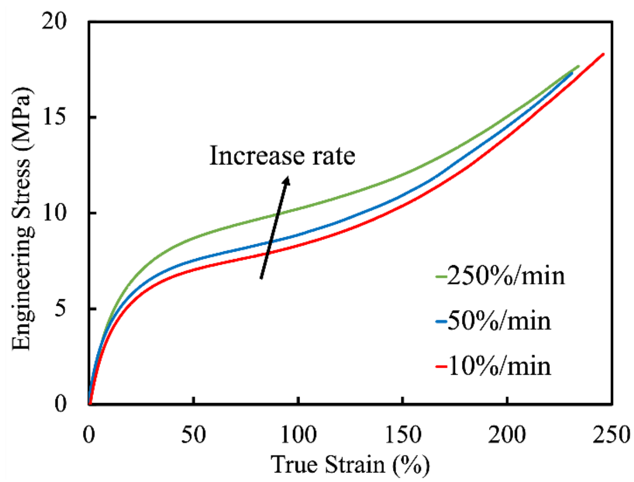

Figure S 2: Uniaxial tensile testing data for polyurethane tubes during loading and unloading at nominal strain rates of 10% per minute to 250% per minute.

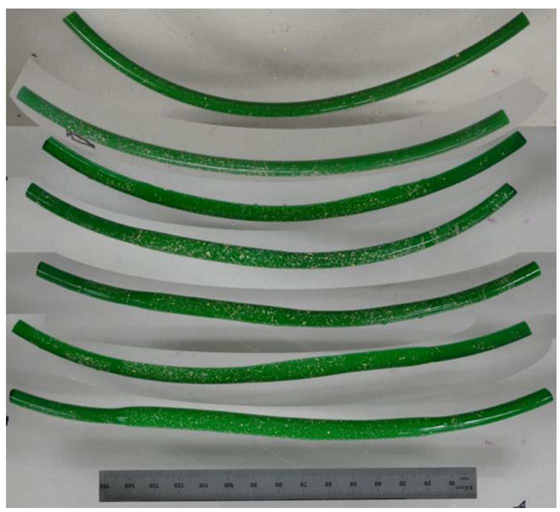

Figure S 3: Inelastic deformation of tubes after being inflated at 2 mL/min to the volumes listed alongside each figure. The ruler at the bottom is 150 mm long. The undeformed specimen (uppermost image) appears bent due to the intrinsic curvature of the tube (see Section 2 in the main text). Significant permanent deformation appears for $\Delta V \geq 10$ mL, both as a gradual straightening of the tube, as well as a permanent change in diameter. Deflation after $\Delta V = 6$ mL (far above $\Delta V = 2.5$ mL at the pressure maximum) induces only slight permanent deformation. This is consistent with Figure 2 and Figure 4 in the main text: at $\Delta V = 6$ mL, the axial stretch is less than 1.25 (Figure 4d), and the circumferential stretch is less than 1.6 (Figure 4c), values at which permanent deformation is modest (Figure 2b).

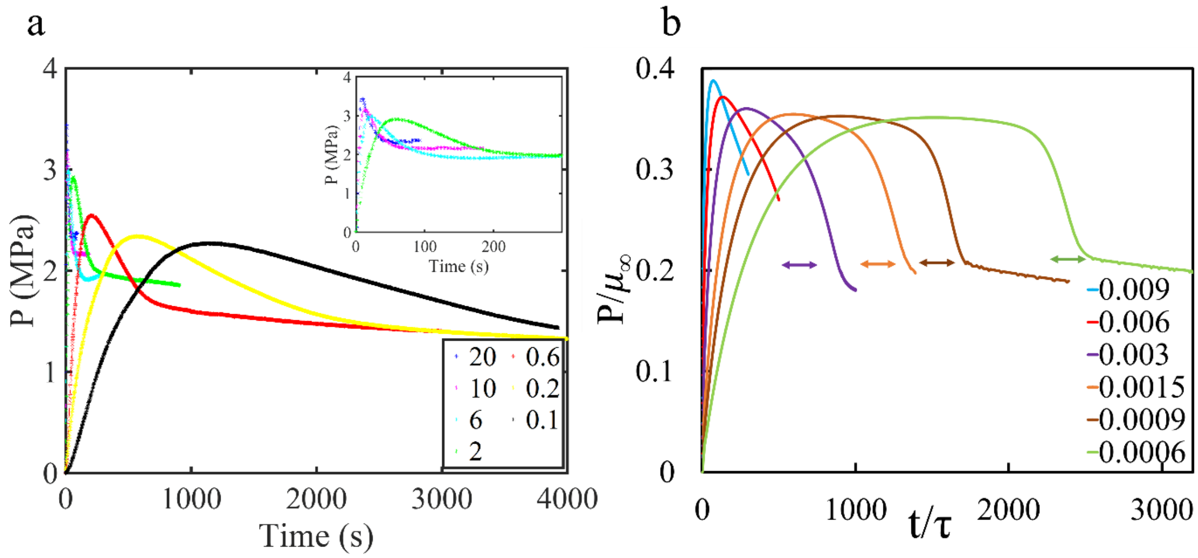

Figure S 4: a. Same data as Figure 4 in the main paper, but with time on the x-axis. Legend lists the various flow rate experiments listed in the legend (all in mL/min). b. Same data as Figure 9 in the main paper, but with nondimensional time in the x-axis. The double arrows (all 50 units wide) indicate that the timescale for pressure-unloading is approximately equal at all *Wi* values.

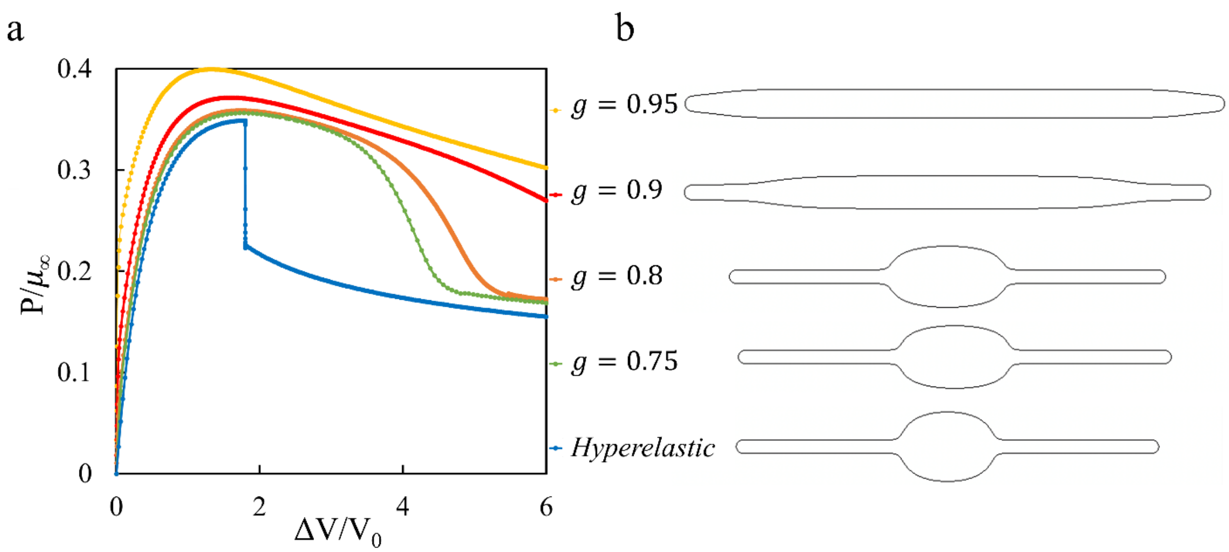

Figure S 5: Effect of the parameter g on tube inflation at one single *Wi* value of 0.006. b. Corresponding profiles of tubes at the same g values (increasing from bottom to top), at a single $\Delta V/V_0$ value of 6.

Section S 2: Hyperelastic tube inflation model

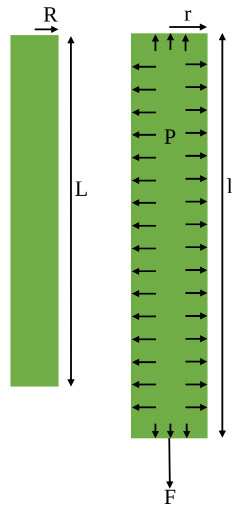

Figure S 6: External force and internal pressure induces axial and circumferential stress as the following equations.

Chang and Kyriakides [7,12] examined the homogeneous inflation of thick-walled tubes, and then derived Eq. 3 and Eq. 4 in the main text in the limiting case of a thin-walled tube. Here we rederive the same equations, but more directly for a thin-walled tube. Consider the inflation of a thin-walled tube of undeformed radius R, undeformed wall thickness H, and undeformed length L, with an external axial force, F (which might be zero). The internal pressure causes both axial and circumferential stresses, whereas the external force causes axial stress only. Assuming homogeneous inflation, a force balance then leads to:

$$\sigma_z = \frac{F}{2\pi r h} + \frac{Pr}{2h} \quad \text{Eq. S. 1}$$

$$\sigma_\theta = \frac{Pr}{h} \quad \text{Eq. S. 2}$$

Here r and h are deformed radius and deformed wall thickness of the tube, respectively.

The deformed and undeformed tube dimensions can be related by incompressibility of the tube walls, i.e. by setting $\pi RHL = \pi rhl$, thus giving:

$$\lambda_z r h = RH \quad \text{Eq. S. 3}$$

where λ_z is axial stretch of the tube. It is useful to rewrite this in the form

$$\frac{h}{r} = \frac{1}{\lambda_z}\frac{RH}{r^2} = \frac{1}{\lambda_z}\frac{H}{R}\frac{R^2}{r^2} = \frac{1}{\lambda_z}\frac{H}{R}\frac{1}{\lambda_\theta^2} \quad \text{Eq. S. 4}$$

Stress components based on derivatives of strain energy function, i. e. $\widehat{W}$, can be written as:

$$\sigma_i = \lambda_i \frac{\partial \widehat{W}}{\partial \lambda_i} \quad \text{Eq. S. 5}$$

Accordingly, Eq. S. 1 and Eq. S. 2 can be combined with Eq. S. 4 and Eq. S. 5 to give:

$$\lambda_\theta \frac{\partial \widehat{W}}{\partial \lambda_\theta} = \frac{Pr}{h} \qquad \Rightarrow \qquad P = \lambda_\theta \frac{\partial \widehat{W}}{\partial \lambda_\theta}\frac{h}{r} = \frac{\partial \widehat{W}}{\partial \lambda_\theta}\frac{H}{R}\frac{1}{\lambda_\theta \lambda_z} \qquad \text{Eq. S. 6}$$

$$\frac{F}{2\pi r h} + \frac{Pr}{2h} = \lambda_z \frac{\partial \widehat{W}}{\partial \lambda_z} \quad \Longrightarrow \quad F = 2\pi r h\left[\lambda_z \frac{\partial \widehat{W}}{\partial \lambda_z} - \frac{Pr}{2h}\right] = 2\pi R H\left[\frac{\partial \widehat{W}}{\partial \lambda_z} - \frac{\lambda_\theta}{2\lambda_z}\frac{\partial \widehat{W}}{\partial \lambda_\theta}\right] \qquad \text{Eq. S. 7}$$

These correspond to Eq. 3 and Eq. 4 in the main text, and also to the equations derived by Chang and Kyriakides [7,12]. We have used these equations (setting *F*=0) to calculate the homogeneous inflation response in the main paper.

Section S 3: Condition to guarantee lengthening when a thin-walled Ogden tube inflates homogeneously

As mentioned in the main text, the experiments show that the tubes stretch when they are inflated, whereas the unconstrained fit to the Ogden model predicts $\lambda_z < 1$. Here we will derive the condition that guarantees that the initial portion of the inflation (i.e. when the inflation just starts) induces lengthening of the tube.

The Ogden model (Eq. 1 in the main text) is substituted into the force equation (Eq. 4 in the main text). The resulting expression is then expanded to first order in $\lambda_z - 1$ to obtain an expression of the form :

$$F = f(\lambda_\theta) + g(\lambda_\theta)(\lambda_z - 1) \qquad \text{Eq. S. 8}$$

Setting F=0, the axial stretch can be written as

$$\lambda_z = 1 - \frac{f(\lambda_\theta)}{g(\lambda_\theta)} \qquad \text{Eq. S. 9}$$

For circumferential stretch close to one (i.e. at the early stages of inflation), it can be shown that the first derivative $\frac{\partial \lambda_z}{\partial \lambda_\theta}$ is always zero. I.e. regardless of the values of the Ogden equation parameters, for small inflations with zero axial force, the tube neither lengthens nor shortens. We must therefore examine the second derivative, which can be shown to be

$$\left.\frac{\partial^2 \lambda_z}{\partial \lambda_\theta^2}\right|_{\lambda_\theta = 1} = \frac{2\sum_{n=1}^{M} \mu_n \alpha_n^2}{3\sum_{n=1}^{M} \mu_n \alpha_n} = \frac{\sum_{n=1}^{M} \mu_n \alpha_n^2}{3\mu} \qquad \text{Eq. S. 10}$$

where the denominator μ is the shear modulus.

Lengthening is guaranteed by

$$\frac{\partial^2 \lambda_z}{\partial \lambda_\theta^2} \geq 0 \qquad \text{Eq. S. 11}$$

Since the shear modulus is necessarily positive, Eq. S. 11, gives the final criterion

$$\sum_{n=1}^{M} \mu_n \alpha_n^2 \geq 0 \qquad \text{Eq. S. 12}$$

This last equation is identical to Eq. 8 in the main text.