

**Supplementary Information:**

**Structural, optical, and magnetic properties with finite-size and interface effects of noncollinear ferrimagnetic  $ACr_2O_4$  ( $A$  = Ni, Co, Mn) single-domain nanocrystals**

Mohamed A. Kassem<sup>1,2,3</sup>, Ahmed M. Nashaat<sup>1,2,3</sup>, Abdulaziz Abu El-Fadl<sup>2,3</sup>

1. Department of Materials Science and Engineering, Kyoto University, Kyoto 606-8501, Japan

2. Department of Physics, Faculty of Science, Assiut University, 71516 Assiut, Egypt

3. Lab. of Smart Materials for Energy Futures, Faculty of Science, Assiut University, Assiut 71516, Egypt

**S1. Crystal structure of bulk polycrystalline and nanoparticles samples**

The powder XRD patterns observed at room temperature for the as-synthesized spinel Ni, Co and Mn chromite nanoparticles and bulk samples (annealed at 1200 °C) are shown with Rietveld refinement results in Fig. S1. Except for the bulk NiCr<sub>2</sub>O<sub>4</sub> that crystalizes in a tetragonal structure with a space group  $I4_1/AMD$  at room temperature, its single-domain nanocrystalline form and the other two spinel cromites in nano and bulk forms crystalize in the cubic  $Fd\bar{3}m$  structure. The inset of Fig. S1(a) demonstrates the splitting of the cubic Ni chromite NCs' (400) peak to tetragonal (004) and (220) peaks after a post annealing at 1200 °C. Table S1 shows the atomic site parameters considered while the XRD pattern analysis for the nano- and bulk spinel chromite NiCr<sub>2</sub>O<sub>4</sub>. The crystal data and refinement results for the nano- and bulk poly-crystalline spinel Ni, Co and Mn chromites are summarized in Table S2. To investigate the cations distribution between the tetrahedrally A and octahedrally B coordinated sites, we compare the experimentally observed lattice constant to the calculated ones for the cases of normal ( $\delta = 0.0$ ) and partially (with  $\delta = 0.5$ ) and fully ( $\delta = 1.0$ ) inversed spinel structures, assuming a general chemical formula of  $(A^{2+})_{1-\delta}Cr^{4+}\delta[A^{2+}\delta Cr^{2+}\delta Cr^{3+}_{2-28}]O_4$ , with the cations in parentheses () and [] locate in the A and B sites, respectively. Figure S2 shows the possible crystal-field splitting schemes of the 3d-levels, and their electronic occupation, for cations available to occupy the A and B sites which are tetrahedrally and octahedrally coordinated by O<sup>2-</sup> anions, shown in Fig. S2(a) and (b), respectively. Figure S2(c) shows the experimentally observed and theoretically calculated lattice constants.

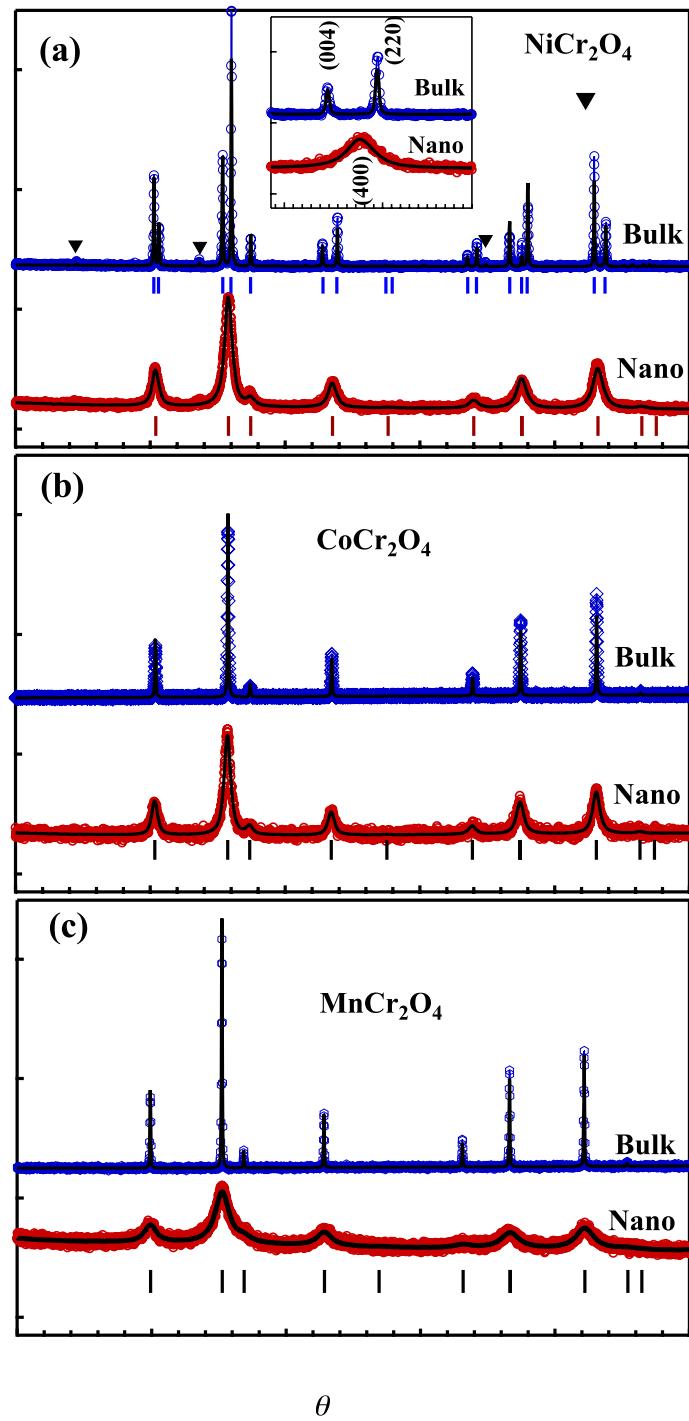
**Table S1:** Atomic site symmetry, occupancy  $g$  and site coordinates ( $x, y, z$ ) of ions in the nano- and bulk spinel  $\text{NiCr}_2\text{O}_4$ .

Atom	Wyck.	$g$	$x$	$y$	$z$
<b>Nano</b> (cubic space group $Fd\bar{3}m$ , setting #2)					
<b>Ni</b>	8a	1.0	1/8		1/8
<b>Cr</b>	16d	1.0	1/2		1/2
<b>O</b>	32e	1.0	$w (\approx 0.258)$	$w$	$w$
<b>Bulk</b> (tetragonal space group $I4_1/amd$ )					
<b>Ni</b>	4b	1.0	0.000	1/4	3/8
<b>Cr</b>	8c	1.0	0.000	0.000	0.000
<b>O</b>	16h	1.0	0.000	$y (\approx 0.55)$	$z (\approx 0.225)$

**Table S2:** Crystal structure data and refinement results for the nano- and bulk poly-crystalline spinel Ni, Co and Mn chromites.

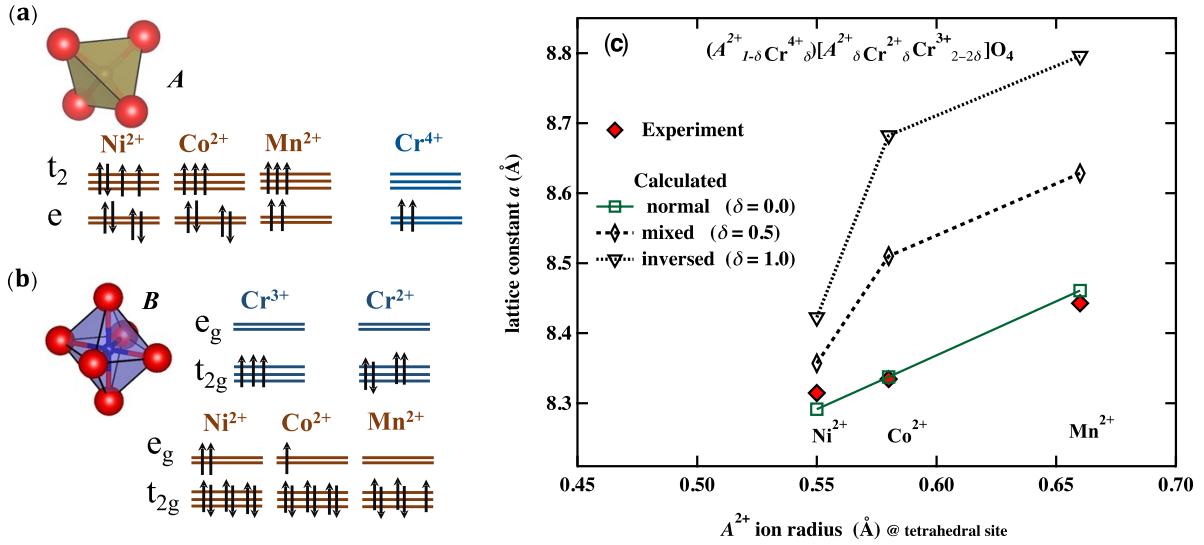
Empirical formula	$\text{NiCr}_2\text{O}_4$		$\text{CoCr}_2\text{O}_4$		$\text{MnCr}_2\text{O}_4$	
Sample form	Nano	Bulk	Nano	Bulk	Nano	Bulk
Space group (No.)	$Fd\bar{3}m$ (227)		$I4_1/amd$ (141)		$Fd\bar{3}m$ (227)	
Temperature (K)	300		300		300	
Formula mass (g/mol)	226.683		226.923		222.928	
# formula unit per unit cell (Z)	8	4	8		8	
# electrons per unit cell F(000)	864	432	856		840	
Crystallite size (nm)	18(2)	> 100	15(2)	> 100	9(2)	> 100
Calculated density (g/cm <sup>3</sup> )	5.230	5.235	5.207	5.210	4.921	4.932
<b>Lattice parameters</b>						
<i>a</i> (Å)	8.321(25)	5.833	8.334(3)	8.333	8.443(5)	8.436
<i>b</i> (Å)	8.321(25)	5.833	8.334(3)	8.333	8.443(5)	8.436
<i>c</i> (Å)	8.321(25)	8.452	8.334(3)	8.333	8.443(5)	8.436
$\alpha (= \beta = \gamma)^\circ$	90	90	90		90	
Unit cell volume (Å <sup>3</sup> )	575(3)	287(1)	578.9(3)	578(1)	601.8(7)	600.4
Profile function	Pseudo Voigt for bulk and a modified Pseudo Voigt [1] for nano					
<b>Reliability Indices</b>						
Expected $R_e$ (%)	2.558	8.186	2.397	7.386	2.830	6.545
Profile $R_p$ (%)	2.585	8.756	1.953	5.916	2.359	5.081
Weighted profile $R_{wp}$ (%)	3.299	11.612	2.441	7.845	2.960	6.584

Goodness-of-fit, $S = R_{wp}/R_e$	1.289	1.42	1.0183	1.062	1.0458	1.006
-----------------------------------	-------	------	--------	-------	--------	-------



**Figure S1:** XRD patterns observed at room temperature (marks) shown with Rietveld refinement results (solid lines) of as-synthesized nanoparticles and powder samples annealed at

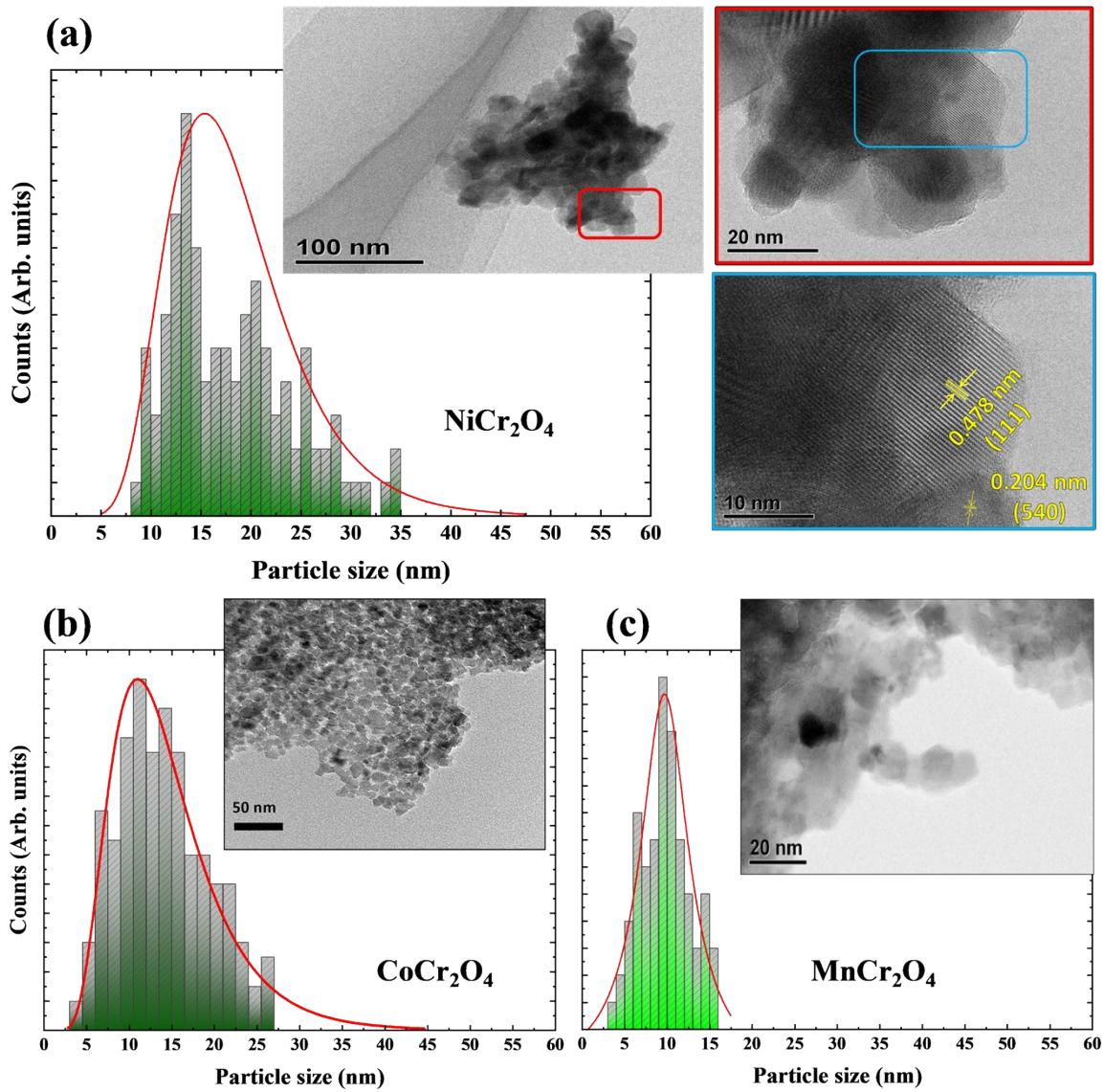
1200 °C for the Ni, Co and Mn spinel chromites. Vertical bars are the Bragg positions of the spinel chromite phase.



**Figure S2:** Polyhedra and crystal-field splitting schemes of the 3d-levels, and their electronic occupation, for the available cations in the tetrahedrally A and octahedrally B sites coordinated by O<sup>2-</sup> anions, shown in (a) and (b), respectively. (c) the experimentally observed lattice constant and that is theoretically calculated for the spinel structure  $(A^{2+})_{1-\delta}Cr^{4+}_{\delta}[A^{2+}_{\delta}Cr^{2+}_{\delta}Cr^{3+}_{2-2\delta}]O_4$ ,  $A^{2+} = Ni^{2+}$ ,  $Co^{2+}$  and  $Mn^{2+}$ , for inversion factor values  $\delta = 0.0$ ,  $0.5$  and  $1.0$ , plotted as functions of the cation radius of  $A^{2+}$  at the tetrahedral site.

## S2. TEM and particle size distribution of the nanoparticles

The particle size distribution and average particle size for the as-synthesized nanoparticles of Ni, Co and Mn spinel chromites were estimated via TEM images analyses as described in the main text, section 3.1. Figure S3 shows TEM micrographs for NiCr<sub>2</sub>O<sub>4</sub>, CoCr<sub>2</sub>O<sub>4</sub> and MnCr<sub>2</sub>O<sub>4</sub> nanoparticles, with an example of high-resolution TEM images at different magnifications showing the interplanar fringes corresponding indexed planes for the NiCr<sub>2</sub>O<sub>4</sub> NCs. The particle size histograms were established from the TEM results and fitted using a Lorentzian function.

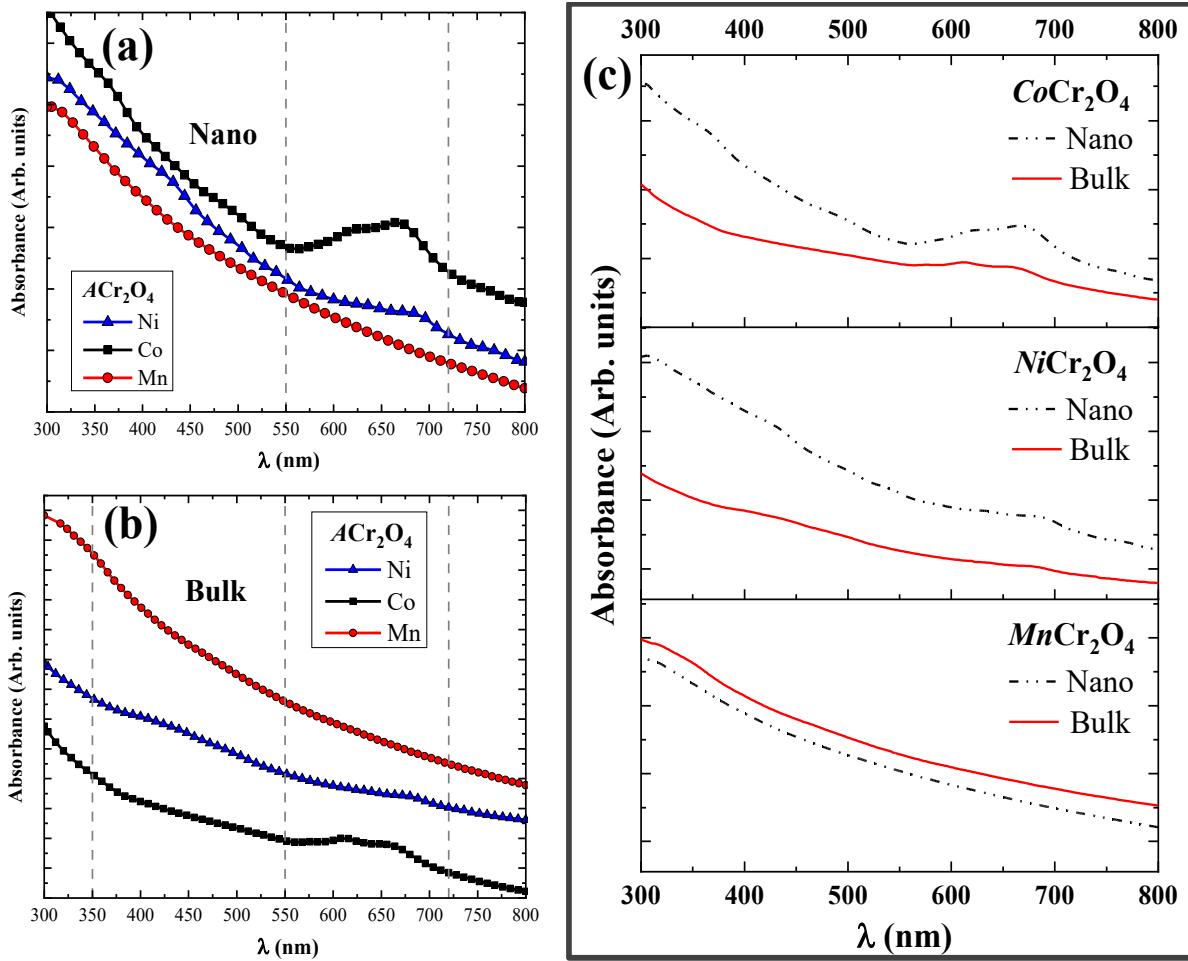


**Figure S3.** (HR)TEM micrographs for (a)  $\text{NiCr}_2\text{O}_4$  NCs at different magnifications showing the interplanar fringes corresponding the indexed planes, and for (b)  $\text{CoCr}_2\text{O}_4$  and (c)  $\text{MnCr}_2\text{O}_4$  NCs, all with established particle size histograms.

### S3. Optical properties of bulk polycrystalline and nanoparticles samples

#### S3.1. General features of the optical absorbance

The optical behavior of both nano and bulk powders is comparatively studied by measuring the optical absorbance of the particles suspended in an aqueous emulsion. As the absorbance behavior is affected by many parameters in this suspension technique, including the homogeneity and the density of the particles suspension, we have optimized the measurement conditions in an energy range of  $\sim 1$  to 5 eV. The absorbance ( $A$ ) spectra of the compounds in the nano and bulk forms in a wavelength range from 300 to 800 nm are shown in Fig. S4. The absorbance spectra of the as-synthesized nanoparticles, Fig. S4(a), and the bulk powders, Fig. S4(b), demonstrate a common maximum absorption (optical edge) in the UV region of the spectrum. There are additional absorption bands observed in the visible light range (between 550 and 725 nm) which vary in position and intensity for the different compounds and different forms of nano and bulk particles, as seen in Fig. S4(a). Main features are observed for the Co chromite spectra including the sharper absorption edge among the three chromite NCs when compared to its bulk form behavior. A deconvolution of its low-energy absorption asymmetric band indicates two peaks around  $\sim 620$  nm and 665 nm with a slight shift to low energy in the NCs form. The band deconvolution is more distinct for the material in the bulk form with almost same intensity of the two peaks, however, the 665-nm peak becomes much intense in the material's NCs form, as seen in Figs. S4 (b) and (c). In case of the Ni chromite, there is more inclination for the NCs absorbance at the absorption edge when compared to its bulk spectrum, with the development of a small band around  $\sim 675$  nm in both the nano and bulk spectra without a peak splitting. In the case of  $\text{MnCr}_2\text{O}_4$  the bulk spectrum is almost identical in behavior with the nano form without any observable additional bands below the absorption edge. These absorption bands are attributed to spin-allowed and intra-atomic d-d transitions of the  $A^{2+}$  cations as described in section S3.3 below.



**Figure S4.** Absorbance spectra of (a) as-synthesized nanoparticles and (b) bulk post-annealed particles, with small vertical offsets for clarity. (c) Comparison of the absorbance behavior of the bulk and nano forms of each compound (spectra are shifted vertically for clarity).

### S3.2. Tauc's plots and the optical energy gap estimation

The absorption coefficient ( $\alpha$ ) can be estimated by employing the Beer-Lambert law [2] with considering the theoretical density  $\rho$  in gm/cm<sup>3</sup> of the material, as:

$$\alpha = \frac{A \rho}{c l}, \quad (S3-1)$$

where  $c$  is the concentration of the sample in the solvent and  $l$  is the distance travelled by the light through the suspension. Corelating  $\alpha$  with the incident photon energy  $h\nu$  gives the general Tauc's expression [3]

$$\alpha h\nu = B(h\nu - E_g)^n. \quad (S3-2)$$

Where  $B$  is a probability constant, and  $n$  is an exponent that determines the transition type. The optical performance of the samples is tested according to the typical allowed direct transition

behavior with  $n=1/2$  and with  $n=2$  for the indirect bandgap transition. To investigate the nature of the optical transitions' nature, two plots of  $(\alpha h\nu)^2$  vs  $h\nu$  and  $(\alpha h\nu)^{1/2}$  vs  $h\nu$  were plotted to test the developing of the absorption band for the nano (Fig. S5) and for the bulk (Fig. S6(a)). The development of the absorption edge was found more consistent for the  $(\alpha h\nu)^2$  plot which implies the direct bandgap nature, while the indirect curve does not exhibit the band edge clearly. Similar observation was reported by Suchomski et al.,[4] for  $\text{CoCr}_2\text{O}_4$  thin films. We observe, however, that the Tauc's plots demonstrated in Figs. 3(b), S4 – S6 of the allowed direct transitions, the samples do not exhibit the expected sudden increase on the onset of the band edge. This behavior can be assumed due to the broad particle size distribution which contributes to the electronic transition but with an overall dominance of the band gap above 3 eV. The effect of the suspension interaction with the particles and the particle interaction in the suspension can affect the behavior as a suspension of ultrafine particles can give better shape of Tauc's plot as reported for the nanosuspension prepared by laser ablation [5]. A direct bandgap for  $\text{CoCr}_2\text{O}_4$  thin films were reported with similar behavior [6].

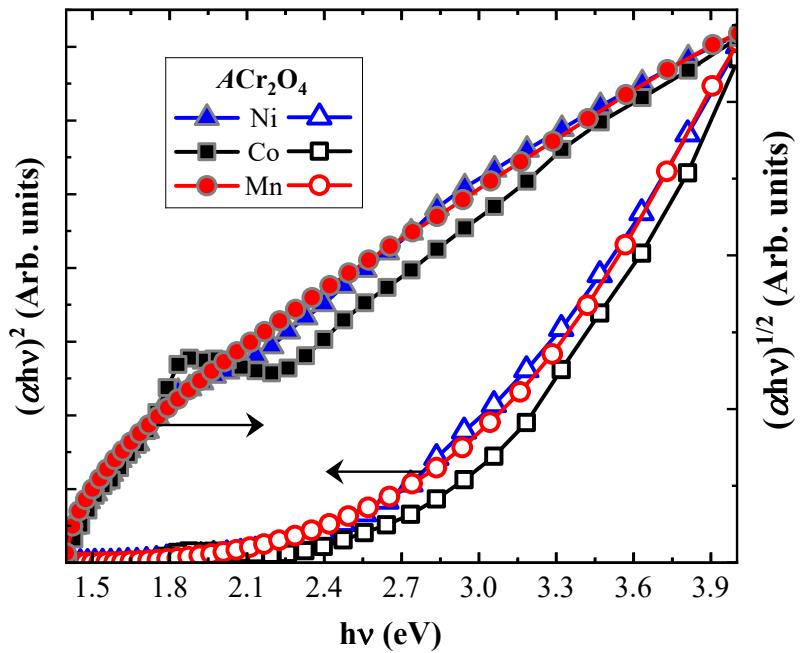
The values of  $E_g$  were estimated by linear extrapolation of data at high photon energy region, where the optical absorption is stable, to  $(\alpha h\nu)^2 = 0$ . The estimated values of  $E_g$ , presented in Table 2 (in the main text), are consistent with results of optical conductivity investigations for bulk chromites with a direct bandgap of  $\text{CoCr}_2\text{O}_4$  nanoparticles were reported as  $\sim 3.1$  eV [4,7,8]. The interpretation of the difference between the bulk and nano curves Fig. S6 especially in case of Co and Ni chromites can be attributed to the change of the band structure in the nanoparticles from that in bulk forms. For example, additional defects are expected in the nano form which can result in tails within the bandgap, this is probably the dominant effect at this particle size rather than an expected quantum confinement effect that results in a wider energy gap.

### S3.3. Intra-atomic d-d transition of $\text{Co}^{2+}$ and $\text{Ni}^{2+}$ ions in tetrahedrally coordinated sites

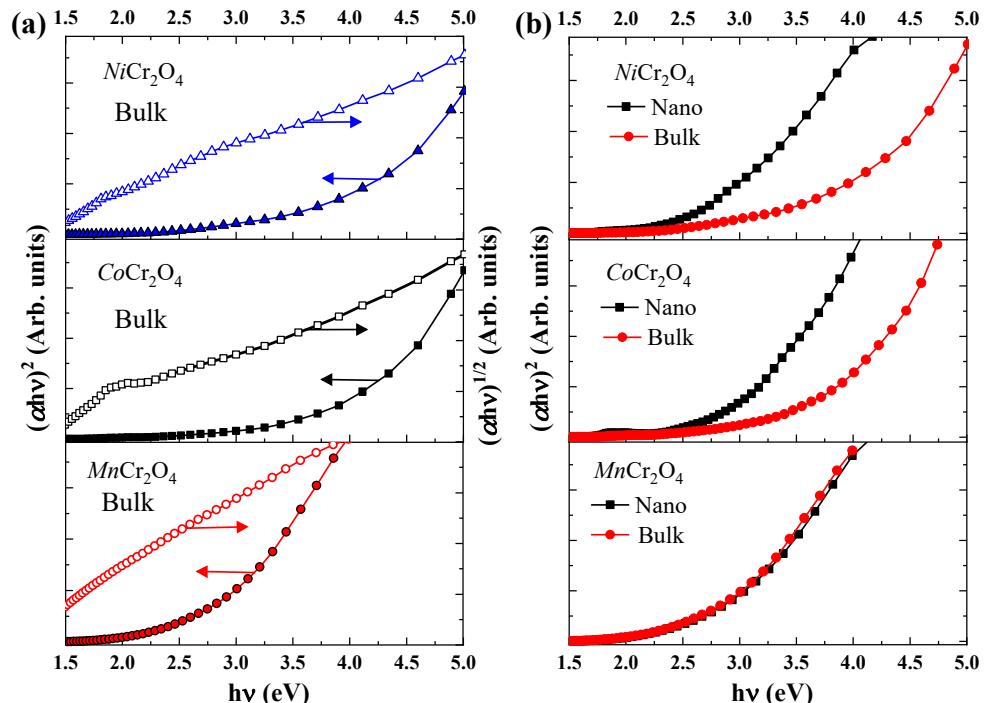
Theoretically, based on the ligand field theory by strong-coupling approximation [9–12], there are three spin allowed intra-atomic d-d transitions for  $\text{Co}^{2+}$  ions in a tetrahedrally site, as the case here of  $\text{CoCr}_2\text{O}_4$ , which are  ${}^4\text{A}_2(e^4t_2^3) \rightarrow {}^4\text{T}_1(e^3t_2^4)$ ,  ${}^4\text{A}_2(e^4t_2^3) \rightarrow {}^4\text{T}_2(e^3t_2^4)$  and  ${}^4\text{A}_2(e^4t_2^3) \rightarrow {}^4\text{T}_1(e^2t_2^5)$ . The latter two excitations occur at low photon energies with the  ${}^4\text{A}_2(e^4t_2^3) \rightarrow {}^4\text{T}_1(e^2t_2^5)$  of a rather small oscillator strength to be experimentally observed. The  ${}^4\text{A}_2(e^4t_2^3) \rightarrow {}^4\text{T}_2(e^3t_2^4)$  transition occurs with a transition energy of  $\omega = \Delta E$  ( $\sim 0.8$  eV) lower than the measurement range [8]. The optical absorption band observed here is assigned to the  ${}^4\text{A}_2(e^4t_2^3) \rightarrow {}^4\text{T}_1(e^3t_2^4)$  excitation with a transition energy of  $\omega = \Delta E + 12B$ , where  $\Delta E$  is the splitting energy between the  $e$  and  $t_2$  orbitals and  $B$  is the Racah parameter [8]. The main two intra-atomic d-d transition of  $\text{Co}^{2+}$  ions in a tetrahedrally site were observed and reported for many compounds such as  $\text{Cs}_3\text{CoCl}_5$  at 0.9 and 2.0 eV [13],  $\text{ZnO:Co}$

at 0.9 and 2.0 eV, ZnS:Co and CdS:Co at 0.8 and 1.8 eV [14]. For chromium spinels were observed in bulk  $\text{CoCr}_2\text{S}_4$  at 0.7 and 1.3 eV [15] and  $\text{CoCr}_2\text{O}_4$  single crystals at 0.8 and 2 eV [8]. The  ${}^4\text{A}_2 (e^4t_2^3) \rightarrow {}^4\text{T}_1(e^3t_2^4)$  transition can split into a dipole-allowed  ${}^4\text{A}_{2g} \rightarrow {}^4\text{T}_{2u}$  and a dipole-forbidden  ${}^4\text{A}_{2g} \rightarrow {}^4\text{T}_{1g}$  transitions by the Davydov splitting [16], which explains the asymmetric band with peaks observed at 1.86 and 2 eV. As well as the visible light asymmetric band, we also observed a very small kink around  $\sim 370$  nm which can be attributed to the additional ligand-metal charge transfer combination with the admissible d-d transitions.

In the case  $\text{Ni}^{2+}$  ions with  $d^8 (e^4t_2^4)$  configuration in a tetrahedrally site, mainly two d-d transitions,  ${}^3\text{T}_1 (e^4t_2^4) \rightarrow {}^3\text{T}_2(e^3t_2^5)$  with a transition energy  $\omega = \Delta E - 3B$  and  ${}^3\text{T}_1 (e^4t_2^4) \rightarrow {}^3\text{T}_1(e^3t_2^5)$  with  $\omega = \Delta E + 9B$ , can occur. Both were observed for a  $\text{NiCr}_2\text{O}_4$  single crystal at 0.94 and 1.95 eV [7]. A weaker broad band is observed here in the absorbance spectrum of  $\text{NiCr}_2\text{O}_4$  with maxima around  $\sim 685$  nm, which is mainly assigned to the  ${}^3\text{T}_1 (e^4t_2^4) \rightarrow {}^3\text{T}_1(e^3t_2^5)$  of the  $\text{Ni}^{2+}$  ions with a transition energy  $\omega = \Delta E + 9B$  [7]. The broadening of the band developed for  $\text{NiCr}_2\text{O}_4$  spectrum may be attributed to the overlapping of the d-d transitions resulted from the splitting in 3d level of the  $\text{Ni}^{2+}$  cation as shown in Fig. S4. Also, a small band is observed around  $\sim 450$  nm, can be attributed to an additional ligand-metal charge transfer combination with the d-d transitions. Similar bands are reported for the  $\text{Ni}^{2+}$  in the tetrahedral coordination of  $\text{NiAl}_2\text{O}_4$  [17]. At room temperature bulk  $\text{NiCr}_2\text{O}_4$  symmetry is tetragonal but for our nano samples the observed room temperature phase is cubic, which may influence the band structure of the material and may introduce different d-d transitions at different energies. As expected, the intensity of the absorption bands in  $\text{NiCr}_2\text{O}_4$  with  $\text{Ni}^{2+}$  are lower in intensity of the optical band when compared to  $\text{CoCr}_2\text{O}_4$  nanoparticles. Furthermore, in the case of  $\text{NiCr}_2\text{O}_4$  the spin-orbit interaction may be involved in the analysis of the energy dependence of the compound. The intra-atomic d-d transitions are spin-forbidden between half-filled orbitals in  $\text{Mn}^{2+}$  ions in the tetrahedrally site with the high-spin  $d^5$  state, which explains the absence of the additional absorption band below the optical edge of  $\text{MnCr}_2\text{O}_4$ . Although a d-d transition of the  $\text{Cr}^{3+}$  ions is possible, with an assigned absorption band observed in unpolarized reflectivity and optical conductivity spectra around 1.6 eV, its oscillator strength is relatively weak [7]. Due to the absence of excitation band below the optical edge in the spectrum of  $\text{MnCr}_2\text{O}_4$ , we exclude the contribution of d-d transition of the  $\text{Cr}^{3+}$  ions in the additional absorption bands observed here for Ni and Co chromites.



**Figure S5.** (a) Tauc's plots to check for the allowed direct and indirect transitions for the three compounds as-synthesized nanoparticles.



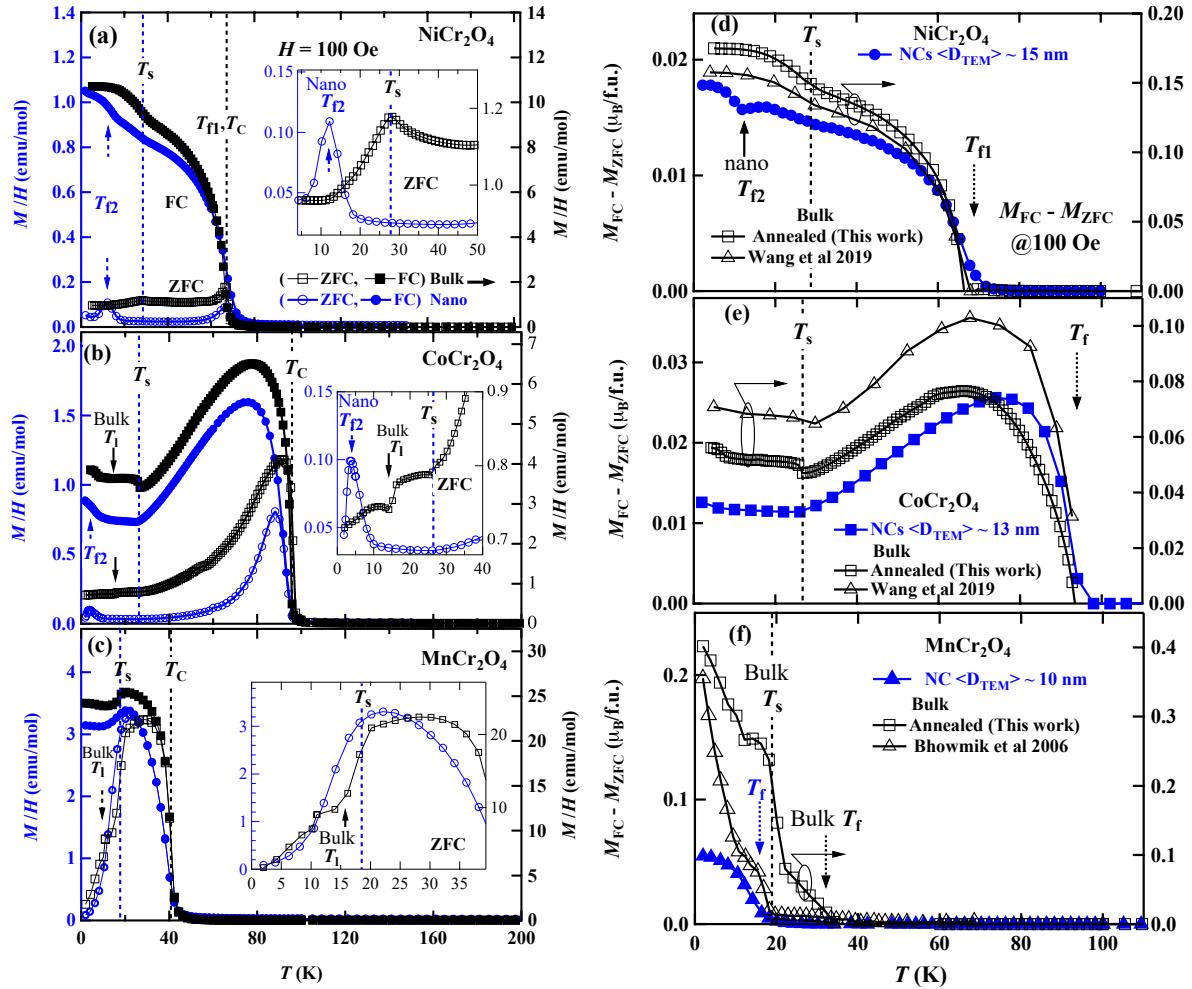
**Figure S6.** (a) Tauc's plot test for the allowed direct and indirect transitions for the three compounds a bulk form. (b) Tauc's plot comparison for the allowed direct transition for the three compounds in bulk and nano forms.

#### S4. Static magnetization of bulk polycrystalline and nanoparticles samples

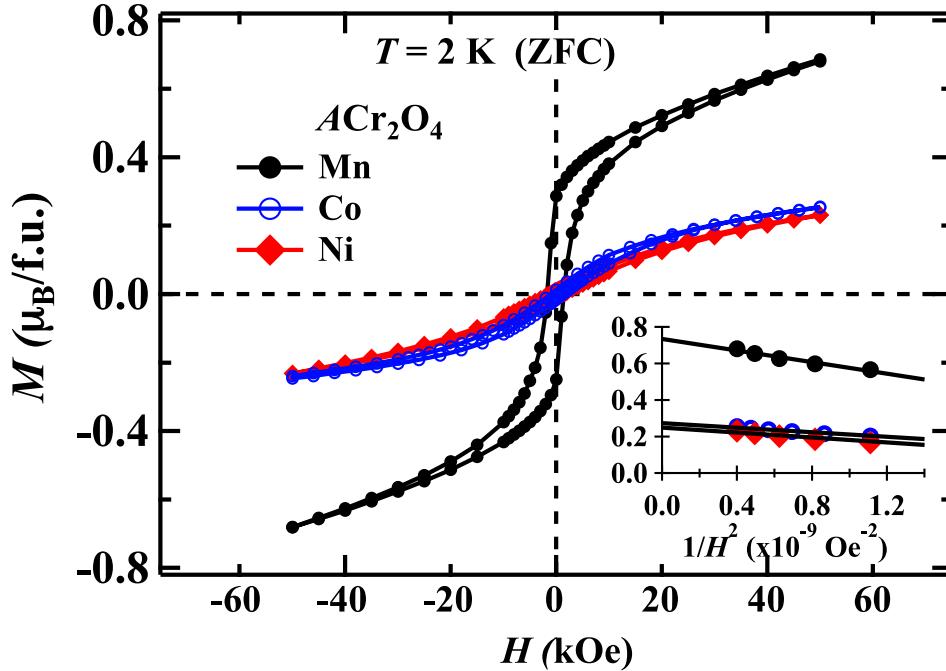
Figures S7 show temperature ( $T$ ) dependence data of the static magnetization ( $M$ ) of as-synthesized nanoparticles and bulk annealed samples of the spinel Ni, Co, and Mn chromites measured at an applied magnetic field  $H = 100$  Oe after ZFC and FC processes. The data are presented as  $M/H$  vs  $T$  in Figs. S7(a) - (c) with the low-temperature ZFC data in the insets, and as the difference ( $M_{\text{FC}} - M_{\text{ZFC}}$ ) vs  $T$  in Figs. S7(d) - (f). The bulk magnetization is about one order of magnitude larger and below the Curie temperature  $T_C$ , anomalies corresponding the bulk noncollinear FIM transition (to canted AFM state in  $\text{NiCr}_2\text{O}_4$ , and spiral FIM order in  $\text{CoCr}_2\text{O}_4$  and  $\text{MnCr}_2\text{O}_4$ ) are observed at  $T_s = 29$  (and 27), 26 (and 24) and 19 K in both  $M(T)/H$  and  $M_{\text{FC}}(T) - M_{\text{ZFC}}(T)$  for the Ni, Co and Mn bulk (and as-synthesized NCs) chromites, respectively. On the other hand, the bulk ‘lock-in’ transition to a commensurate spiral order in Co and Mn chromites is observed at  $T_s = 15$  and 14 K, respectively, but is suppressed in their nanoparticles with 13 and 10 nm particle sizes, respectively. We can not observe a transition anomaly corresponding to the spiral order in the Mn chromite NCs. The observed magnetic transitions in the bulk chromites agree with the reported magnetization and neutron diffraction results. The magnetic transitions in the NCs form are described in detail in section 3.3.1 in the main text. One notes that the noncollinear FIM transitions at  $T_s$  are largely hidden in the ZFC magnetization of the NCs, probably by a dominant surface disorder. However, unlike the bulk behavior, a low-temperature spin freezing is indicated by a blocking peak in the  $M_{\text{ZFC}}(T)$  at a temperature  $T_{\text{f2}} \approx 13$  and 5 K for the Ni and Co Chromite NCs, respectively, these are in addition to the ZFC magnetization peak at  $T_{\text{f1}}$  just below  $T_C$ .

#### S5. Core-shell magnetic spinel chromite nanoparticles.

We assume that each particle magnetically consists of a bulk magnetic core and a surface disordered shell. Above the Curie temperature  $T_C$ , the whole particle exhibits a homogeneous spin disorder in a paramagnetic behavior. With decreasing the temperature below  $T_C$ , a magnetic transition to a collinear Néel-type FIM order occurs in the particle’s core while a spin disorder due to lack of a spin compensation is maintained in each particle’s surface shell, as shown in inset of Fig. 5(i). About 20, 35, and 28% of the nanoparticle’s magnetization is a bulk magnetization maintained in its magnetic core, which is surrounded by a magnetically dead layer of an average thickness of  $\sim 3.4$ , 1.9 and 1.8 nm for the Ni, Co, and Mn as-synthesized chromite NCs, respectively, as described in the sub-section 3.3.2 in the main text. With a further decrease of temperature below  $T_s$ , the core exhibits a noncollinear FIM order with a relatively unstable state below  $T_s$  and subsequent additional spin-spin interaction (magnetic exchange) at the core-shell interface and spin-charge (magnetoelectric) interaction within the noncollinear FIM core.



**Figure S7:** Temperature dependence of the static magnetic susceptibility ( $M/H$ ) measured at 100 Oe after ZFC (open) and FC (solid) processes; (a), (b), and (c); and of the FC magnetization splitting from ZFC magnetization,  $M_{FC} - M_{ZFC}$ , measured at 100 Oe; (d), (e) and (f); for NiCr<sub>2</sub>O<sub>4</sub>, CoCr<sub>2</sub>O<sub>4</sub> and MnCr<sub>2</sub>O<sub>4</sub> bulk powder and NCs.



**Figure S8:** Isothermal magnetic ZFC hysteresis loops  $M(H)$  of  $\text{NiCr}_2\text{O}_4$ ,  $\text{CoCr}_2\text{O}_4$  and  $\text{MnCr}_2\text{O}_4$  NCs measured at 2 K. Inset shows the corresponding  $M(H)$  vs.  $1/H^2$  plots

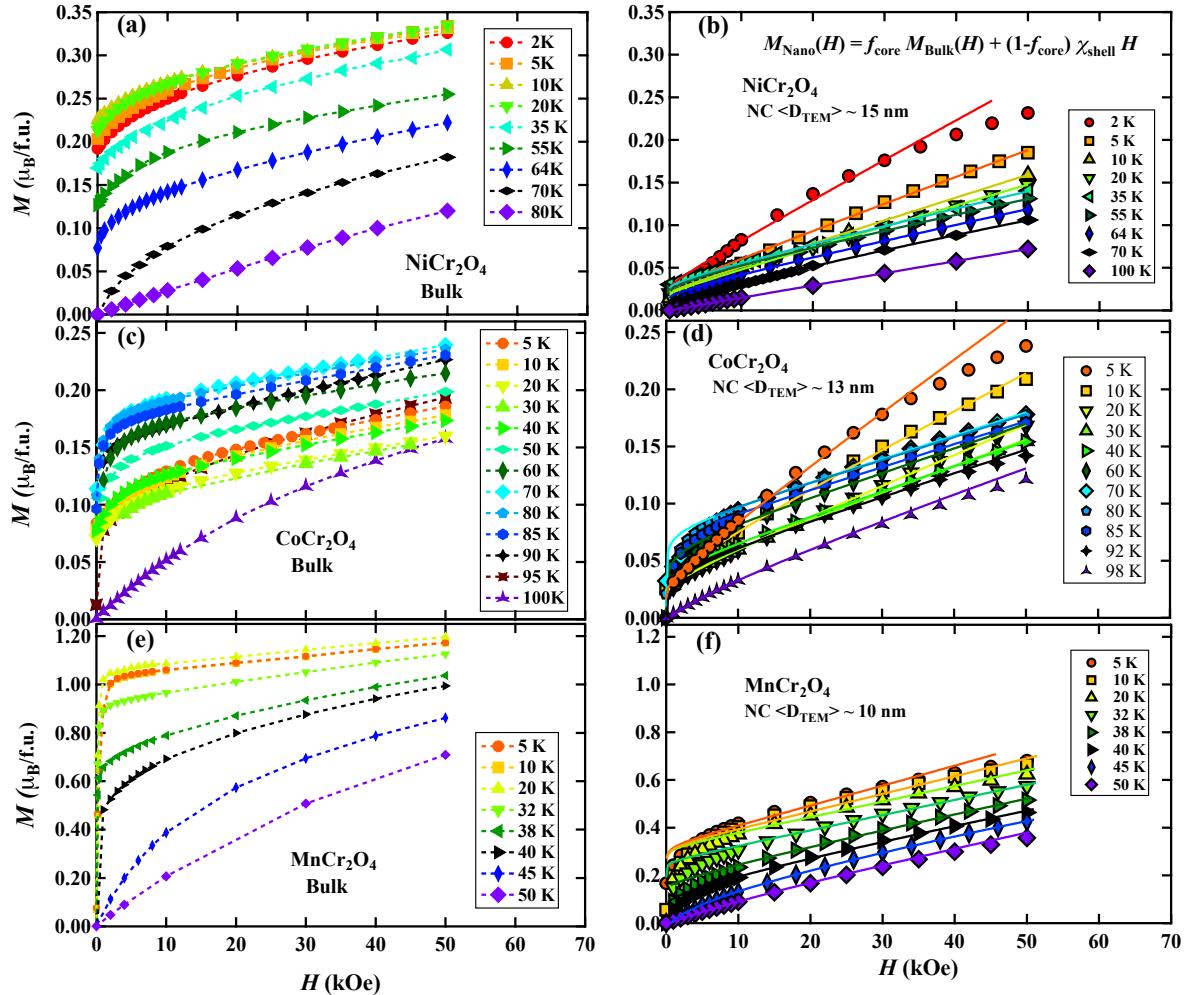
## S6. The single domain critical size

Small magnetic nanoparticles below a certain critical size behave as single domain particles and they do not benefit energetically from the wall formation. Assuming almost spherical particles, we can approximately estimate the single-domain maximum size as [18],

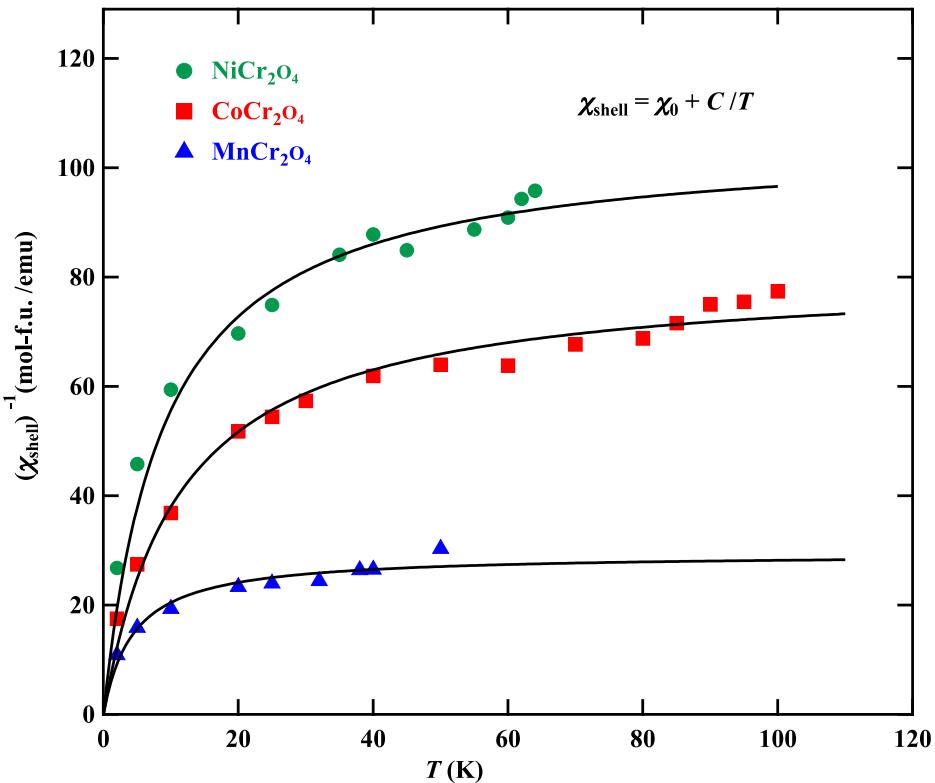
$$D_{SD} \approx \frac{18\sqrt{A K_1}}{\mu_0 M_s^2}, \quad (\text{S6-1})$$

with  $A \approx \frac{3 k_B T_C}{z a}$  is the exchange stiffness and  $K_1 \approx \frac{H_c M_s}{2}$  (at least for the Co chromite with cubic symmetry and  $<100>$  easy axes down to low temperatures) is the anisotropy constant, respectively, assuming the Stoner–Wohlfarth model for randomly oriented identical particles[19,20]. Here,  $M_s$  is the saturation magnetization in emu/g,  $k_B$  is the Boltzmann constant, and  $z$  is the effective number of the nearest neighbors,  $T_C$  is the Curie temperature and  $H_c$  is the coercive field, all in cgs units. Applying these criteria with values of  $H_c$  and  $M_s$  at 2 K results in critical single-domain sizes of about 110, 200 and 56 nm for the  $\text{NiCr}_2\text{O}_4$ ,  $\text{CoCr}_2\text{O}_4$  and  $\text{MnCr}_2\text{O}_4$  NCs, respectively. This approximate estimation indicates that our

spinel chromite nanoparticles with particles sizes up to 35, 25 and 15 nm, respectively as shown in Fig. S3, are single-domain magnetic particles free from domain walls.



**Figure S9.** Isothermal initial magnetization curves  $M(H)$  measured using annealed bulk samples and as-synthesized nanocrystals for  $\text{NiCr}_2\text{O}_4$  in (a), (b);  $\text{CoCr}_2\text{O}_4$  in (c), (d); and  $\text{MnCr}_2\text{O}_4$  in (e), (f); respectively. Solid lines in (b), (d) and (f) are fits of the nanoparticles experimental data (symbols) to Eq. (7) in main text employing isothermal bulk data in (a), (c) and (e), respectively.



**Figure S10.** The temperature dependence of the distinguished shell susceptibility of  $ACr_2O_4$  ( $A=$  Ni, Co, Mn) NCs. Solid lines represent fittings to a modified Curie law

$$\chi_{Shell} = \chi_0 + \frac{C}{T}$$

## References:

- [1] F. Izumi, T. Ikeda, A rietveld-analysis programm RIETAN-98 and its applications to zeolites, in: Materials Science Forum, Trans Tech Publ, 2000: pp. 198–205.
- [2] D.F. Swinehart, The beer-lambert law, J Chem Educ 39 (1962) 333.
- [3] J. Tauc, Optical properties and electronic structure of amorphous Ge and Si, Mater Res Bull (1968). [https://doi.org/10.1016/0025-5408\(68\)90023-8](https://doi.org/10.1016/0025-5408(68)90023-8).
- [4] C. Suchomski, C. Reitz, K. Brezesinski, C. Tavares De Sousa, M. Rohnke, K.I. Iimura, J.P. Esteves De Araujo, T. Brezesinski, Structural, optical, and magnetic properties of highly ordered mesoporous  $MCr_2O_4$  and  $MCr_{2-x}Fe_xO_4$  ( $M = Co, Zn$ ) spinel thin films with uniform 15 nm diameter pores and tunable nanocrystalline domain sizes, Chemistry of Materials 24 (2012) 155–165. <https://doi.org/10.1021/CM2026043>.
- [5] J.H. Ryu, G.S. Park, K.M. Kim, C.S. Lim, J.W. Yoon, K.B. Shim, Synthesis of  $CaWO_4$  nanocolloidal suspension via pulsed laser ablation and its optical properties, Appl Phys A Mater Sci Process 88 (2007). <https://doi.org/10.1007/s00339-007-4051-4>.
- [6] C. Suchomski, C. Reitz, K. Brezesinski, C. Tavares de Sousa, M. Rohnke, K. Iimura, J.P. Esteves de Araujo, T. Brezesinski, Structural, Optical, and Magnetic Properties of Highly Ordered Mesoporous  $MCr_2O_4$  and  $MCr_{2-x}Fe_xO_4$  ( $M=Co, Zn$ ) Spinel Thin Films with Uniform 15 nm Diameter Pores and Tunable Nanocrystalline Domain Sizes, Chemistry of Materials 24 (2012) 155–165.

- [7] V. Kocsis, S. Bordács, D. Varjas, K. Penc, A. Abouelsayed, C.A. Kuntscher, K. Ohgushi, Y. Tokura, I. Kézsmárki, Magnetoelasticity in  $ACr_2O_4$  spinel oxides (A= Mn, Fe, Co, Ni, and Cu), *Phys Rev B Condens Matter Mater Phys* 87 (2013) 064416. <https://doi.org/10.1103/PHYSREVB.87.064416/FIGURES/11/MEDIUM>.
- [8] K. Ohgushi, Y. Okimoto, T. Ogasawara, S. Miyasaka, Y. Tokura, Magnetic, Optical, and Magnetooptical Properties of Spinel-Type  $A Cr_2 X_4$  ( A =Mn, Fe, Co, Cu, Zn, Cd; X =O, S, Se), *J. Phys. Soc. Jpn.* 77 (2008) 034713. <https://doi.org/10.1143/JPSJ.77.034713>.
- [9] R.G. Burns, Mineralogical applications of crystal field theory. Second edition, *Mineralogical Applications of Crystal Field Theory*. Second Edition (1993).
- [10] C.K. Jørgensen, Absorption spectra and chemical bonding in complexes, Elsevier, 2015.
- [11] H. Kamimura, S. Sugano, Y. Tanabe, Ligand field theory and its applications, Syokabo, Tokyo 224 (1969).
- [12] J.S. Griffith, L.D. Roberts, The Theory of the Transition-Metal Ions , *Phys Today* 15 (1962). <https://doi.org/10.1063/1.3057810>.
- [13] J. Ferguson, Crystal spectra of metal coordination compounds. I. Tetrahedral cobalt(II), *J Chem Phys* 32 (1960). <https://doi.org/10.1063/1.1730729>.
- [14] H.A. Weakliem, Optical spectra of  $Ni^{2+}$ ,  $Co^{2+}$ , and  $Cu^{2+}$  in tetrahedral sites in crystals, *J Chem Phys* 36 (1962). <https://doi.org/10.1063/1.1732840>.
- [15] R.K. Ahrenkiel, T.H. Lee, S.L. Lyu, F. Moser, Giant magneto-reflectance of  $CoCr_2S_4$ , *Solid State Commun* 12 (1973). [https://doi.org/10.1016/0038-1098\(73\)90124-5](https://doi.org/10.1016/0038-1098(73)90124-5).
- [16] A.S. Davydov, Theory of Molecular Excitons, 1971. <https://doi.org/10.1007/978-1-4899-5169-4>.
- [17] M. Ulrich, J. García-Otero, J. Rivas, A. Bunde, Slow relaxation in ferromagnetic nanoparticles: Indication of spin-glass behavior, *Phys Rev B Condens Matter Mater Phys* 67 (2003).
- [18] J.M.D. Coey, Magnetism and Magnetic Materials, Cambridge University Press, 2010. [https://doi.org/10.1016/0160-9327\(88\)90178-0](https://doi.org/10.1016/0160-9327(88)90178-0).
- [19] N.A. Usov, S.E. Peschany, Theoretical hysteresis loops for single-domain particles with cubic anisotropy, *J Magn Magn Mater* 174 (1997) 247–260. [https://doi.org/10.1016/S0304-8853\(97\)00180-7](https://doi.org/10.1016/S0304-8853(97)00180-7).
- [20] E.C. Stoner, E. Wohlfarth, A mechanism of magnetic hysteresis in heterogeneous alloys, . *Phil. Trans. R. Soc. Lond A* 240 (1948) 599–642.