Electronic Supplementary Information (ESI)

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S1 Power Requirements

In actuating the variable modulus (VM) particles, we need to determine the appropriate power setting to use. Using too little power is not enough to melt the actuated particles, whereas too much power will result in significant thermal cross talk, leading to melting of un-actuated particles. We characterize this behavior by placing a single un-actuated particle in the middle of six surrounding actuated particles. Using different power settings, we measure the temperature of the actuated and non-actuated particles as a function of time. The power values are calculated using constant voltage and the measured mean resistances ($P = V^2/R$), as shown in Fig. S1.

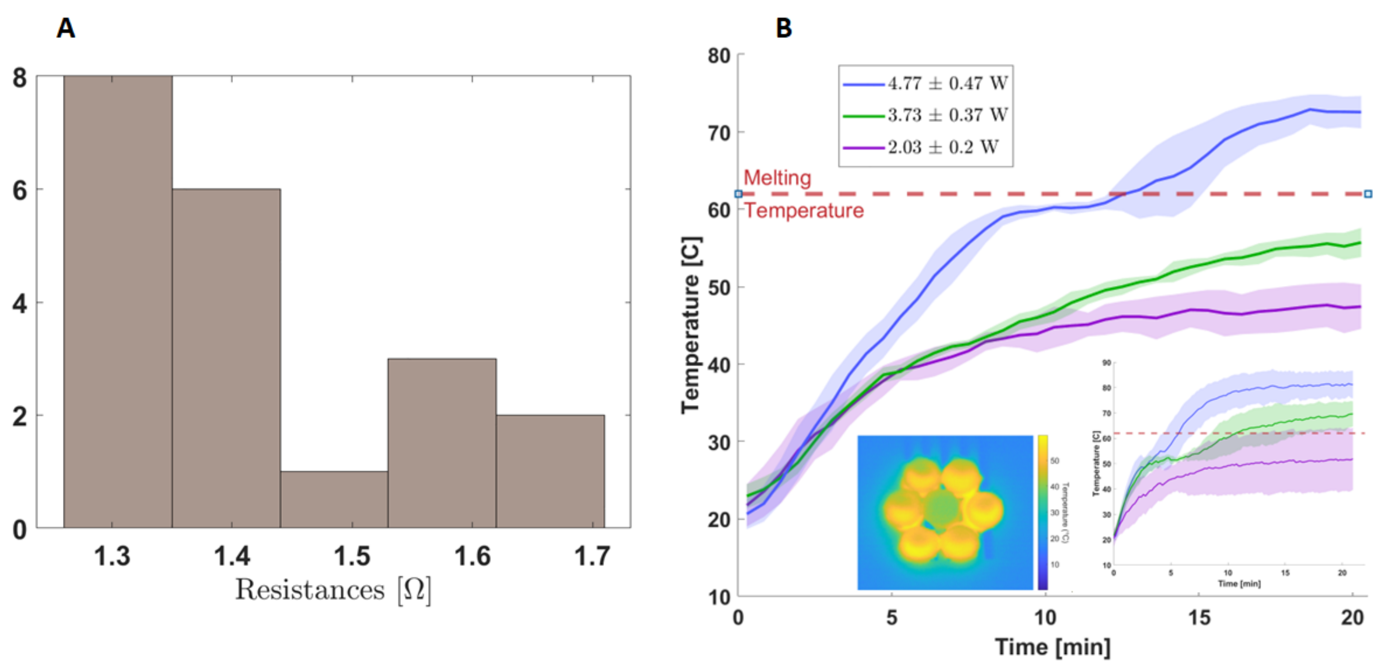

Figure S1: A) Histogram of resistance values for variable modulus particles. B) Temperature as a function of time for a non-actuated particle that is surrounded by actuated particles (inset) for different power inputs.

S2 Normalization Procedure

We extract the output forces $F_{o,p}$ from the polariscope by chi-square fitting the intensity image (e.g. in Fig. S2A) using Eqs. 5 and 6 in the main text. This procedure allows us to obtain $2tKF_{o,p}/\lambda$. We can determine the proportionality constant $2tK/\lambda$ by equating the total output force to the total input force we impart the system: $\sum_{p=1}^{5} F_{o,p} = F_i$. The output force on the bottom plate versus the input force is shown in Fig. S2B.

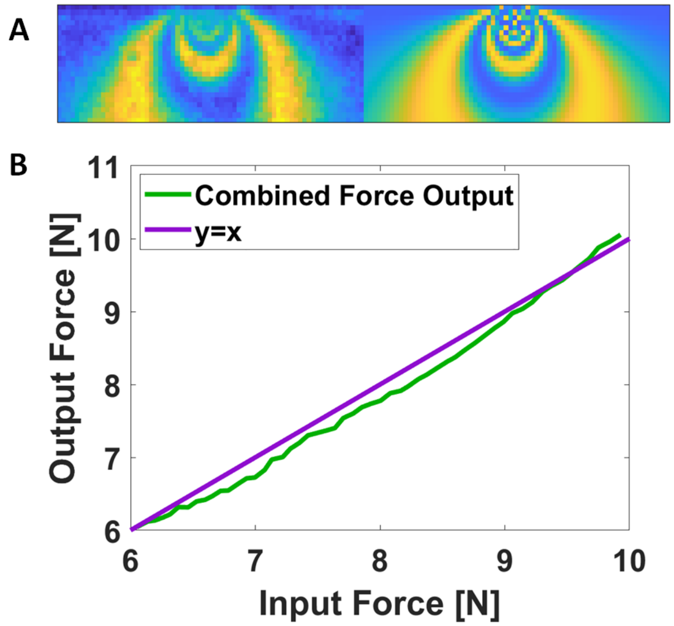

Figure S2: Procedure for obtaining the force on the boundary $F_{o,p}$ from the photoelastic image: A) (left) Photoelastic fringe pattern from the experiments. (right) Photoelastic pattern generated after fitting the output force to Eqs. 5 and 6 in the main text. B) The output force $F_{o,p}$ on the boundary from a particle on the bottom row plotted versus the input force R_i applied to the top boundary.

S3 Linear Relationship between $F_{o,p}$ and F_i

We show through DEM simulations that the output force $F_{o,3}$ (*i.e.*, the force that the middle particle in the bottom row exerts on the boundary) is roughly proportional to the input force F_i over a wide range of input forces as long as the interparticle contact network does not change, even though the interparticle force law is non-linear as a function of the particle deformation. In Fig. S3, we plot $F_{o,3}$ versus F_i for the three example configurations, the "Best Solution", "Random 1", and "Random 2" in Fig. 4E in the main text, where the forces have been normalized by the compressive modulus of the soft particle $k_{soft}D^2$ and D is the diameter of both stiff and soft particles. We show that $F_{o,3}$ is proportional to F_i for rescaled $F_i/(k_{soft}D^2) \leq 0.1$ until a discontinous drop occurs for the "Best Solution," which signals a particle rearrangement. In experiments, the largest input force is $F_i \sim 10$ N, which corresponds to $F_i/(k_{soft}D^2) \sim 0.2$. In the experiments, we do not observe particle rearrangements over the full range of input forces. Particle rearrangements occur in the DEM simulations at lower input forces than in the experiments since the DEM simulations do not include interparticle friction and friction between the particles and the boundaries.

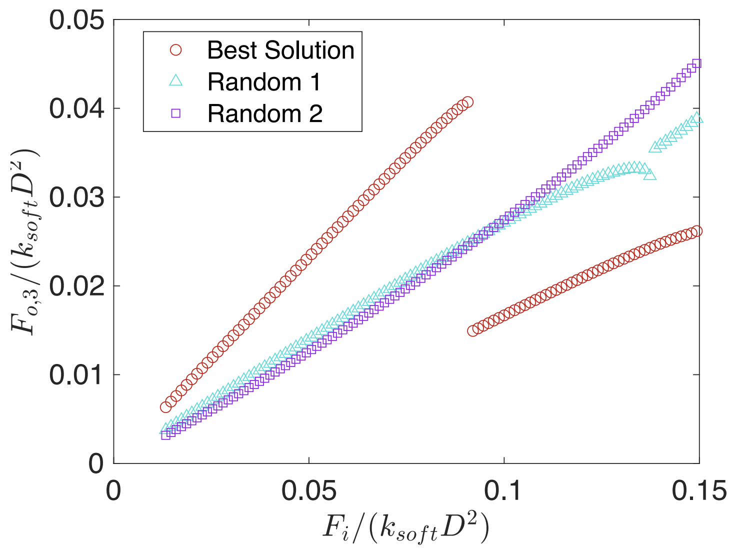

Figure S3: Output force that the third particle in the bottom row exerts on the boundary $F_{o,3}$ plotted versus the input force F_i for the three configurations ("Best Solution", "Random 1", and "Random 2" in Fig. 4 of the main text). Both F_i and $F_{o,3}$ are normalized by the compressive modulus $k_{soft}D^2$, where D is the diameter of the hard and soft particles.