Supplementary Materials: Stacking-Controlled Magnetic Anisotropy

Switching in bilayer Janus Mn₂Cl₃Br₃

Table S1. Calculated lattice constant (a), Mn-X-Mn bond angle, Mn-Y-Mn bond angle, and energy difference between FM and Néel AFM for the monolayers $Mn_2X_3Y_3$.

| $Mn_2X_3Y_3$ | a (Å) | ∠Mn-X-Mn | ∠Mn-Y-Mn | $E_{N\acute{e}el-AFM}$ - E_{FM} (meV) |
|---|-------|----------|----------|---|
| Mn ₂ Cl ₃ Br ₃ | 6.32 | 98.67 | 88.91 | 593.16 |
| $Mn_2Br_3I_3$ | 6.79 | 95.99 | 85.97 | 468.17 |
| Mn ₂ Cl ₃ I ₃ | 6.60 | 101.34 | 80.55 | 416.81 |

Table S2. Bilayer Mn₂Br₃I₃: theoretical lattice parameter (a = b), distance between Mn layers of bilayer Mn-Mn length (d_{Mn-Mn}), energy difference from the ground state (AB-II) $\Delta Z_{X(Y)} = (Z_{Mn-X(Y)}-Z_{Mn-X0(Y0)})/Z_{Mn-X0(Y0)}$ and MAE.

| Stacking | a (Å) | E-E ₀ | d _{Mn-Mn} | ۸7 | ۸7 | MAE |
|----------------|--------------|------------------|--------------------|----------------------------------|----------------------------------|--------|
| configurations | <i>u</i> (A) | (meV) | (Å) | $\Delta \mathbf{Z}_{\mathrm{X}}$ | $\Delta \mathbf{Z}_{\mathrm{Y}}$ | (meV) |
| AA-II | 6.71 | 36.10 | 7.281 | 1.09 | -9.25 | -39.30 |
| AA-BrI | 6.79 | 60.87 | 6.744 | -0.91/1.24 | -5.60/-0.22 | -40.13 |
| AA-BrBr | 6.67 | 251.74 | 5.964 | -1.44 | -4.41 | -33.63 |
| AB-II | 6.70 | 0 | 7.326 | 0.41 | -4.63 | -48.08 |
| AB-BrI | 6.71 | 44.34 | 6.548 | 0.55/1.34 | -3.53/-3.12 | -56.44 |
| AB-BrBr | 6.66 | 184.76 | 5.959 | -0.87 | -1.13 | -39.12 |

Table S3. Bilayer Mn₂Cl₃I₃: theoretical lattice parameter (a = b), distance between Mn layers of bilayer Mn-Mn length (d_{Mn-Mn}), energy difference from the ground state (AB-II), $\Delta Z_{X(Y)} = (Z_{Mn-X(Y)}-Z_{Mn-X0(Y0)})/Z_{Mn-X0(Y0)}$ and MAE.

| Stacking configurations | a (Å) | E-E ₀ (meV) | d _{Mn-Mn} (Å) | $\Delta Z_{\rm X}$ | $\Delta Z_{\rm Y}$ | MAE (meV) |
|-------------------------|-------|---------------------------|---------------------------|--------------------|--------------------|--------------|
| AA-II | 6.48 | 21.40 | 6.010 | 1.19 | -2.39 | -9.21 |
| AA-ClI | 6.48 | 57.82 | 6.075 | -0.54/2.31 | -1.81/-0.22 | -9.88 |
| AA-ClCl | 6.48 | 180.99 | 5.914 | -0.32 | -0.64 | -8.12 |
| AB-II | 6.47 | 0 | 6.815 | -0.31 | -1.99 | -4.97 |
| AB-ClI | 6.47 | 32.98 | 5.891 | -1.43/0.28 | -1.87/0.12 | -8.37 |
| AB-ClCl | 6.48 | 74.78 | 5.646 | -1.69 | -1.08 | -8.00 |

Table S4. MAE for the bilayer MnX₃(X=Cl, Br, or I) with AA and AB stackings.

| Stacking | MnCl ₃ | MnBr ₃ | MnI_3 |
|----------|-------------------|-------------------|---------|
| AA | -1.08 | -19.36 | -30.55 |

| AB | -2.28 | -22.90 | -34.76 |
|----|-------|--------|--------|
|----|-------|--------|--------|

| Stacking | J_{inter}^{1x} | J_{inter}^{2x} | J^{x}_{inter} | J_{inter}^{1z} | J_{inter}^{2z} | J^{Z}_{inter} |
|----------------|------------------|------------------|-----------------|------------------|------------------|-----------------|
| configurations | (meV) | (meV) | (meV) | (meV) | (meV) | (meV) |
| AA-BrBr | 4.52 | 0.09 | 4.78 | 4.77 | 0.15 | 5.22 |
| AA-ClBr | 2.83 | 0.08 | 3.06 | 3.03 | 0.02 | 3.10 |
| AA-ClCl | 1.72 | 0.01 | 1.75 | 1.68 | 0.02 | 1.75 |
| AB-BrBr | 3.98 | 0.33 | 2.99 | 4.01 | 0.28 | 2.86 |
| AB-ClBr | 2.23 | 0.27 | 1.93 | 2.00 | 0.26 | 1.79 |
| AB-ClCl | 1.03 | 0.15 | 0.95 | 0.88 | 0.13 | 0.84 |

Table S5. Interlayer exchange constants of AA/AB-stacking bilayer Mn₂Br₃I₃

Table S6. Interlayer exchange constants of AA/AB-stacking bilayer Mn₂Br₃I₃

| Stacking | J_{inter}^{1x} | J_{inter}^{2x} | J^{x}_{inter} | J_{inter}^{1z} | J_{inter}^{2z} | J^{z}_{inter} |
|----------------|------------------|------------------|-----------------|------------------|------------------|-----------------|
| configurations | (meV) | (meV) | (meV) | (meV) | (meV) | (meV) |
| AA-II | 0.58 | 0.01 | 0.62 | 0.81 | -0.11 | 0.47 |
| AA-BrI | 3.95 | 1.30 | 7.85 | 3.52 | 1.00 | 6.52 |
| AA-BrBr | 0.59 | -0.04 | 0.47 | 0.53 | -0.03 | 0.43 |
| AB-BrBr | 0.03 | 0.01 | 0.05 | -0.04 | -0.01 | -0.06 |
| AB-BrI | 0.25 | 0.23 | 0.81 | 0.20 | 0.16 | 0.57 |
| AB-BrBr | 0.15 | 0.09 | 0.34 | 0.08 | 0.11 | 0.36 |

Table S7. Intralayer exchange parameters, interlayer exchange parameters of z and x directions and the anisotropy between the z and x directions (ΔJ_{intra} and ΔJ_{inter}) of AA/AB-stacking bilayer Mn₂Br₃I_{3.}

| Stacking | $J^x_{\scriptscriptstyle intra}$ | $J^x_{\scriptscriptstyle inter}$ | $J^z_{\it intra}$ | $J^z_{\scriptscriptstyle inter}$ | ΔJ_{intra} | ΔJ_{inter} |
|----------------|----------------------------------|----------------------------------|-------------------|----------------------------------|--------------------|--------------------|
| configurations | (meV) | (meV) | (meV) | (meV) | (meV) | (meV) |
| AA-II | 1.58 | 0.62 | 0.45 | 0.47 | -1.14 | -0.15 |
| AA-BrI | 3.18 | 7.85 | 2.01 | 6.52 | -1.17 | -1.34 |
| AA-BrBr | 1.31 | 0.47 | 0.31 | 0.43 | -1.00 | -0.04 |
| AB-BrBr | 1.41 | 0.05 | 0.16 | -0.06 | -1.25 | -0.12 |
| AB-BrI | 1.57 | 0.81 | 0.12 | 0.57 | -1.45 | -0.24 |
| AB-BrBr | 1.53 | 0.34 | 0.39 | 0.36 | -1.14 | 0.02 |

Table S8. Interlayer exchange constants of AA/AB-stacking bilayer $Mn_2Cl_3I_3$

| Stacking | J_{inter}^{1x} | J_{inter}^{2x} | J^{x}_{inter} | J_{inter}^{1z} | J_{inter}^{2z} | J^{Z}_{inter} |
|----------------|------------------|------------------|-----------------|------------------|------------------|-----------------|
| configurations | (meV) | (meV) | (meV) | (meV) | (meV) | (meV) |
| AA-II | 0.19 | 0.09 | 0.47 | 0.25 | 0.19 | 0.83 |

| AA-ClI | 0.23 | 0.07 | 0.45 | 0.22 | 0.10 | 0.52 |
|---------|-------|-------|-------|------|-------|-------|
| AA-ClCl | 0.09 | 0.00 | 0.08 | 0.20 | 0.09 | 0.48 |
| AB-II | -0.01 | -0.02 | -0.06 | 0.07 | -0.17 | -0.48 |
| AB-ClI | 0.03 | 0.20 | 0.60 | 0.03 | 0.18 | 0.56 |
| AB-ClCl | 0.01 | 0.02 | 0.08 | 0.01 | 0.01 | 0.05 |

Table S9. Intralayer exchange parameters, interlayer exchange parameters of z and x directions and the anisotropy between the z and x directions (ΔJ_{intra} and ΔJ_{inter}) of AA/AB-stacking bilayer Mn₂Cl₃I₃

| Stacking | J^x_{intra} | $J^x_{\scriptscriptstyle inter}$ | $J^z_{\it intra}$ | $J^z_{\scriptscriptstyle inter}$ | ΔJ_{intra} | ΔJ_{inter} |
|----------------|---------------|----------------------------------|-------------------|----------------------------------|--------------------|--------------------|
| configurations | (meV) | (meV) | (meV) | (meV) | (meV) | (meV) |
| AA-II | 0.53 | 0.47 | 0.29 | 0.83 | -0.25 | 0.36 |
| AA-ClI | 0.67 | 0.45 | 0.41 | 0.52 | -0.26 | 0.07 |
| AA-ClCl | 0.50 | 0.08 | -0.11 | 0.48 | -0.61 | 0.40 |
| AB-II | 0.38 | -0.06 | 0.14 | -0.48 | -0.23 | -0.43 |
| AB-ClI | 0.16 | 0.60 | -0.23 | 0.56 | -0.39 | -0.05 |
| AB-ClCl | 0.53 | 0.08 | 0.30 | 0.05 | -0.23 | -0.03 |

Table S10. Intralayer exchange parameters, interlayer exchange parameters of z and x directions and the anisotropy between the z and x directions (ΔJ_{intra} and ΔJ_{inter}) of AA/AB-stacking bilayer MnX₃

| Stacking | $J^x_{\scriptscriptstyle intra}$ | $J^x_{\scriptscriptstyle inter}$ | $J^z_{\it intra}$ | $J^z_{\scriptscriptstyle inter}$ | ΔJ_{intra} | ΔJ_{inter} |
|----------------------|----------------------------------|----------------------------------|-------------------|----------------------------------|--------------------|--------------------|
| configurations | (meV) | (meV) | (meV) | (meV) | (meV) | (meV) |
| AA-MnCl ₃ | 4.40 | 1.89 | 5.29 | 3.38 | 0.89 | 1.49 |
| AB-MnCl ₃ | 2.09 | 0.14 | 2.12 | 0.27 | 0.03 | 0.13 |
| AA-MnBr ₃ | 4.34 | 2.18 | 3.68 | 2.86 | -0.66 | 0.68 |
| AB-MnBr ₃ | 3.86 | 1.56 | 3.32 | 1.54 | -0.54 | -0.01 |
| AA-MnI ₃ | 2.28 | 1.01 | 1.62 | 1.03 | -0.65 | 0.02 |
| AB-MnI ₃ | 2.13 | 0.18 | 1.53 | 0.09 | -0.60 | -0.09 |

Table S11 MAE of Janus monolayers and bilayer $Mn_2Cl_3Br_3$ under AA-BrBr stacking with U= 3.3, 3.6, and 3.9 eV.

| Structures | MAE (meV) | | |
|---|--------------------|--------------------|--------------------|
| | <i>U</i> = 3.3 eV. | <i>U</i> = 3.6 eV. | <i>U</i> = 3.9 eV. |
| Monolayer Mn ₂ Cl ₃ Br ₃ | -5.06 | -5.32 | -5.75 |
| AA-BrBr bilayer Mn ₂ Cl ₃ Br ₃ | 2.17 | 2.39 | 2.49 |

| Mn ₂ X ₃ Y ₃ | $\Delta A_{11} (1/\text{eV})$ | $\Delta A_{12} (1/\text{eV})$ | $\Delta A_{13} (1/\text{eV})$ |
|---|-------------------------------|-------------------------------|-------------------------------|
| Mn ₂ Cl ₃ Br ₃ | 277.79 | -305.75 | -288.13 |
| $Mn_2Br_3I_3$ | 322.47 | -285.75 | -691.37 |
| $Mn_2Cr_3I_3$ | 20.85 | -101.18 | 82.57 |

Table S12. ΔAij between the monolayer and the top layer in the AA-YY stacked bilayer for different Mn₂X₃Y₃ systems.



Figure S1 Effect of the reduction in the distance between the Y layer and the Mn layer on orbitalresolved MAE of Mn-*d* orbitals in (a) Mn₂Cl₃Br₃, (b) Mn₂Br₃I₃, and (c) Mn₂Cl₃I₃.



Figure S2 Effect of the reduction in the distance between the Y layer and the Mn layer on orbitalresolved MAE of X-*p* orbitals in (a) Mn₂Cl₃Br₃, (b) Mn₂Br₃I₃, and (c) Mn₂Cl₃I₃.



Figure S3 PDOS of the monolayers (a) $Mn_2Cl_3Br_3$, (b) $Mn_2Br_3I_3$, and (c) $Mn_2Cl_3I_3$. PDOS of one layer from AA-YY stacking bilayer of (d) $Mn_2Cl_3Br_3$, (e) $Mn_2Br_3I_3$, and (f) $Mn_2Cl_3I_3$.



Figure S4 Spin-polarized band structures of the $Mn_2Cl_3Br_3$ monolayer at U= (a) 3.3, (b) 3.6, and (c)3.9 eV.



Fig S5 MAE with spin rotation of bilayer $Mn_2Cl_3Br_3$ under (a) AA-BrBr and (b) AB-BrBr stacking order.