Appendix A. Design and Assembly of the Thermal Module

Figure S1 provides a conceptual schematic of the thermal module assembly used in this study. The core of the module is the custom-designed PCB-based heater, which integrates both the resistive heating element and the temperature sensing trace into a single component.

This PCB-based heater is bonded to a Ceramics block using a thermally conductive adhesive. The ceramic block provides structural support and electrical insulation while ensuring efficient heat transfer towards the sample cartridge.

On the opposing side of the heater, Heat dissipation fins are attached, also using a thermally conductive adhesive to ensure a low-resistance thermal path. During operation, these fins are actively cooled by circulating cooling water, which enables the rapid temperature reduction required for the fast cooling ramps demonstrated in the manuscript. This complete assembly allows for precise and rapid bidirectional temperature control.

Fig. S1. Conceptual schematic of the multi-layer PCB-based heater-sensor

Appendix B. Design of the Fuzzy Gain-Tuned PID Controller

The PID controller is a widely used feedback control algorithm. Its basic principle is to employ the error between the output and the setpoint, together with the integral and derivative of that error, as the control quantity so as to enable automatic adjustment of the control action. The discrete incremental form of the PID controller can be defined as:

$$T^{fb}(k) = T^{fb}(k-1) + \Delta T^{fb}(k)$$

$$\Delta u(k) = \begin{bmatrix} K_p & K_i \cdot T & \frac{K_d}{T} \end{bmatrix} \cdot \begin{bmatrix} e(k) - e(k-1) \\ e(k) \\ e(k) - 2e(k-1) + e(k-2) \end{bmatrix}$$

$$e(k) = T_{liquid}(k) - r(k)$$

Where $T^{fb}(k)$ represents the controller output at discrete time k, and $\Delta T^{fb}(k)$ represents its incremental change. The present control output $T^{fb}(k)$ is determined by the previous output $T^{fb}(k-1)$ and the increment $\Delta T^{fb}(k)$. K_p, K_i, and K_d are the proportional, integral and derivative gains of the controller, T is the sampling interval, and $e(k)$ is the control error, i.e. the difference between the current reference $r(k)$ and the system output $T_{liquid}(k)$.

The gains K_p, K_i, and K_d of a conventional PID controller are typically based on the system's static operating point. Consequently, they are unable to effectively adapt to situations where system parameters vary over time. This leads to poor control performance for nonlinear, time-varying systems subject to uncertainties such as external disturbances and parameter perturbations.

To enhance the adaptability and robustness of the outer-loop control under rapidly changing thermal conditions, the controller gains $K_p(k)$, $K_i(k)$ and $K_d(k)$ require dynamic adjustment that follows the system variations.

Fuzzy logic overcomes the limitation of fixed gains inherent in conventional PID controllers by implementing a dual-input, triple-output real-time feedback system. Based on the current tracking error and its rate of change, this system dynamically adjusts these gain parameters in real-time. This real-time adjustment based on error and error change rate effectively balances the trade-off between rapid response and stability.

B.1 Fuzzy Inference Inputs and Outputs

Fuzzification converts input and output quantities into fuzzy sets according to membership functions. The fuzzy gain-scheduled PID controller functions as a real-time feedback system with dual inputs and triple outputs. The inputs to its fuzzy inference system are the current tracking error(e_1) and the rate of change of error(e_2), which are defined as:

$$e_1 = e(k)$$
$$e_2 = e(k) - e(k-1)$$

The system outputs are the increments to the PID gains: $\Delta K_p(k)$, $\Delta K_i(k)$ and $\Delta K_d(k)$. The updated system gains are defined as:

$$K_p(k) = K_p(k-1) + \Delta K_p(k)$$
$$K_i(k) = K_i(k-1) + \Delta K_i(k)$$
$$K_d(k) = K_d(k-1) + \Delta K_d(k)$$

The five fuzzy variables $e_1(k)$, $e_2(k)$, $\Delta K_p(k)$, $\Delta K_i(k)$, $\Delta K_d(k)$ are normalized. They are scaled to the interval [−9,9] by multiplying with quantization factors ϵ to eliminate dimensional differences (parameters detailed in Table B1).

Table B1. Fuzzy Inference Input and Output Variables

Variable	Basic Domain	Fuzzy Domain	Quantization Factor(ϵ)
e_1	[-6,6]	[-9,9]	1.5
e_2	[-12,12]	[-9,9]	0.75
$\Delta K_p(k)$	[-3,3]	[-9,9]	3
$\Delta K_i(k)$	[-1.5,1.5]	[-9,9]	6
$\Delta K_d(k)$	[-3,3]	[-9,9]	3

B.2 Membership Functions

The choice of membership functions directly affects the controller's performance. Gaussian membership functions offer several advantages in fuzzy PID control:
(a) Smooth Output Characteristics. The infinite differentiability of Gaussian functions results in smoother variations in the controller's output, preventing abrupt signal changes.
(b) Greater Flexibility in Parameter Tuning. Describing a Gaussian function requires only two parameters (mean μ and standard deviation σ), compared to trapezoidal or triangular membership functions.
(c) Enhanced Noise Immunity. Membership degrees converge rapidly to zero outside the $\pm\sigma$ interval, preventing noise from triggering anomalous fuzzy rules.
Using Gaussian membership functions (function parameters detailed in Table B2, and the distribution of the functions illustrated in Fig. S2), each variable is partitioned into 7 fuzzy grades(L_i, $i = 1,2,\cdots,7$): NB (Negative Big), NM (Negative Medium), NS (Negative Small), ZO (Zero), PS (Positive Small), PM (Positive Medium), and PB (Positive Big).

Table A2. Gaussian Membership Function Parameters

Fuzzy Level	Membership Function	u	σ	Domain Range
NB		-9	1	[-9, -6)
NM		-6	1	[−9,−3)
NS		-3	1	[-6, 0)
ZO	$f_{Li}(x) = exp\frac{-(x-u_i)^2}{2\cdot\sigma_i}$	0	1	[-3, 3)
PS		3	1	[0, 6)
PM		6	1	[3, 9]
PB		9	1	[6, 9]

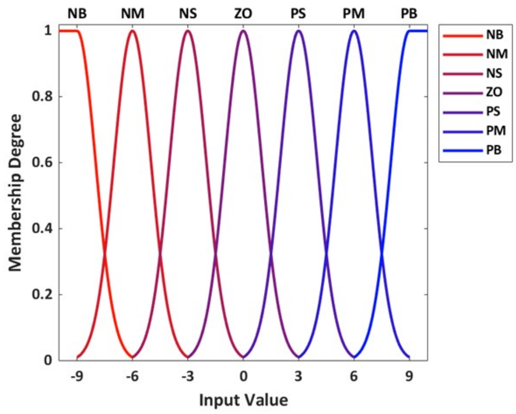

Fig. S2 Gaussian-type Membership Function Distribution

B.3 Fuzzy Rules

Based on the discrete PID controller form and considering the goals of fast system response and stable output, the rules for adjusting the parameters K_p, K_i, K_d are formulated as follows:
(a) Initial Regulation Phase. Adopt a larger K_p value to increase response speed, adopt a smaller K_i to prevent integral windup, and slightly increase K_d to prevent excessive overshoot.
(b) Mid Regulation Phase. Adopt a smaller K_p to prevent excessive overshoot, appropriately increase K_i to accelerate system stabilization, and appropriately decrease K_d to enhance system stability.
(c) Final Regulation Phase. Adopt larger K_p and K_i to reduce steady-state error and increase stability, and decrease K_d to diminish the braking effect during the control process.
Based on the inputs e_1, e_2 and the 7 fuzzy grades, a total of 49 rules are constructed. The rule format is presented in Table B3.

Table B3. Fuzzy Rule Base for Gain Increments

ΔK_p,ΔK_i,ΔK_d		e_2 NB	NM	NS	ZO	PS	PM	PB
e_1	NB	PB,NB,PS	PB,NB,NS	PM,NM,NB	PM,NM,NB	PS,NS,NB	ZO,ZO,NM	ZO,ZO,ZO
	NM	PB,NB,PS	PB,NB,NS	PM,NM,NB	PS,NS,NM	PS,NS,NM	ZO,ZO,NS	NS,ZO,ZO
	NS	PM,NB,ZO	PM,NM,NS	PM,NS,NM	PS,NS,NM	ZO,ZO,NS	NS,PS,NS	NS,PS,ZO
	ZO	PM,NM,ZO	PM,NM,NS	PS,NS,NS	ZO,ZO,NS	NS,PS,NS	NM,PM,NS	NM,PM,ZO
	PS	PS,NM,ZO	PS,NS,ZO	ZO,ZO,ZO	NS,PS,ZO	NS,PS,ZO	NM,PM,ZO	NM,PB,ZO
	PM	PS,ZO,PB	ZO,ZO,NS	NS,PS,PS	NM,PS,PS	NM,PM,PS	NM,PB,PS	NB,PB,PB
	PB	ZO,ZO,PB	ZO,ZO,PM	NM,PS,PM	NM,PB,PB	NM,PM,PS	NB,PB,PS	NB,PB,PB

B.4 Fuzzy Inference and Defuzzification

The Mamdani inference method is employed. Combining the min-max aggregation strategy and the center of gravity(COG) defuzzification strategy, the results from the fuzzy sets are mapped into definite values for $\Delta K_p(k)$, $\Delta K_i(k)$, $\Delta K_d(k)$.
The interpolation formula for ΔK_p is defined as:

$$\Delta K_p = \frac{1}{\epsilon_{k_p}} \cdot \frac{\sum_{i\in L}\sum_{j\in L} u_{\Delta K_p}(i,j) w_{ij}}{\sum_{i\in L}\sum_{j\in L} w_{ij}}$$

$$w_{ij} = f_{L_i}(e_1) \cdot f_{L_j}(e_2)$$

Where w_{ij} represents the product of the weights for e_1 at fuzzy grade L_i and e_2 at fuzzy grade L_j(the weight computation function f_{Li} is provided in Table B2), ϵ_{kp} is the quantization factor for ΔK_p (provided in Table A1), and $u_{\Delta K_p}(i,j)$ is the centroid value u of the fuzzy grade for ΔK_p corresponding to e_1 at grade L_i and e_2 at grade L_j(provided in Table B3).