

## Electronic Supplementary Information

# Moiré-Pattern-Assisted Thermoelectric Enhancement in Tungsten Diselenide Bilayer

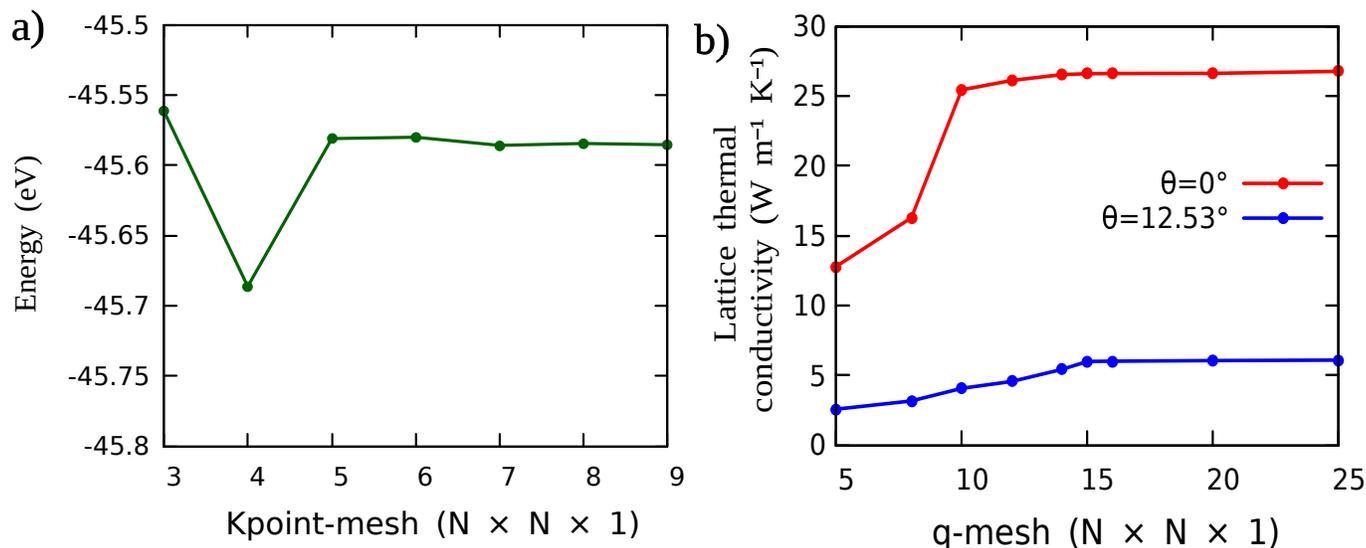
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### I. CONVERGENCE OF ELECTRONIC AND PHONON PROPERTIES

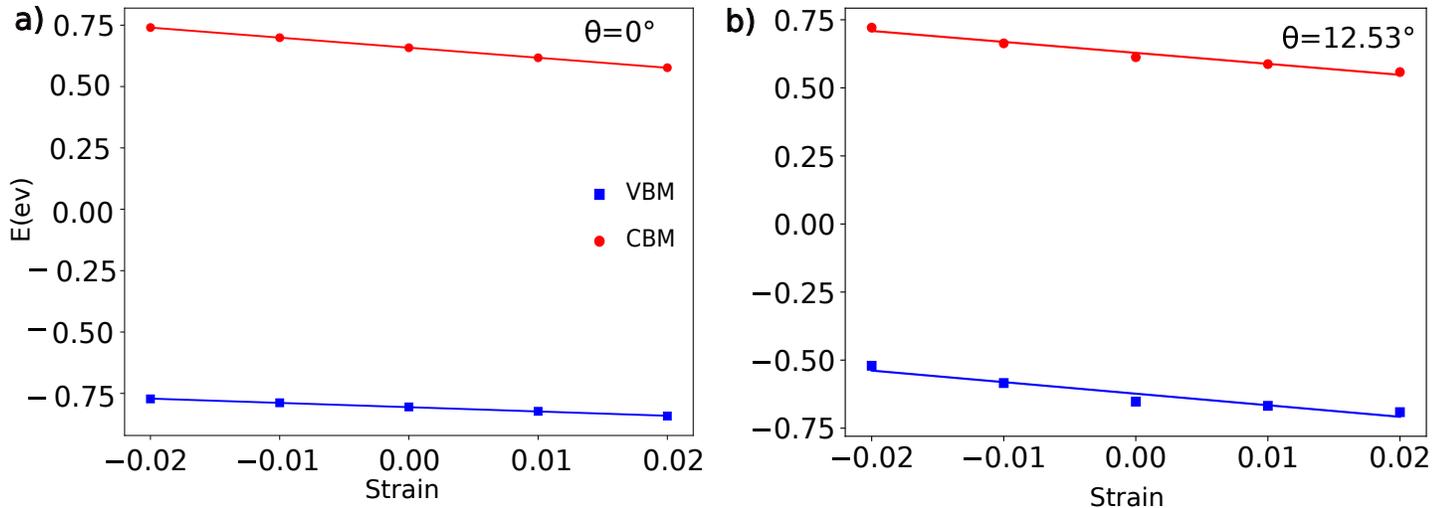
The convergence of the ground-state energy with respect to the k-point mesh and the convergence of the lattice thermal conductivity with respect to the q-point mesh are shown in **Fig. S1**. The ground-state energy is found to converge beyond a  $7 \times 7 \times 1$  k-point mesh, while the lattice thermal conductivity converges for q-point meshes denser than  $15 \times 15 \times 1$ .



**Figure S1.** Convergence of (a) electronic ground state energy with k-point mesh and (b) lattice thermal conductivity with q-point mesh.

## II. DEFORMATION POTENTIAL CALCULATIONS

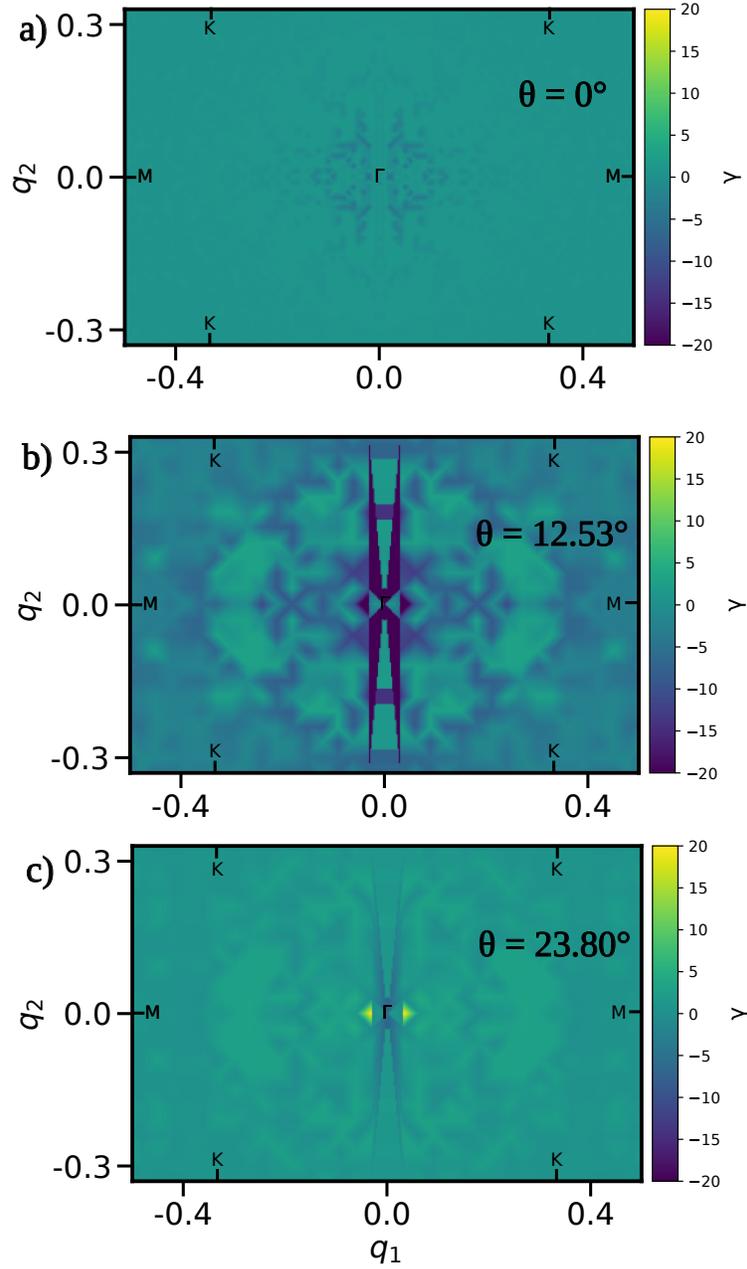
The calculated deformation potential by applying uniaxial strain is shown in **Fig. S2**. The structures are isotropic, therefore, the deformation potential along both the in-plane directions is same.



**Figure S2.** Deformation potential calculations for (a) tungsten diselenide bilayer ( $0^\circ$ ) (b)  $12.53^\circ$  twisted tungsten diselenide superlattice. VBM and CBM represent valence band maxima and conduction band minima, respectively.

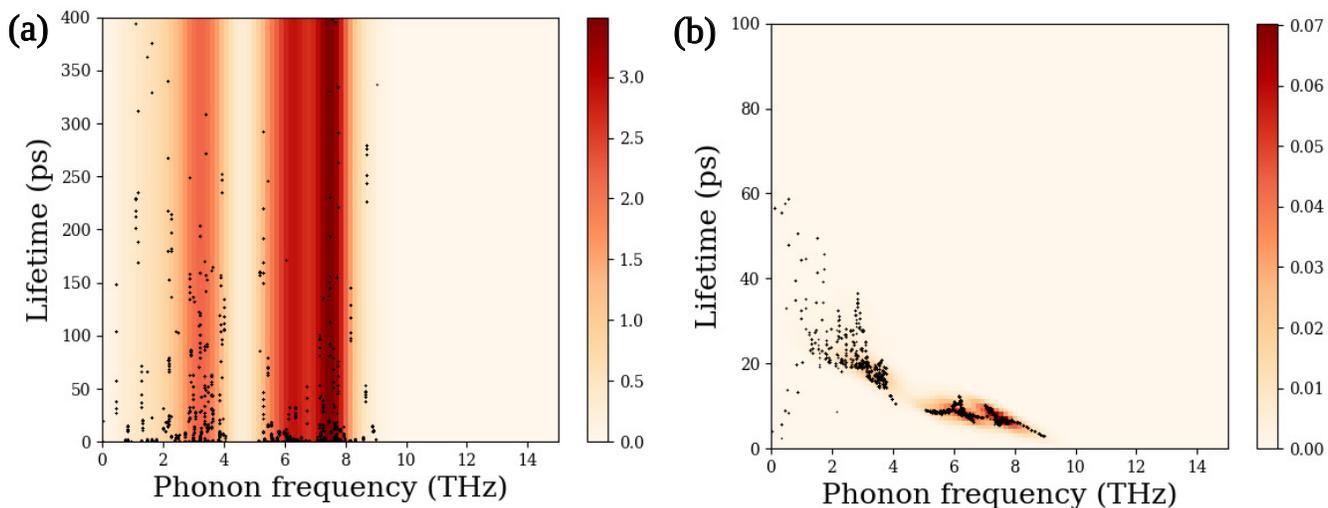
## III. PHONON TRANSPORT PROPERTIES

The Grüneisen-parameter contour maps clearly demonstrate the strong influence of the moiré potential on the anharmonic behavior of twisted  $WSe_2$  bilayer is shown in **S3** for all the three structures. The untwisted structure exhibits relatively uniform and moderate anharmonicity, while increasing twist leads to pronounced variations across the mini-Brillouin zone.

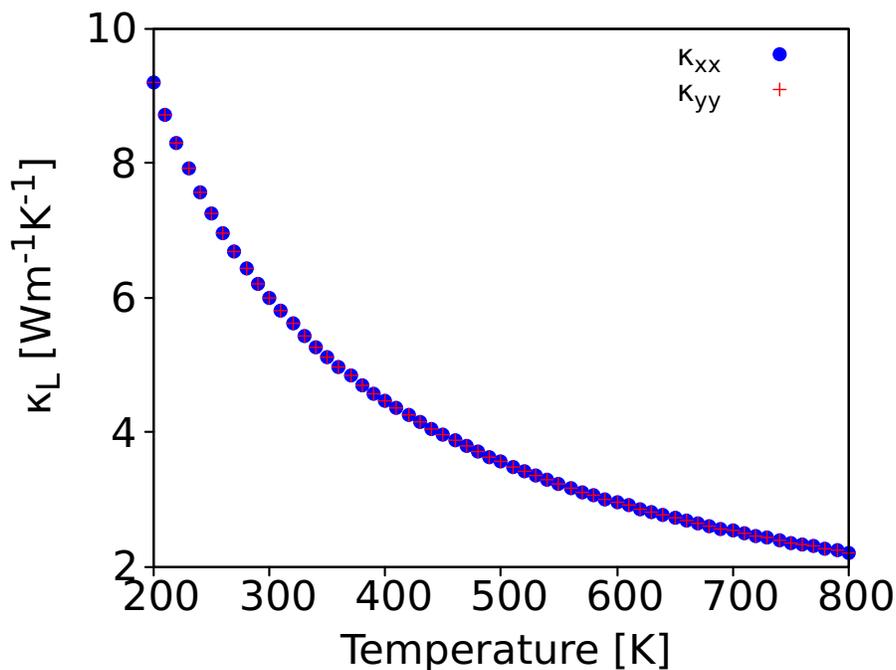


**Figure S3.** Grüneisen-parameter contour maps plots for (a) tungsten diselenide bilayer ( $0^\circ$ ) (b)  $12.53^\circ$  twisted, and (c)  $23.80^\circ$  twisted tungsten diselenide superlattice. The range of color bar has been taken same for comparison.

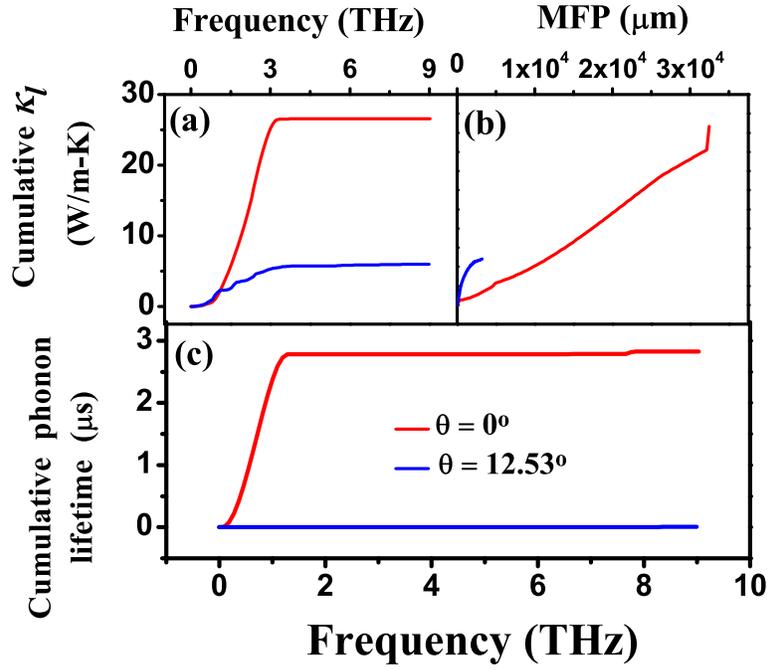
The variation of phonon lifetime with the low frequencies is shown in S4. It is observed that the untwisted bilayer configuration exhibits comparatively longer phonon relaxation times. In contrast, introducing a  $12.53^\circ$  twist markedly enhances phonon scattering, leading to a substantial reduction in relaxation time for phonons.



**Figure S4.** Phonon lifetime vs frequency plots for (a) tungsten diselenide bilayer ( $0^\circ$ ) (b)  $12.5^\circ$  twisted tungsten diselenide superlattice. The heatmap shows the probability density (how many phonon modes exist in that frequency–lifetime region) and the black-dots represent the actual data points for each phonon mode.



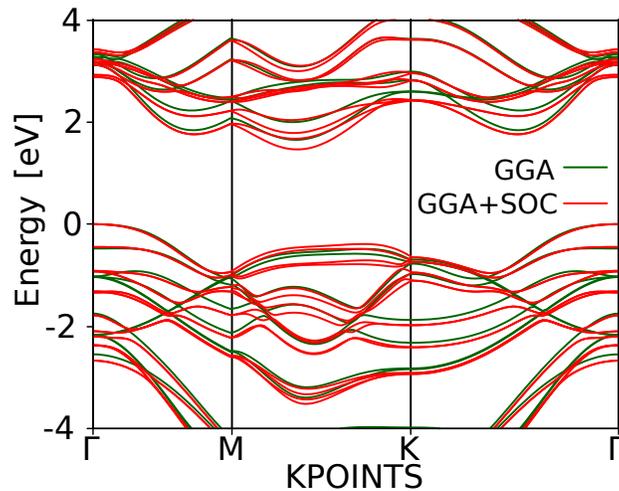
**Figure S5.** Lattice thermal conductivity along the  $xx$  (solid blue circles) and  $yy$  (open red squares) directions as a function of temperature for  $12.5^\circ$  twisted tungsten diselenide superlattice. These two directions are effectively symmetry-equivalent with respect to in-plane thermal transport



**Figure S6.** (a) Cumulative lattice thermal conductivity vs frequency plot. (b) cumulative lattice thermal conductivity vs phonon mean-free path and (c) cumulative phonon lifetime vs frequency plot for tungsten diselenide bilayer ( $0^\circ$ ) and  $12.53^\circ$  twisted tungsten diselenide superlattice. The Y-axis for both figures (a) and (b) is kept identical.

#### IV. SOC EFFECT ON ELECTRONIC BAND STRUCTURE

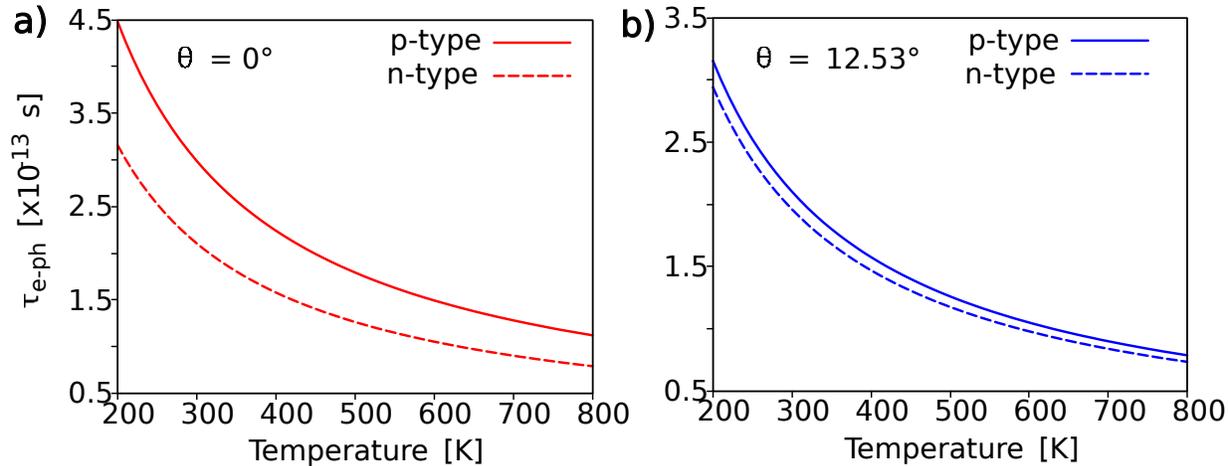
The comparison of electronic band structure for  $0^\circ$  twisted  $WSe_2$  bilayer with and without spin orbit coupling effect is shown in **Fig. S7**. It clearly demonstrates that SOC induces a pronounced splitting of the electronic bands near the Fermi level.



**Figure S7.** Comparison of the electronic band structures of twisted  $WSe_2$  bilayer calculated without spin-orbit coupling (GGA) and with spin-orbit coupling (GGA+SOC).

## V. ELECTRON-PHONON SCATTERING RELAXATION TIME

The temperature dependent electron-phonon scattering relaxation time for  $WSe_2$  bilayer and  $12.53^\circ$  twisted  $WSe_2$  bilayer for both types of charge carriers is shown in **Fig. S8**



**Figure S8.** Relaxation time for electron-phonon scattering of (a) tungsten diselenide bilayer ( $0^\circ$ ) (b)  $12.53^\circ$  twisted tungsten diselenide superlattice. p-type and n-type refers to holes and electrons concentrations respectively.