Supplementary Information: Short time spreading dynamics of elastic drops

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Within this Supplemental Material, we present additional experimental details like rheology, detailed derivations of the scaling laws mentioned in the main text, and a additional plot.

Strain amplitude measurements

For the prepared PAAm, strain amplitude sweep was performed on a dynamic shear rheometer (Kinexus Rotational Rheometer, Malvern Instruments) at 1 Hz frequency (see Fig. S1). From the measured G' and G'', we observed that for PAAm 6.5 wt%, the Linear Viscoelastic Region (LVR) is sustained till approximately 0.1% strain, beyond which the material response deviates due to the relatively weak polymer network of the PAAm (Fig. S1a). Further, beyond 20% strain, G'' overshoots G' and the material eventually ruptures around 100% strain. For PAAm 30 wt%, the LVR is observed for approximately 1% strain due to the relatively stiffer polymer network (Fig. S1b). Although the material response deviates beyond 1% strain, a drastic drop in both G'' and G'' is only observed at 120% strain, highlighting close to rupture scenario.

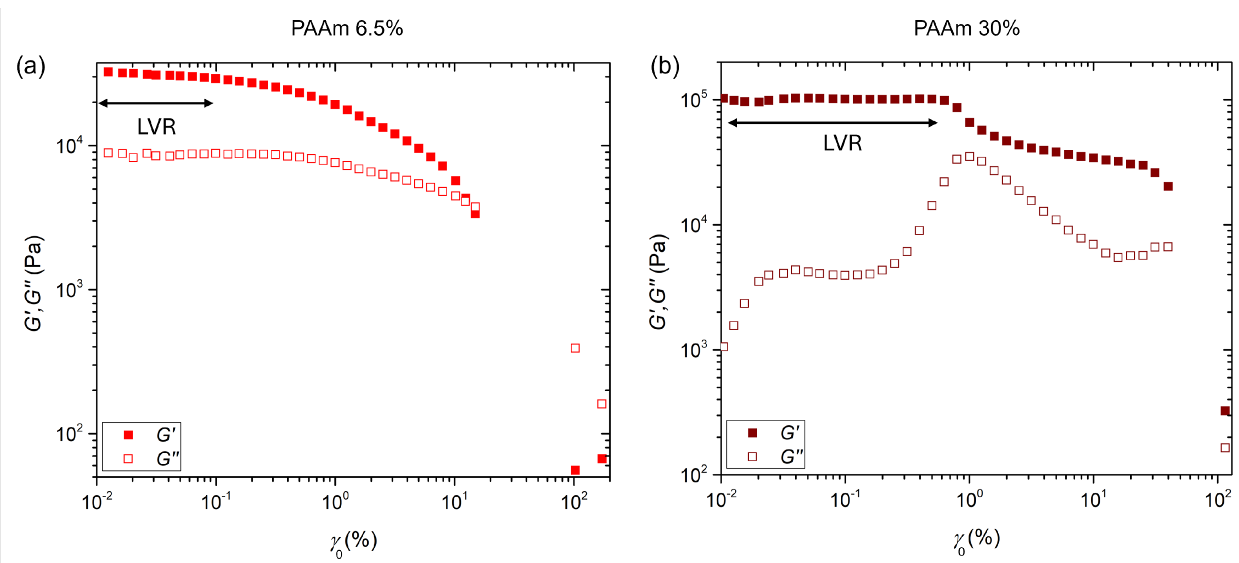

FIG. S1. Variation of shear storage modulus (G') and shear loss modulus (G'') with shear strain γ_0 (%) for (a) PAAm 6.5 wt.% monomer and (b) PAAm 30 wt.% monomer. The Linear Viscoelastic Regions (LVR) are marked for clarity. The measurements are performed at a frequency of 1 Hz.

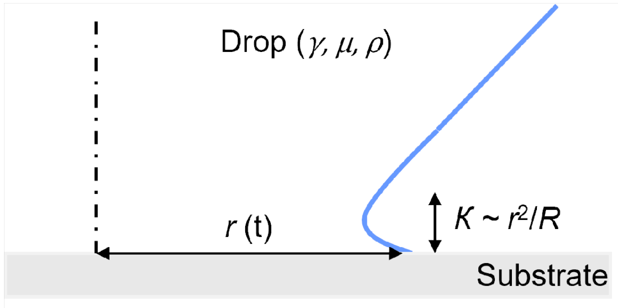

FIG. S2. Scheamtic of a spreading drop. r is the time-dependent drop-substrate footprint radius, i.e., spreading radius. The curvature driving the CL motion is represented as $\kappa \sim r^2/R$, where R is the initial drop radius.

Derivation of scaling laws

1. Inertial-Capillary spreading: When a liquid drop makes contact with any surface, a large unbalanced surface tension force, i.e., γ/κ drives the contact line (CL) motion, where $\kappa \sim r^2/R$ is the curvature (Fig. S2) [1, 2]. Here r is the time-dependent spreading radius and R is the initial drop radius. For early time spreading, when liquid inertia opposes capillarity, the force balance per unit area of CL can be expressed as,

$$p_{\text{in}} \sim p_{\text{cap}} \tag{1}$$

$$\Rightarrow \rho v^2 \sim \frac{\gamma R}{r^2} \tag{2}$$

$$\Rightarrow \rho \left(\frac{dr}{dt}\right)^2 \sim \frac{\gamma R}{r^2} \tag{3}$$

$$\Rightarrow \left(r\frac{dr}{dt}\right)^2 \sim \frac{\gamma R}{\rho} \tag{4}$$

$$\Rightarrow r\frac{dr}{dt} \sim \sqrt{\frac{\gamma R}{\rho}} \tag{5}$$

$$\Rightarrow \int r dr \sim \sqrt{\frac{\gamma R}{\rho}} \int dt \tag{6}$$

$$\Rightarrow r^2 \sim \sqrt{\frac{\gamma R}{\rho}} t \tag{7}$$

$$\Rightarrow r \sim \left(\frac{\gamma R}{\rho}\right)^{1/4} t^{1/2} \tag{8}$$

In the above derivation ρ is the liquid density and $v = dr/dt$ is the CL velocity.

2. Viscous-Capillary spreading: Similarly, for early time spreading, when viscosity opposes capillarity, the force balance per unit area of CL can be expressed as,

$$p_{\text{vis}} \sim p_{\text{cap}} \tag{9}$$

$$\Rightarrow \mu \frac{\partial u}{\partial r} \sim \frac{\gamma R}{r^2} \tag{10}$$

$$\Rightarrow \mu \frac{v}{r} \sim \frac{\gamma R}{r^2} \tag{11}$$

$$\Rightarrow \mu \frac{1}{r}\frac{dr}{dt} \sim \frac{\gamma R}{r^2} \tag{12}$$

$$\Rightarrow \int r dr \sim \frac{\gamma R}{\mu} \int dt \tag{13}$$

$$\Rightarrow r^2 \sim \frac{\gamma R}{\mu} t \tag{14}$$

$$\Rightarrow r \sim \left(\frac{\gamma R}{\mu}\right)^{1/2} t^{1/2} \tag{15}$$

In the above derivation μ can either be the high shear rate viscosity for weakly elastic polymeric liquids obtained from shear rate ramp measurements or the viscosity obtained from rheology fitting for drops/spheres with moderate or high Young's modulus.

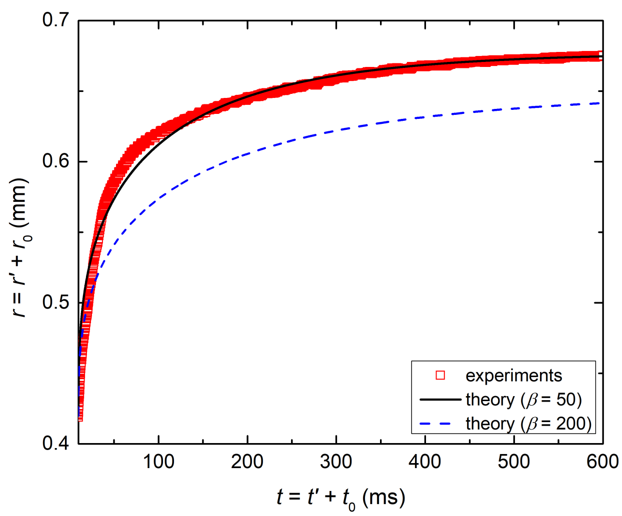

FIG. S3. Evolution of the spreading radius r for 1mm radius PAAm 6.5% drops ($E_0 = 0.16\,\text{kPa}$) on glass substrates in regime II. The symbols represent experimental data, whereas the solid lines represent the theoretical solution for two different β values. Here, the origin of the x-axis is at $t = 10$ms.

[1] A. Eddi, K. G. Winkels, and J. H. Snoeijer, Short time dynamics of viscous drop spreading, Phys. Fluids **25**, 10.1063/1.4788693 (2013).

[2] S. Mitra and S. K. Mitra, Understanding the early regime of drop spreading, Langmuir **32**, 8843 (2016).