Supporting Information: Boundary effects on Turing pattern formation in a spiral growing domain

Leonardo Silva-Dias† and Milos Dolnik‡

†*Faculty of Mathematics, Natural Sciences, and Materials Engineering: Institute of Physics, University of Augsburg, Universitätsstraße 1, 86159 Augsburg, Germany*

‡*Department of Chemistry and Center for Complex Systems, MS015, Brandeis University, Waltham, Massachusetts 02454-9110, USA*

E-mail:

Time scales

Here we estimate the ratio between the diffusion time scale of the activator (τ_a) and the growth time scale (τ_g) for the reacting domain.

The diffusion time scale is the time it takes for the activator to diffuse across a reference area A, where the reference area is defined as $A = \lambda^2$. This time scale is obtained from the expression: $\tau_a = \lambda^2/D_a$. Using $\lambda = 7.25$ s.u., and $Da = 1.0$ (s.u.)^2/t.u, then $\tau_a \approx 52.6$ t.u..

The growth time scale refers to the time it takes for the reacting domain to grow from time zero size to the reference area A. This area is expressed through the double integral:

$$A = \int_{\theta_0}^{\theta_F} \int_{r_0}^{r_F} r dr d\theta \tag{1}$$

As defined in the main text, $\mathbf{r(t)}$, and $\theta(t)$ are time-dependent polar coordinates. Using these expressions, we obtain the following:

$$A = \dot{\theta}|\dot{\mathbf{r}}|(r_0 + |\dot{\mathbf{r}}|\frac{t}{2})t^2 \tag{2}$$

Now, assuming that $A = \lambda^2$ and $\dot{\theta} = \frac{360|\dot{\mathbf{r}}|}{n\lambda}$ are known, we can solve the equation above to find the growth time scale.

Therefore, we can verify that $\tau_g = 2.58$ $t.u.$ for $|\dot{\mathbf{r}}| = 0.5$ $s.u./t.u.$, and $\tau_g = 25.8$ $t.u.$ for $|\dot{\mathbf{r}}| = 0.05$ $s.u./t.u.$.

Videos

The videos display some examples of the the spiral growth process and Turing pattern formation obtained from the numerical simulations.

- video1.mp4 - It shows the pattern formation for $\ddot{\mathbf{r}} = 0.01$, $\dot{\mathbf{r}} = 0.5$, and $n_0 = 1.0$ in an IEB system with **NaDi** boundary conditions.

- video2.mp4 - It shows the pattern formation for $\dot{\mathbf{r}} = 0.5$, and $n = 2.0$ in an IEB system with **DaNi** boundary conditions.

- video3.mp4 - It shows the pattern formation for $\dot{\mathbf{r}} = 0.5$, and $n = 1.5$ in an IEB system with **N** boundary conditions.

- video4.mp4 - It shows the pattern formation for $\dot{\mathbf{r}} = 0.5$, and $n = 1.0$ in an EB system with **NaDi** boundary conditions.

Simulations with Dirichlet boundary conditions for both chemicals

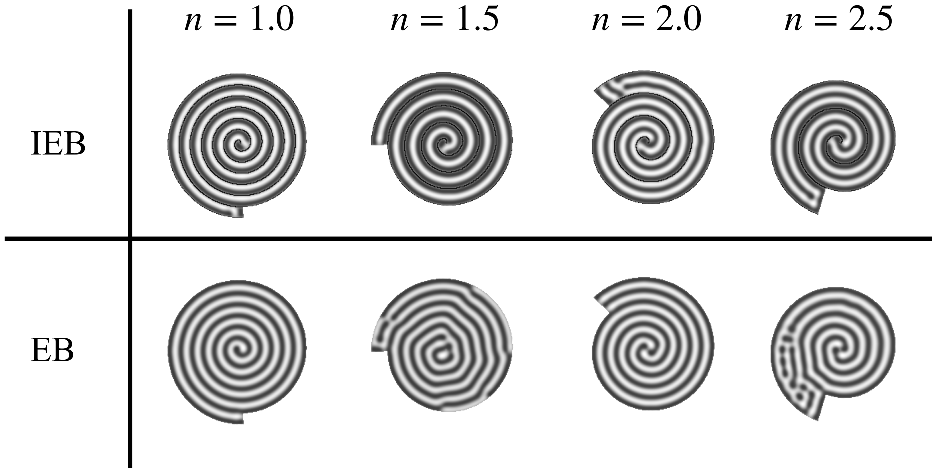

Figure 1: Turing patterns on rotating spiral growing domain with Dirichlet boundary conditions for both activator and inhibitor

Different boundary conditions for the internal and external boundaries

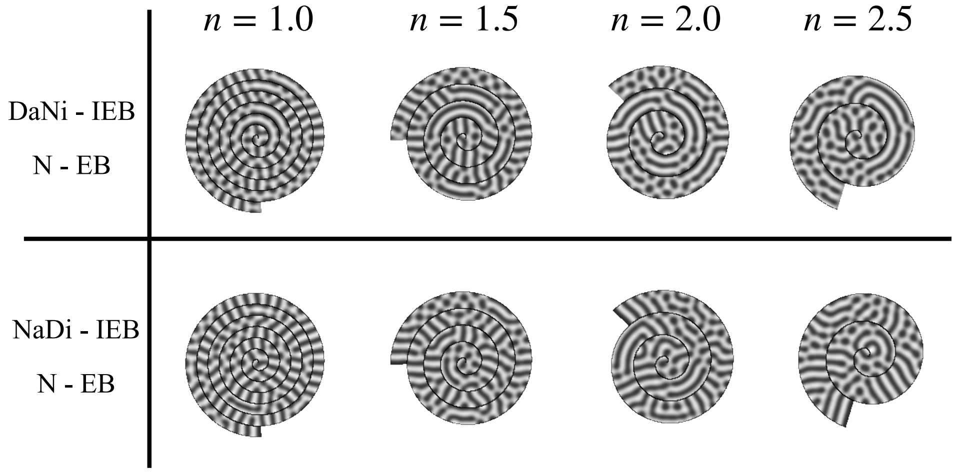

Figure 2: Turing patterns on rotating spiral growing domain with with different boundary conditions for internal and external boundaries.

Parameter n - acceleration

In this section, we derive Eq. (10) from the main text. We start from the definition of the parameter n for linear growth. The parameter n represents the number of pattern wavelengths that the radius of the system grows after one 360° rotation:

$$n = \frac{\Delta \mathbf{r}}{\lambda} \tag{3}$$

where, $\Delta r = |\mathbf{r}_F| - |\mathbf{r}_0|$, $\mathbf{r}_F = (\beta, 0)$ is the radius vector after a 360^o rotation, and λ is the wavelength of the natural Turing pattern. We assume here $\mathbf{r}_0 = (0, 0)$. The time taken to complete a 360^o rotation is $t = 360/\dot{\theta}$, which leads to $\beta = \frac{|\dot{\mathbf{r}}|360^o}{\dot{\theta}} + \frac{|\ddot{\mathbf{r}}|(360^o)^2}{2\dot{\theta}^2}$. Hence, we obtain the following:

$$n = \frac{360^o|\dot{\mathbf{r}}|}{\lambda\dot{\theta}} + \frac{|\ddot{\mathbf{r}}|}{2\lambda}\left(\frac{360^o}{\dot{\theta}}\right)^2. \tag{4}$$