Supplementary Material

Alkali metal doped two dimensional Janus Cr$_2$Br$_3$I$_3$ monolayers with quantum anomalous Hall effect

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Fig. S1 The phonon dispersion spectra of the (a) Li- and (c) K doped ML Cr$_2$Br$_3$I$_3$. Total energy fluctuation at AIMD simulations of the (b) Li- and (d) K doped ML Cr$_2$Br$_3$I$_3$. The insets in (b) and (d) shows snapshots of the Li- and K doped ML Cr$_2$Br$_3$I$_3$ at 6 ps.
The strain dependence of the distance (d) between two Cr atoms is drawn in Fig. S2(a). The d significantly enlarges when ε is from -1% to 0%. As shown in Fig. S2(b), both the angles of α₁ and α₂ also suddenly increase from -1% to 0%. The results clearly prove the enhancement of FM superexchange interaction. Therefore, the change of d, α₁, and α₂ under various strains likely induces a sharp change of energy difference at ε = 0 through affecting the magnetic interactions in Figs. 2(a) through 2(c).
**Fig. S3** Band structures of pristine ML Cr$_2$Br$_3$I$_3$. (a) without SOC. (b) with SOC. Density of states (DOS) and Projected density of states (PDOS) of pristine ML Cr$_2$Br$_3$I$_3$. 
**Fig. S4** DOS of (a) Li doped, (c) Na doped, and (e) K doped ML Cr$_2$Br$_3$I$_3$. PDOS of (b) Li doped, (d) Na doped, and (f) K doped ML Cr$_2$Br$_3$I$_3$. 
**Fig. S5** Band structures of Li doped ML Cr_{2}Br_{3}I_{3} with SOC at different U values. (a) U = 0 eV. (b) U = 1 eV. (c) U = 2 eV.