Supplementary Information for

Elasto-inertial instabilities in the merging flow of viscoelastic fluids

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S-1 Numerical and experimental results of the merging water flow

The three-dimensional numerical simulation of the merging flow of Newtonian water in the T-shaped microchannel was performed in COMSOL®. Figure S-1 shows the computational domain, where only a length of 2 mm in the main branch and a length of 0.75 mm in each side branch were considered to reduce the computational cost. The width and depth of the channel were used as measured. The flow field was solved from the continuity and Navier-Stokes equations,

$$\nabla \cdot \mathbf{v} = 0, \tag{S-1}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \eta \nabla^2 \mathbf{v}, \tag{S-2}$$

where $\mathbf{v}$ is the velocity vector, p is the pressure, ρ is the water density (1000 kg/m³), and η is the water viscosity (1 mPa·s). A no-slip boundary condition, $\mathbf{v} \cdot \mathbf{t} = 0$, was imposed to all the walls with $\mathbf{t}$ being the tangential unit vector. The inlet boundary of each side branch was imposed with the volumetric flow rate under test. The outlet boundary of the main branch was imposed with a fully developed flow condition. The computational domain was discretized using tetrahedral elements. A grid independence study was conducted to ensure the solution accuracy. A grid with the smallest mesh size of 6 μm near the T-junction but the overall mesh size not exceeding 10 μm in the entire domain was found sufficient to produce mesh-size insensitive results. Figure S-2 compares the experimental images with the numerical predictions for the merging flow of water in a range of flow rates.
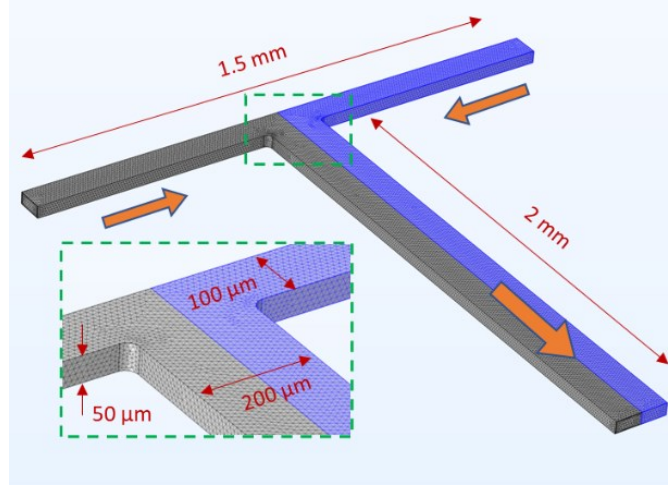
Fig. S-1. Isometric view of the three-dimensional computational domain used for the numerical simulation of water flow in a T-shaped micorchnanel, where the inset shows the zoom-in view of the meshes in the T-junction region.
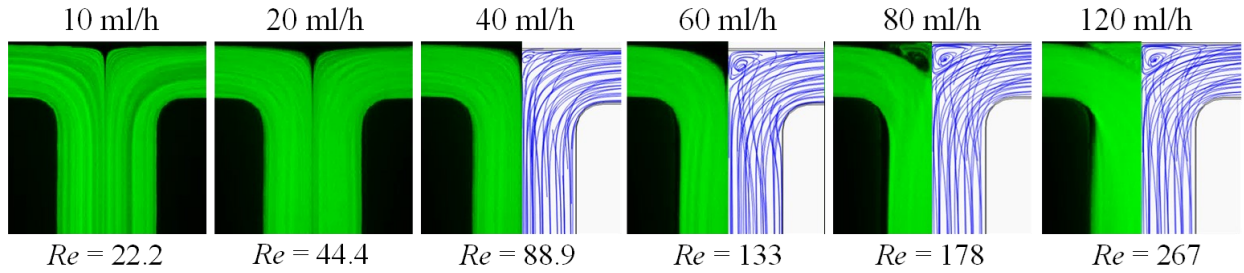

Fig. S-2. Comparison of the experimentally obtained streaklines of tracer particles (left half) and the numerically predicted streamlines (right half) for the merging flow of water in a T-shaped microchannel under a range of flow rates through the main branch.

S-2 Additional analysis of the merging flow of PEO solutions based on the McKinley-Pakdel criterion

We considered the contribution of the solvent viscosity in the McKinley-Pakdel criterion,[1,2]

$$M_{crit} = \sqrt{2(1-\beta)}\left(ReEl\frac{w}{2\mathcal{R}}\right)_{crit} \qquad (S\text{-}3)$$

where β is the viscosity ratio of the solvent (i.e., water) to polymer solution. Fig. S-3 plots the experimentally observed threshold Re against $El\sqrt{2(1-\beta)}$ in the PEO solutions with varying polymer concentrations and molecular weights, respectively. The data points from each parametric study can be best fitted to a power trendline like Fig. 6(a) in the main text. However, the power indices in both cases (see the equations displayed on the chart) still deviate significantly from the prediction of Eq. (S-3).
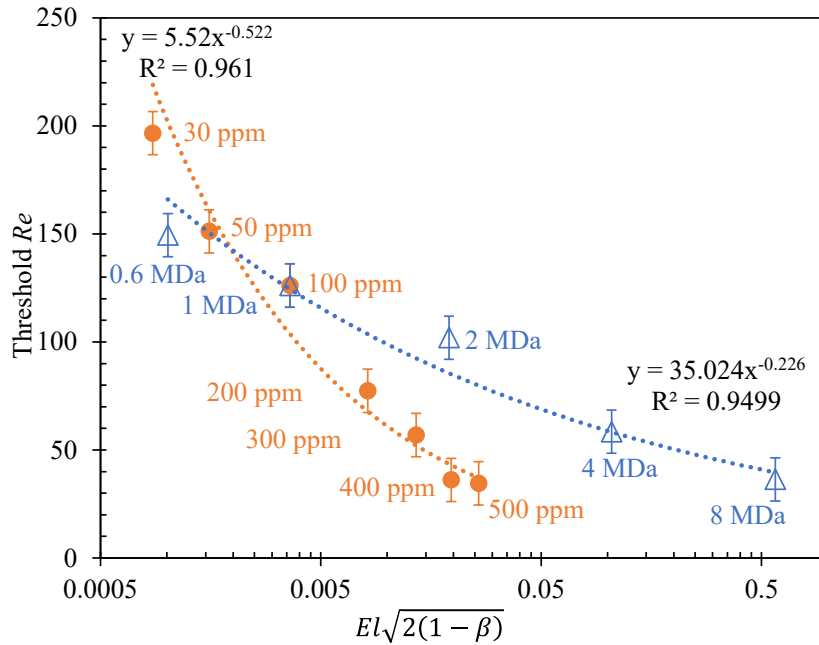

Fig. S-3. Summary of the observed threshold Re for the onset of elastic instabilities against $El\sqrt{2(1-\beta)}$ in the merging flow of PEO solutions with varying polymer concentrations (for $M_w = 1$ MDa) or M_w (at 100 ppm). The dashed lines are each a power trendline fitted to the data points, for which the equation and R-squared value are displayed on the chart.

We also attempted to rewrite the McKinley-Pakdel criterion[1] as,

$$M_{crit} = \left(\frac{2\lambda V}{w}\frac{w}{2\mathcal{R}}\right)_{crit} = \left(Wi\frac{w}{2\mathcal{R}}\right)_{crit} \tag{S-4}$$

which indicates the existence of a single value for Wi_{crit} in our experiment. Fig. S-4 plots the experimentally observed critical Wi as a function of El in the PEO solutions with varying polymer concentrations and molecular weights, respectively. The data points for the former case of varying polymer concentrations appear to fluctuate around a constant value of $Wi_{crit} = 1.2$. However, those for the case of varying polymer molecular weights exhibit a significant increase (nearly one order of magnitude) with the increase of El.
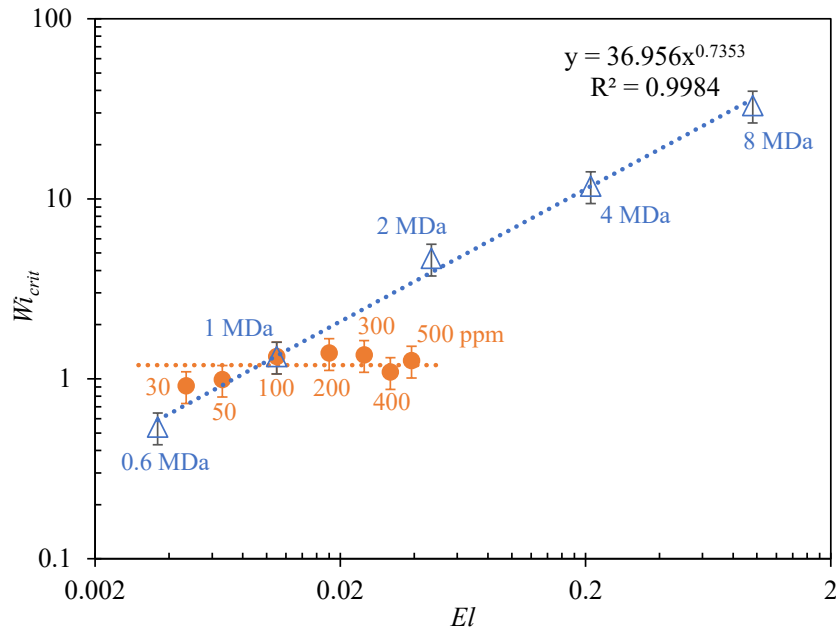

Fig. S-4. Summary of the observed critical Wi for the onset of elastic instabilities against El in the merging flow of PEO solutions with varying polymer concentrations (for $M_w = 1$ MDa) or M_w (at 100 ppm). The dashed line for the case of varying polymer M_w is a power trendline fitted to the data points, for which the equation and R-squared value are displayed on the chart. The dashed line for the case of varying polymer concentrations is a straight horizontal line with $Wi_{crit} = 1.2$, which represents a simple average of all the observed values.

References

1   P. Pakdel and G. H. McKinley, *Phys. Rev. Lett.*, 1996, **77**, 2459–2462.

2   G. H. McKinley, P. Pakdel and A. Oztekin, *J. Non-Newton. Fluid Mech*. 1996, **67**, 19-

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