

Supplemental Material:

**Unraveling the lattice thermal conductivity and thermoelectric properties of
monolayer Mg₃Bi₂**

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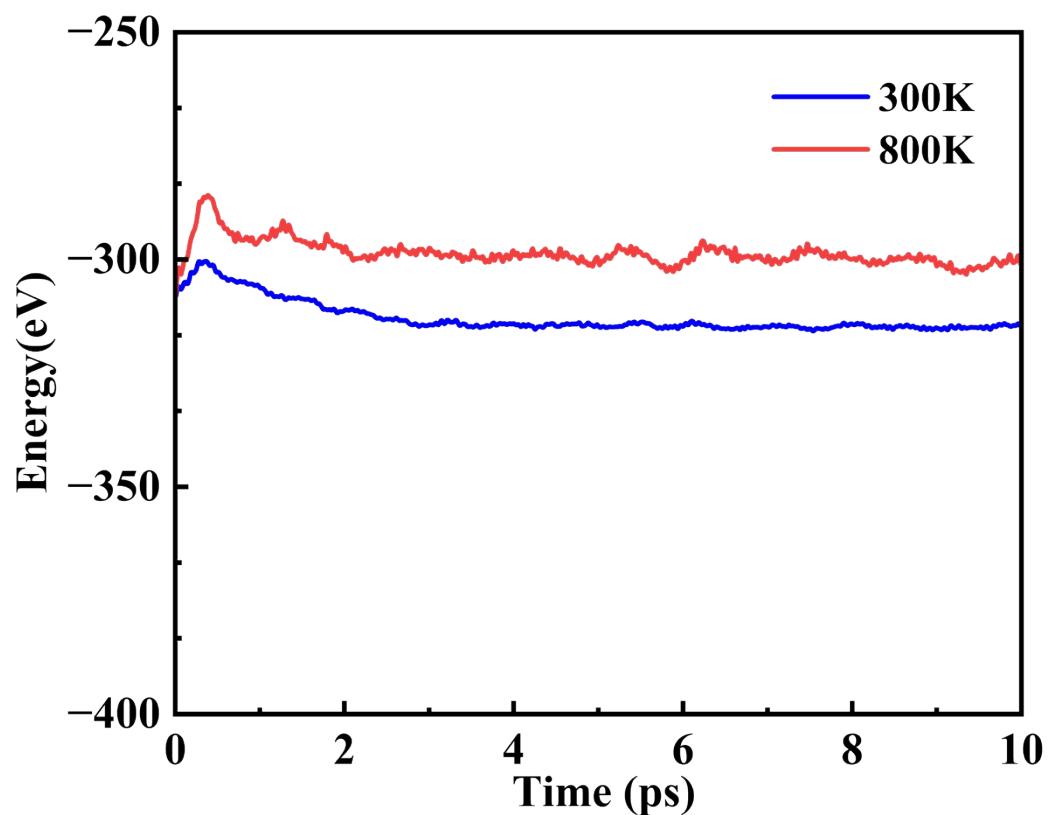


Fig. S1. Energy fluctuations of monolayer Mg_3Bi_2 in AIMD simulations at 300K and 800K.

The formation energy (E_f) of each structure can be calculated by the following formula:

$$E_f = (E_{total} - n_{Mg}E_{Mg} - n_{Bi}E_{Bi})/(n_{Mg} + n_{Bi}) \quad (1)$$

where E_{total} , E_{Mg} and E_{Bi} are the total energy of the system, the energy of Mg and Bi atoms, respectively. n_{Mg} and n_{Bi} are the number of Mg and Bi atoms in the unit cell, respectively.

The elastic energy $U(\varepsilon)$ of 2D materials using standard Voigt notation can be expressed as [1]:

$$U(\varepsilon) = \frac{1}{2}C_{11}\varepsilon_{xx}^2 + \frac{1}{2}C_{22}\varepsilon_{yy}^2 + C_{12}\varepsilon_{xx}\varepsilon_{yy} + 2C_{66}\varepsilon_{xy}^2, \quad (2)$$

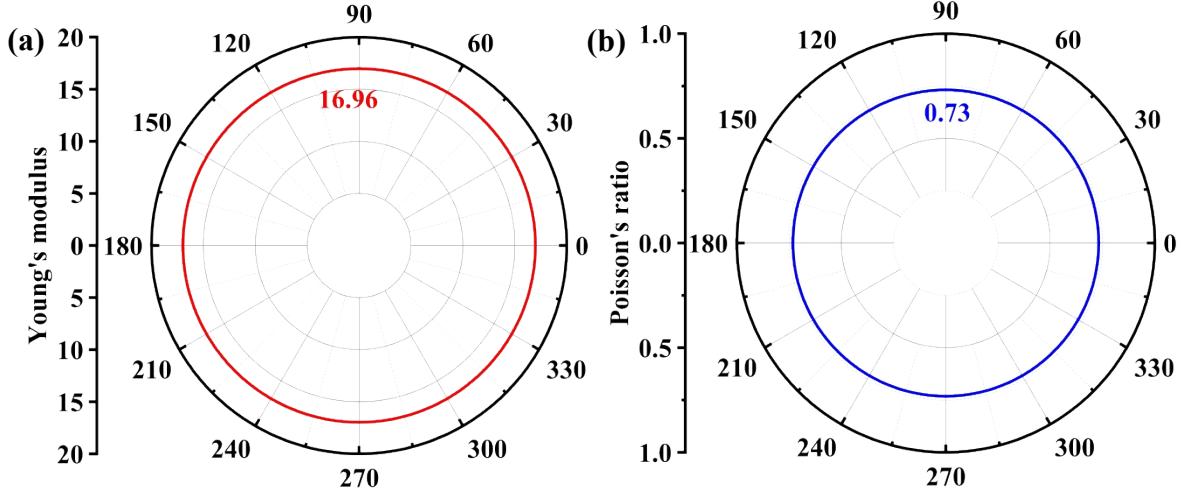
where the ε_{ij} ($i,j=x,y$) and C_{ij} ($i,j=1, 2, 6$) are the strain tensors and elastic constants, respectively. The C_{ij} can be gained from the second partial derivative of strain energy with respect to strain ($C_{ij} = (1/S_0)(\partial^2 U(\varepsilon)/\partial \varepsilon_i \partial \varepsilon_j)$, where S_0 is the area of the equilibrium unit cell. The strain range is from -2% to 2% with the step of 0.05%. Meanwhile, the orientation-dependent mechanical properties, including the in-plane Young's modulus $E(\theta)$ and Poisson's ratio $v(\theta)$, can be calculated based on the elastic constants via the following equations [2]:

$$E(\theta) = \frac{C_{11}C_{22} - C_{12}^2}{C_{11}\sin^4\theta + C_{22}\cos^4\theta + (\frac{C_{11}C_{22} - C_{12}^2}{C_{66}} - 2C_{12})\sin^2\theta\cos^2\theta} \quad (3)$$

$$v(\theta) = \frac{C_{12}(\sin^4\theta + \cos^4\theta) - \left(C_{11} + C_{22} - \frac{C_{11}C_{22} - C_{12}^2}{C_{66}}\right)\sin^2\theta\cos^2\theta}{C_{11}\sin^4\theta + C_{22}\cos^4\theta + \left(\frac{C_{11}C_{22} - C_{12}^2}{C_{66}} - 2C_{12}\right)\sin^2\theta\cos^2\theta} \quad (4)$$

The orientation-dependent $Y(\theta)$ and $v(\theta)$ and the corresponding polar diagram are

plotted in Fig. S1. The Voigt and Reuss estimated bulk (B^V, B^R) and shear moduli (G^V, G^R) of 2D materials can be calculated from the following formulas [3, 4]:



$$B^V = B^R = \frac{C_{11} + C_{12}}{2}, \quad (5)$$

$$G^V = G^R = \frac{C_{11} - C_{12}}{2}, \quad (6)$$

$$B = (B^V + B^R)/2, \quad (7)$$

$$G = (G^V + G^R)/2, \quad (8)$$

where B and G represent bulk (B) and shear modulus, respectively. The calculated results are shown in Table S1.

Fig. S2. Orientation-dependent (a) Young's modulus and (b) Poisson's ration for monolayer Mg_3Bi_2 .

Table S1. Elastic constants, elastic compliance constants, bulk modulus, shear modulus, Young's modulus, and Poisson's ratio for monolayer Mg_3Bi_2 .

Structure	$C_{11}=C_{22}$ (N/m)	C_{12} (N/m)	C_{66} (N/m)	S_{11} (m/N)	S_{12} (m/N)	S_{66} (m/N)	B (N/m)	G (N/m)	ν	E (N/m)
Mg_3Bi_2	36.49	26.69	4.90	0.058	-0.043	0.204	31.59	4.89	0.73	16.96

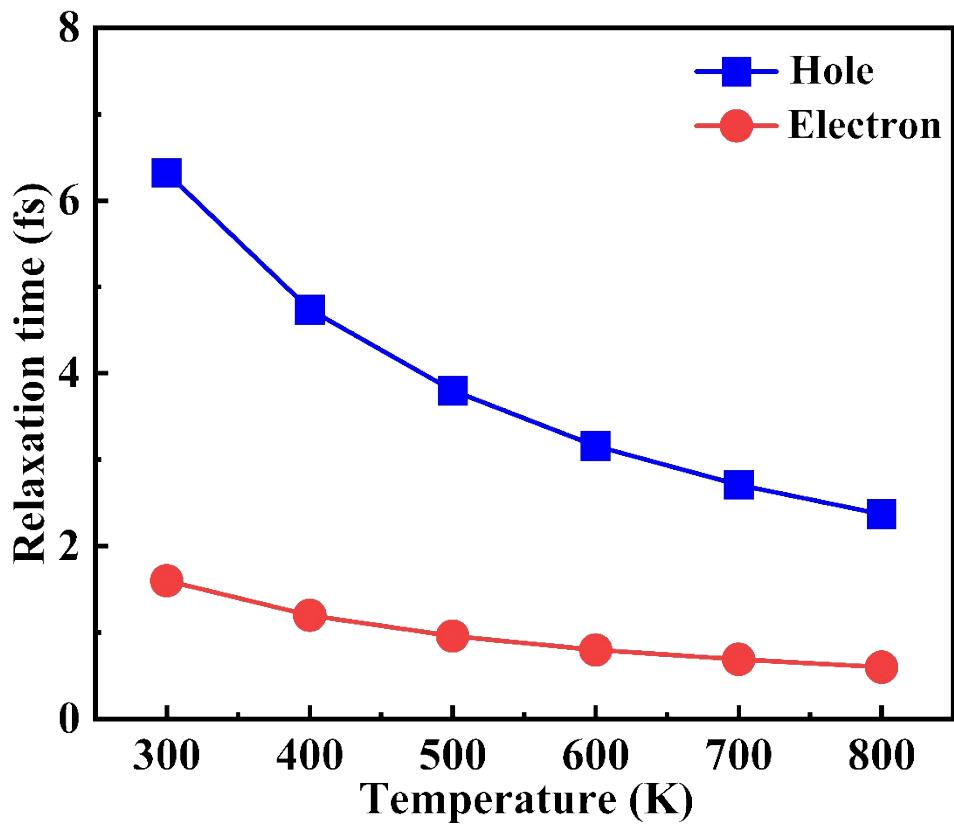


Fig. S3. Relaxation time for monolayer Mg₃Bi₂.

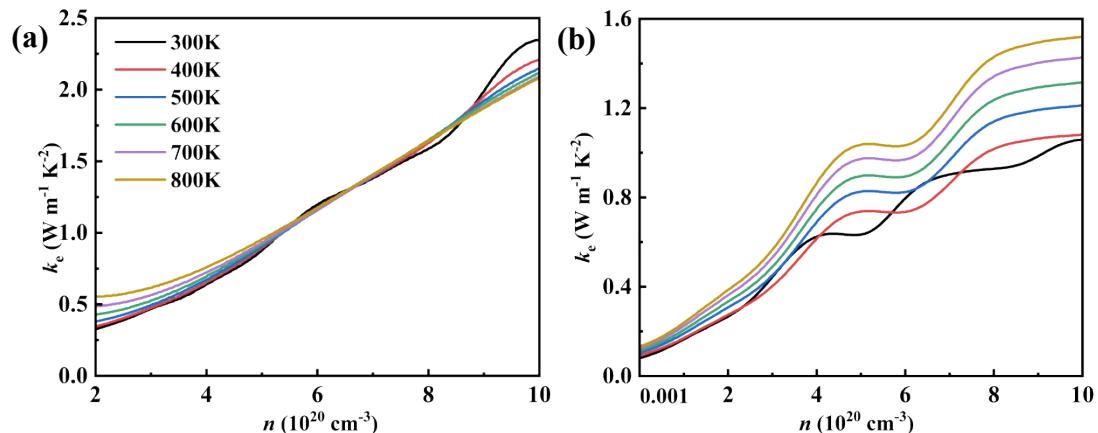


Fig. S4. Electrical thermal conductivity k_e as a function of carrier concentration for (a) p -type and (b) n -type monolayer Mg_3Bi_2 .

Table S2. Comparison of electrical properties of p -type monolayer Mg_3Bi_2 .

Methods	n (10^{20} cm^{-3})	σ (10^4 S m^{-1})	S (μVK^{-1})	PF ($\text{mWm}^{-1}\text{K}^{-2}$)	ZT	T (K)	Refs.
DFT	5.07	12.26	50.60	0.32	0.08	300	This work
Magnetron sputtering	4.24	17.20	45.76	0.32	0.08	300	[5]
Magnetron sputtering	/	39.46	52.8	1.1	/	393	[6]
Magnetron sputtering	4	29.41	82	1.97	/	300	[7]
Magnetron sputtering	/	32.05	53	0.89	/	565	[8]
Thermal evaporation	4.5	83.5	60.1	3.02	0.11	473	[9]

References

- [1] B. Peng, H. Zhang, H. Shao, Y. Xu, G. Ni, and R. Zhang, Phonon transport properties of two-dimensional group-IV materials from ab initio calculations, Phys. Rev. B 94, (2016): 245420.
- [2] Y. Xu, Y. Liu, Y. Chen, Y. Zhang, C. Ma, H. Zhang, S. Sun, and Y. Ji, ACS Appl. Mater. Interfaces 12, (2020): 58349-58359.
- [3] R. Li, Q. Shao, E. Gao, and Z. Liu, Elastic anisotropy measure for two-dimensional

- crystals, Extreme Mech. Lett. 34, (2020): 100615.
- [4] Z. Wu, E. Zhao, H. Xiang, X. Hao, X. Liu, and J. Meng, Crystal structures and elastic properties of superhard IrN_2 and IrN_3 from first principles, Phys. Rev. B 76, (2007): 054115.
- [5] Y. Ran, W. Ma, H. Yu, W. Li, D. Zhou, F. Wang, N. Gao, Z. Yu, and K. Tai, Enhanced thermoelectric performance of $\text{Mg}_3\text{Sb}_{2-x}\text{Bi}_x$ thermoelectric thin films through carrier concentration modulation by Bi alloying, J. Alloys Compd. 985, (2024): 174028.
- [6] W. Fang, W. Zhu, Y. Shao, P. Zheng, J. Si, Formation of metastable cubic phase and thermoelectric properties in Mg_3Bi_2 films deposited by magnetron sputtering, Appl. Surf. Sci. 596, (2022) 153602.
- [7] G. Sadowski, Y. Zhu, R. Shu, T. Feng, A. Febvrier, D. Music, W. Liu, P. Eklund, Epitaxial growth and thermoelectric properties of Mg_3Bi_2 thin films deposited by magnetron sputtering, Appl. Phys. Lett. 120, (2022) 051901.
- [8] J. Tani, H. Ishikawa, Fabrication and analysis of Mg_3Bi_2 thin films by post annealing Mg/Bi bilayer thin films, Mater. Lett. 331, (2022) 133460.
- [9] J. W. C. Reinders, C. Roldán-Carmona, H.J. Bolink, F. Palazon, Tunable p- and n-Type Tellurium-Free Mg_3Bi_2 Thermoelectric Thin Films by Thermal Coevaporation, ACS Appl. Energy Mater. 6, (2023) 10327-10332.