Supplementary Information (SI) for Lab on a Chip.

Supporting Material for

Oscillatory flow for contactless particle trapping

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Table of variables

Variable name	Description	Unit
ρ	Fluid density	Kg/m^3
μ	Dynamic viscosity of the fluid	kg/(m·s)
V_{sf}	Mean velocity of the bias flow	m/s
V_1	Mean oscillatory fluid velocity	m/s
V_{Osc}	Apparent mean oscillatory velocity	m/s
ϕ	Phase offset	-
B	Bias ratio of forward to backward movement	-

Supplementary table 1: table of variables.

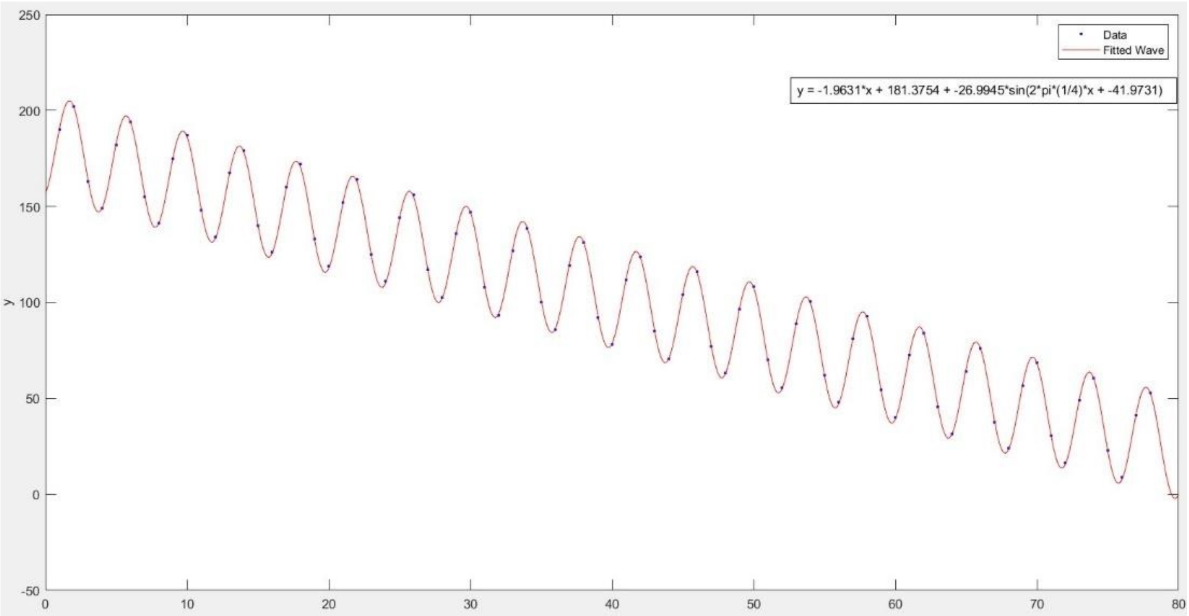
Forward and backward particle displacement calculation method.

The data shown in Supplementary Fig1.A corresponds to the data from the trap strength experiment, with 40 µm beads, 25 Hz and 25% speaker volume. The data was extracted with ImageJ, allowing to obtain the position of the center of mass of a particle for each frame in which the particle can be seen. The X coordinate was used to fit a sinewave to the oscillatory position of the particle as seen in Supplementary Fig 1.B. Four parameters were used to fit the model to the data, these parameters being the coefficients of the sinewave equation: $y = mt + b + A\sin(\frac{2\pi}{4}t + \phi)$. Here, A is amplitude, ϕ is phase offset, and t is time at which the image was taken. The $mt + b$ linear function accounted for lateral drift of the untrapped particle due to the syringe-pump steady flow, while the sinewave accounted for the oscillatory flow velocity. Once the equation was obtained, local extrema of the function were found by performing a curve fit of the expression for the sinewave with y =-2(Arcos(-2m/(API))+P)/PI + 2n with n being the frame number. The y coordinates were then found for adjacent frames, giving coordinates for a peak, a trough, and a peak respectively. The forward and backward motions were then found by subtracting the trough to the first peak and then the trough to the second peak respectively.

A.

Frame number	Particle X position	Frame number	Particle X position	Frame number	Particle X position	Frame number	Particle X position	Frame number	Particle X position	Frame number	Particle X position
1	190.0	16	126.2	31	107.8	46	116.0	61	72.5	76	8.8
2	202.0	17	160.0	32	93.2	47	77.0	62	84.0	77	41.2
3	163.0	18	172.0	33	126.8	48	63.2	63	45.5	78	52.8
4	149.0	19	133.0	34	138.5	49	96.5	64	31.5		
5	182.0	20	118.8	35	100.0	50	108.2	65	64.0		
6	194.0	21	152.0	36	85.8	51	70.0	66	76.0		
7	155.0	22	164.0	37	119.2	52	55.5	67	37.5		
8	141.3	23	125.0	38	131.2	53	88.8	68	24.0		
9	174.8	24	111.0	39	92.0	54	100.5	69	56.5		
10	187.0	25	144.2	40	78.0	55	62.0	70	68.5		
11	148.0	26	156.0	41	111.7	56	48.0	71	30.5		
12	134.0	27	117.2	42	123.8	57	81.0	72	16.5		
13	167.5	28	102.5	43	85.0	58	92.8	73	49.0		
14	179.0	29	135.8	44	70.5	59	54.5	74	60.5		
15	140.0	30	147.0	45	104.0	60	40.0	75	22.8		

B.



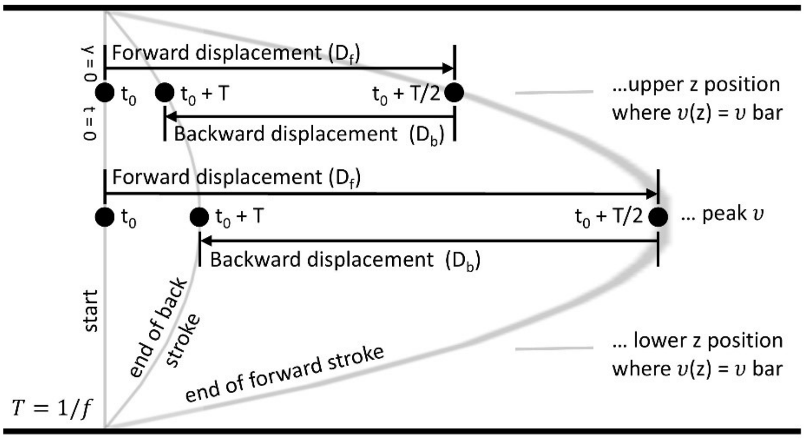
Supplementary figure 1: Example data and resulting fit of x coordinate of particle center of mass.

Computation of V1 from micro particle image velocimetry.

To understand how the characteristic oscillatory velocity magnitude (V_1) may be computed from knowing the forward displacement (D_f) and backward displacement (D_b) of an observed particle, let us first establish that:

$$D_f = \int_0^{T/2} [v_{osc}(z,t) + v_{sf}(z)]dt \quad , \quad D_b = \int_{T/2}^{T} [v_{osc}(z,t) - v_{sf}(z)]dt \quad , \text{ and } \quad B = \frac{D_f}{D_b} .$$

These single-oscillation displacements are directly measured from PIV, upstream of the oscillatory trap, from one or more 10 µm particles that travel within any particular z-position. As in the associated supplementary figure 2, note that although the magnitudes of D_f and D_b approach zero near the walls and become largest at the midplane, their ratio (B) is invariant owing to the parabolic flow profile that scales with the current pressure gradient.



<u>Supplementary figure 2:</u> Micro-particle image velocimetry enables measurement of the characteristic oscillatory velocity magnitude V1. Channel cross-section is depicted (H_1 = 100 μm), along with a single oscillation in the y-z plane (symmetric into the page, in x direction).

However, there are two special positions above and below the midline where the instantaneous velocities (oscillatory and steady flow, which does not vary) are each equal to the current mean velocity, defined as:

$$\bar{\upsilon}_{osc}(t) + \bar{\upsilon}_{sf} = \frac{1}{H_1}\int_0^{H_1} [\upsilon_{osc}(z,t) + \upsilon_{sf}(z)]dz \; .$$

Taking the further step of averaging over time, for one half period (T/2), and multiplying by the channel cross-sectional area, the flow rate may be directly expressed in their terms as:

$$Q_f = WH_1\frac{2}{T}\int_0^{T/2} [\bar{\upsilon}_{osc}(t) + \bar{\upsilon}_{sf}]dt = Q_{osc} + Q_{sf}$$

, and similarly

$$Q_b = WH_1\frac{2}{T}\int_{T/2}^{T} [\bar{\upsilon}_{osc}(t) - \bar{\upsilon}_{sf}]dt = Q_{osc} - Q_{sf} \; ,$$

where all velocities positive. Let us then define the characteristic oscillatory velocity V_1 as:

$$V_1 = \frac{2}{T}\int_0^{T/2} \bar{\upsilon}_{osc}(t)dt = \frac{2}{T}\int_{T/2}^{T} \bar{\upsilon}_{osc}(t)dt \; .$$

In other words, V_1 is the average oscillatory speed at one of the two special streamlines where the instantaneous velocity equals the mean velocity across the channel height H_1. Conveniently, this allows us to write the following, realizing also that υ_{sf}(z) and its average is constant:

$$Q_{osc} + Q_{sf} = WH_1(V_1 + \bar{\upsilon}_{sf}) \; , \quad Q_{osc} - Q_{sf} = WH_1(V_1 - \bar{\upsilon}_{sf})$$

, and finally

$$\frac{Q_{osc} + Q_{sf}}{Q_{osc} - Q_{sf}} = \frac{V_1 + \bar{\upsilon}_{sf}}{V_1 - \bar{\upsilon}_{sf}} = \frac{(V_1 + \bar{\upsilon}_{sf})\left(T/2\right)}{(V_1 - \bar{\upsilon}_{sf})\left(T/2\right)} = \frac{D_f}{D_b} = B \; ,$$

which again is directly measured from the particle velocimetry. Because the syringe pump flow rate is known, and υ_{sf} bar is that divided by WH_1, and T is the known speaker oscillation period, V_1 may be directly computed using the above established equivalency of B.

Captions for supplementary videos

- "MOAT_trapped_20um_particles.avi" file: 20 µm polystyrene beads trapped in an expanding channel (from 100 µm depth to 200 µm depth over a 400 µm long transition zone). 75 Hz oscillation frequency, 12 µL/min steady bias flow rate, 80 frames per second video recording.
- "MOAT_trapped_cells.mp4" file: 19.3 µm diameter MGH-BRx-142 cells trapped in an expanding channel (from 100 µm depth to 200 µm depth over a 400 µm long transition zone). 75 Hz oscillation frequency, 25 µL/min steady bias flow rate, 76 frames per second video recording

Data supporting this article, used to generate trapping efficiency plots in figure 2.

		40%		60%	
		Re	Q µL/min	Re	Q µL/min
10µm	25 Hz	1,31E+00	0,00E+00	1,31E+00	0,00E+00
		5,22E-01	0,00E+00	5,22E-01	0,00E+00
		4,57E+00	1,00E+01	4,57E+00	0,00E+00
		2,08E+00	0,00E+00	2,08E+00	0,00E+00
	50 Hz	3,58E+00	0,00E+00	3,58E+00	0,00E+00
		4,98E+00	1,00E+01	4,98E+00	1,00E+01
		5,73E+00	1,00E+01	5,73E+00	0,00E+00
		4,77E+00	1,00E+01	4,77E+00	0,00E+00
	75 Hz	3,58E+00	0,00E+00	3,58E+00	0,00E+00
		8,25E+00	4,00E+01	8,25E+00	0,00E+00
		1,19E+01	5,00E+01	1,19E+01	0,00E+00
		9,24E+00	6,00E+01	9,24E+00	0,00E+00
	100 Hz	1,88E+00	0,00E+00	1,88E+00	0,00E+00
		2,56E+00	0,00E+00	2,56E+00	0,00E+00
		2,58E+00	0,00E+00	2,58E+00	0,00E+00
		2,55E+00	0,00E+00	2,55E+00	0,00E+00
	125 Hz	1,40E+00	0,00E+00	1,40E+00	0,00E+00
		1,09E+00	0,00E+00	1,09E+00	0,00E+00
		3,01E+00	0,00E+00	3,01E+00	0,00E+00
		2,74E+00	0,00E+00	2,74E+00	0,00E+00

		40%		60%	
		Re	Q µL/min	Re	Q µL/min
20µm	25 Hz	1,31E+00	0,00E+00	1,31E+00	0,00E+00
		5,22E-01	0,00E+00	5,22E-01	0,00E+00
		4,57E+00	1,00E+01	4,57E+00	1,00E+01
		2,08E+00	1,00E+01	2,08E+00	1,00E+01
	50 Hz	3,58E+00	2,00E+01	3,58E+00	2,00E+01
		4,98E+00	3,00E+01	4,98E+00	2,00E+01
		5,73E+00	3,00E+01	5,73E+00	3,00E+01
		4,77E+00	3,00E+01	4,77E+00	3,00E+01
	75 Hz	3,58E+00	1,00E+01	3,58E+00	1,00E+01
		8,25E+00	1,20E+02	8,25E+00	1,00E+02
		1,19E+01	1,30E+02	1,19E+01	1,10E+02
		9,24E+00	1,30E+02	9,24E+00	1,20E+02
	100 Hz	1,88E+00	0,00E+00	1,88E+00	0,00E+00
		2,56E+00	2,00E+01	2,56E+00	1,00E+01
		2,58E+00	1,00E+01	2,58E+00	0,00E+00
		2,55E+00	1,00E+01	2,55E+00	0,00E+00
	125 Hz	1,40E+00	0,00E+00	1,40E+00	0,00E+00
		1,09E+00	0,00E+00	1,09E+00	0,00E+00
		3,01E+00	0,00E+00	3,01E+00	0,00E+00
		2,74E+00	0,00E+00	2,74E+00	0,00E+00

		40%		60%	
		Re	Q µL/min	Re	Q µL/min
40µm	25 Hz	1,31E+00	0,00E+00	1,31E+00	0,00E+00
		5,22E-01	0,00E+00	5,22E-01	0,00E+00
		4,57E+00	0,00E+00	4,57E+00	0,00E+00
		2,08E+00	1,00E+01	2,08E+00	1,00E+01
	50 Hz	3,58E+00	2,00E+01	3,58E+00	1,00E+01
		4,98E+00	2,00E+01	4,98E+00	1,00E+01
		5,73E+00	2,00E+01	5,73E+00	2,00E+01
		4,77E+00	2,00E+01	4,77E+00	1,00E+01
	75 Hz	3,58E+00	2,00E+01	3,58E+00	1,00E+01
		8,25E+00	7,00E+01	8,25E+00	7,00E+01
		1,19E+01	1,00E+02	1,19E+01	8,00E+01
		9,24E+00	8,00E+01	9,24E+00	8,00E+01
	100 Hz	1,88E+00	0,00E+00	1,88E+00	0,00E+00
		2,56E+00	1,00E+01	2,56E+00	1,00E+01
		2,58E+00	1,00E+01	2,58E+00	1,00E+01
		2,55E+00	2,00E+01	2,55E+00	1,00E+01
	125 Hz	1,40E+00	0,00E+00	1,40E+00	0,00E+00
		1,09E+00	1,00E+01	1,09E+00	1,00E+01
		3,01E+00	1,00E+01	3,01E+00	1,00E+01
		2,74E+00	1,00E+01	2,74E+00	1,00E+01

<u>Data supporting this article, used to generate trap strength plots in figure 3.</u>

	Re	Q µL/min		Re	Q µL/min		Re	Q µL/min
10µm	4,60E-01	3,00E+00	20µm	1,69E+00	9,44E+00	40µm	1,29E+00	7,87E+00
	3,20E-01	3,00E+00		3,00E-01	3,00E+00		9,50E-01	8,21E+00
	1,76E+00	3,89E+00		1,59E+00	1,75E+01		1,21E+00	8,86E+00
	3,45E+00	8,20E+01		4,41E+00	7,11E+01		2,73E+00	2,28E+01
	5,04E+00	1,02E+02		5,48E+00	8,09E+01		2,78E+00	3,10E+01
	5,31E+00	1,04E+02		4,97E+00	6,57E+01		3,47E+00	3,44E+01
	2,90E-01	3,00E+00		2,95E+00	3,24E+01		1,10E+00	7,20E+00
	6,00E+00	1,44E+02		4,45E+00	8,94E+01		2,24E+00	1,77E+01
	5,00E+00	1,36E+02		3,93E+00	7,61E+01		5,54E+00	6,09E+01
	2,05E+00	1,64E+01		1,92E+00	1,62E+01		4,20E-01	3,00E+00
	1,49E+00	3,00E+00		2,42E+00	2,50E+01		5,30E-01	4,09E+00
	1,40E+00	3,00E+00		1,84E+00	1,34E+01		6,50E-01	4,83E+00
	1,17E+00	3,00E+00		1,00E+00	3,00E+00		3,90E-01	3,00E+00
	2,67E+00	2,97E+01		2,31E+00	2,00E+01		5,40E-01	4,50E+00
	2,78E+00	3,33E+01		2,03E+00	2,11E+01		6,50E-01	4,91E+00