Supplementary Information (SI) for Soft Matter.

Supporting information

Energy Balance

The fracture condition (21b) can be obtained directly from energy balance. Let us fix the displacement at the peel arm and move the crack forward to a unit distance, then the strain energy release is:

$$\frac{E^*\left(\frac{\partial\delta}{\partial x}\Big|_{x=0}\right)^2 h}{2} = \frac{E^*\left(\sqrt{\frac{\tau_0}{E^* h\delta_0}}\frac{\partial\delta}{\partial X}\right)^2\Bigg|_{X=0} h}{2} = \frac{\tau_0\delta_0}{2}\left(\frac{\partial\Delta}{\partial X}\Big|_{X=0}\right)^2 = \frac{\tau_0\delta_0}{2}(L)^2 = \tau_0\delta_0\Delta_0 \tag{S1}$$

At fracture, this equals the cohesive zone energy which is $\tau_0\delta_c = \tau_0\delta_0\,\phi\left(\dot{\delta}/\dot{\delta}_0\right)$
Equating both sides gives:

$$\Gamma = \tau_0\delta_0\Delta_0 = \tau_0\delta_c = \tau_0\delta_0\,\phi\left(\dot{\delta}/\dot{\delta}_0\right) \Rightarrow \Delta_0 = \phi\left(\dot{\delta}/\dot{\delta}_0\right) = \phi\left(d\Delta_{tip}/dT\right) \tag{S2}$$

Thus, the LHS of this equation can be viewed as the energy input and the RHS as the intrinsic toughness which depends on the rate.
Calculation of (21b):

$$\Delta = \frac{(X + C(T))^2}{2} + (X + C(T))L(T) + \Delta_{tip}(T) \tag{S3}$$

$$\begin{aligned}\frac{\partial\Delta}{\partial T}\Big|_X &= (X + C(T))\frac{dC}{dT} + (X + C(T))\frac{dL}{dT} + L(T)\frac{dC}{dT} + \frac{d\Delta_{tip}(T)}{dT}\\ \Rightarrow \frac{\partial\Delta}{\partial T}\Big|_{X=-C(T)} &= L(T)\frac{dC}{dT} + \frac{d\Delta_{tip}(T)}{dT}\end{aligned} \tag{S4}$$

The last term can be computed using (19), i.e.,

$$\frac{d\Delta_{tip}(T)}{dT} = 2\left(1 - \frac{\beta}{\sqrt{\beta^2 + \alpha T}}\right)\left[\frac{\alpha}{2} - \left(\sqrt{\beta^2 + \alpha T} - \beta\right)\frac{d\beta}{dT}\right] \tag{S5}$$

Hence

$$\begin{aligned}\dot{\Delta}_{tip} &= 2\left(\sqrt{\beta^2 + \alpha T} - \beta\right)\left\{1 - \frac{\left(\sqrt{\beta^2 + \alpha T} - \beta\right)}{\sqrt{\beta^2 + \alpha T}}\right\}\frac{d\beta}{dT} + \alpha \ldots\\ &\left(1 - \frac{\beta}{\sqrt{\beta^2 + \alpha T}}\right)\left[2\beta\frac{d\beta}{dT} + \alpha\right]\end{aligned} \tag{S6}$$

Non-steady state crack growth

Figures S1-S3 plot the normalized crack length β and shear displacement at crack tip Δ_{tip} with parameter $\alpha = 0.2, 0.8, 1.5$ respectively. All cases are with $\beta_0 = 1$ and employ quadratic function ϕ in eq. (43) with $\eta_{max} = 1$, and $\phi_{max} = 2$.

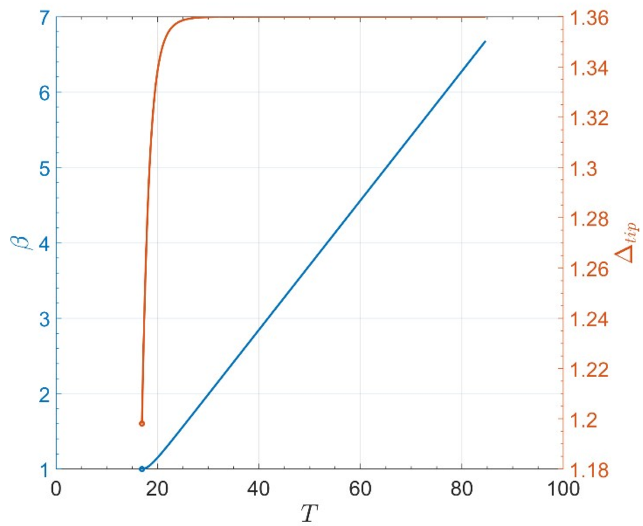

Figure S1 Normalized crack length (blue line) and (red line) versus normalized time *T* for . Simulations β used , , .

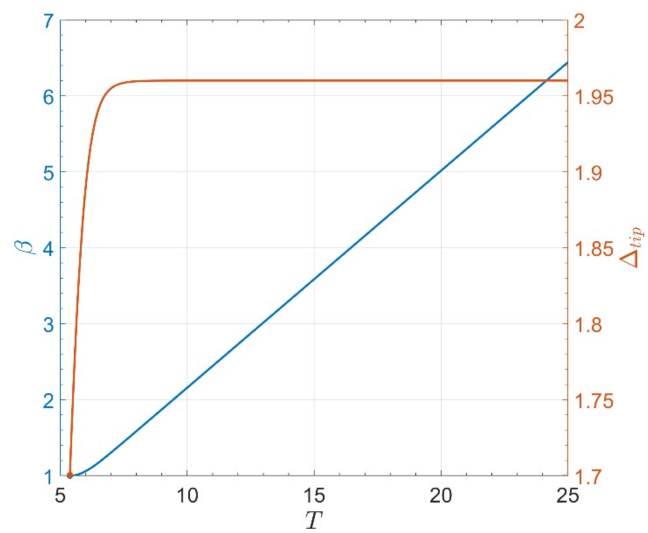

Figure S2 Normalized crack length (blue line) and (red line) versus normalized time *T* for β . Simulations used , , .

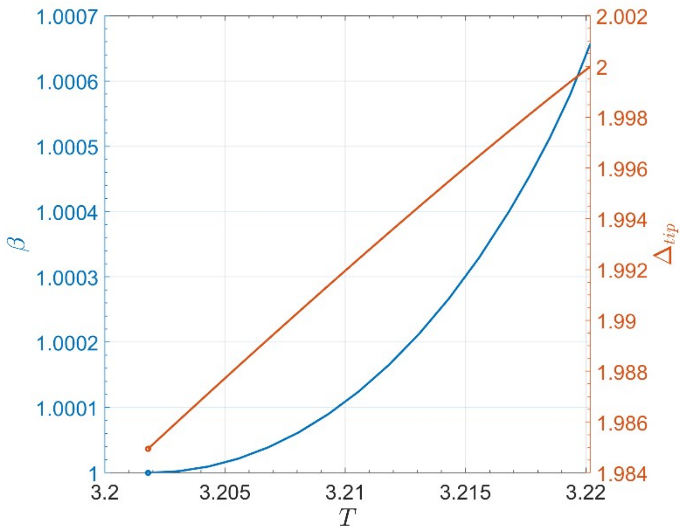

Figure S3 Normalized crack length (blue line) and (red line) versus normalized time *T* for . Simulations β used , , .