Supplementary information

Similarity Modulation Mechanism in Triboelectric Nanogenerators

SI 1.1 Derivation of the Dynamic Governing Equations

The fundamental dynamics of triboelectric nanogenerators (TENGs) are governed by the circuit equation:

$$R\frac{dQ}{dt} = V_{OC} - \frac{Q}{C}$$

Substituting the short-circuit charge relation $Q_{SC} = V_{OC}C$ yields:

$$\frac{1}{RC}Q + \frac{dQ}{dt} = \frac{1}{RC}Q_{SC}$$

We introduce dimensionless variables using characteristic parameters: charge $q = \sigma WL$, time period T, and capacitance $C_0 = \frac{\epsilon_0 WL}{\delta}$. Defining:

- Dimensionless charge: $\hat{Q} = Q/q$
- Dimensionless short-circuit charge: $\hat{j} = Q_{SC}/q$
- Dimensionless capacitance inverse: $\hat{g}^{-1} = C/C_0$
- Dimensionless time: $\hat{t} = t/T$

The governing equation transforms to:

$$\frac{1}{S_m}\frac{d\hat{Q}}{d\hat{t}} + \hat{g}\hat{Q} = \hat{g}\hat{j}$$

where $S_m = \omega_0 T = \frac{T}{RC_0}$ represents the fundamental similarity number.

The general solution is expressed as:

$$\hat{Q}(\hat{t}) = \frac{1}{\hat{K}}\int \hat{j}\,d\hat{K}$$

with the integration kernel $\hat{K} = exp(S_m\hat{G}) = exp(S_m\int \hat{g}\,d\hat{t})$.

The output current follows as:

$$I = \frac{dQ}{dt} = \frac{q\,d\hat{Q}}{T\,d\hat{t}} = \omega_0 q\hat{g}(\hat{j} - \hat{Q})$$

The electrostatic dynamics are derived from energy considerations. The electrode voltage is:

$$V = -\frac{Q}{C} + V_{OC}$$

leading to electrostatic energy components:

$$U_{e1} = -\int V dQ = \frac{1}{2C}Q^2 - V_{OC}Q$$

$$U_{e2} = \frac{1}{2}V_{OC}q$$

The total electrostatic energy becomes:

$$U_e = U_{e1} + U_{e2} = \frac{1}{2}RC_0 q^2 \hat{g}(\hat{Q}^2 - 2\hat{j}\hat{Q} + \hat{j})$$

The electrostatic force is obtained from energy conservation:

$$-F\frac{dx}{dt} = \frac{dU_e}{dt} + I^2R$$

yielding:

$$F = -\frac{1}{2D}\omega_0 R q^2 (\frac{d\hat{g}}{d\hat{x}}\hat{Q}^2 + \frac{d(\hat{g}\hat{j})}{d\hat{x}}(1 - 2\hat{Q}))$$

For the lateral sliding mode TENG, specific relations apply:

$$C_0 = \frac{\epsilon_0 WL}{\delta}, C = \frac{\epsilon_0 W(L - x)}{\delta}, Q_{SC} = \sigma x W$$

with dimensionless functions:

$$\hat{g} = \frac{1}{1 - \hat{x}}, \hat{j} = \hat{x}, \omega_0 = \frac{\delta}{\epsilon_0 WLR}$$

The current and force expressions simplify to:

$$I = \frac{\delta\sigma}{\epsilon_0 R}\frac{\hat{x} - \hat{Q}}{1 - \hat{x}}, F = -\frac{\delta\sigma^2 W}{2\epsilon_0}\frac{(1 - \hat{Q})^2}{(1 - \hat{x})^2}$$

SI 1.2 Numerical Method and Experimental Validation

We implement a discrete computational scheme:

$$\hat{Q}_n = \frac{1}{\hat{K}_n}\sum_{i=1}^{n}\hat{j}_i \Delta\hat{K}_i$$

with cumulative terms:

$$\hat{K}_i = \exp(S_m \hat{G}_i), \hat{G}_i = \sum_{l=1}^{i} \hat{g}_l \Delta\hat{t}_l$$

Initial conditions: $\hat{Q}_0 = 0$, $\hat{K}_0 = 1$, $\hat{G}_0 = 0$

The numerical model is validated against COMSOL Multiphysics simulations using the TENG parameters in Table S1 and three distinct motion profiles:
Intermittent Harmonic:

$$x(t) = \begin{cases} 8[1 - \cos(4\pi t)], & N \le t \le N + 0.5 \\ 0, & N + 0.5 < t \le N + 1 \end{cases}$$

Piecewise Linear:

$$x(t) = \begin{cases} 64t, & N \le t \le N + 0.25 \\ 32 - 64t, & N + 0.25 \le t \le N + 0.5 \\ 0, & N + 0.5 < t \le N + 1 \end{cases}$$

High-Frequency Harmonic:

$$x(t) = \begin{cases} 8[1 - \cos(12\pi t)], & N \le t \le N + 1/6 \\ 0, & N + 1/6 < t \le N + 1 \end{cases}$$

Comparative results for current output and electrostatic force across all motion patterns demonstrate excellent agreement between our numerical method and finite element simulations, confirming the validity of the proposed dynamic model.

SI 1.2 Electromagnetic Generator Similarity Analysis

The electromagnetic generator (EMG) with a structure analogous to the lateral sliding TENG is schematically represented in Fig. S4. The EMG maintains identical principal dimensions to the TENG configuration, with both the permanent magnets and rectangular coil characterized by length L and width W. The system operates with a uniform magnetic flux density B across the air gap, while the external circuit is represented by a purely resistive load R_E. The relative displacement between the magnetic field and the coil is denoted by x.

The fundamental governing equation for the EMG system is derived from Faraday's law:

$$I = -\frac{1}{R_E}\frac{d\phi}{dt}$$

We introduce dimensionless parameters using the characteristic magnetic flux $\phi_0 = BWL$ and the characteristic time scale T (typically taken as the motion period for periodic excitation):

$$\hat{I} = \frac{I}{BWL/(R_E T)}, \hat{\phi} = \frac{\phi}{BWL}, \hat{t} = \frac{t}{T}$$

The dimensionless governing equation becomes:

$$\hat{I} = -\frac{d\hat{\phi}}{d\hat{t}}$$

Critical Observation: Equation (S29) contains no dimensionless similarity parameters, indicating that EMG operation exhibits unconditional similarity. This fundamental property implies that for any two EMGs with geometrically similar structures and kinematically analogous motion inputs, their current outputs, power generation characteristics, and electromagnetic forces can be mutually converted through simple scaling factors, without requiring matching of any additional dimensionless groups.

For the specific EMG configuration shown in Fig. S4, where the magnetic flux variation arises primarily from translational motion, the current output is expressed as:

$$I = \frac{BWL}{R_E T}\frac{d\hat{x}}{d\hat{t}}$$

The corresponding electromagnetic braking force is given by:

$$F = -\frac{B^2 W^2 L}{R_E T}\frac{d\hat{x}}{d\hat{t}}$$

This analysis reveals a fundamental distinction between TENG and EMG dynamics: while TENG operation is governed by the similarity parameter S_m that couples electrical and mechanical time scales, EMGs exhibit scale-invariant behavior that enables direct performance prediction across different system sizes and operating conditions through simple geometric and material scaling.

SI 2: For *Periodicity*

We have

$$\hat{x}(\hat{t}+1) = \hat{x}(\hat{t})$$

In general, the capacitance C and the short circuit transfer charge Q_{SC} are simple functions of the dimensionless displacement $\hat{x}$, so we have

$$\hat{g}(\hat{t}+1) = \hat{g}(\hat{t})$$

$$\hat{J}(\hat{t}+1) = \hat{J}(\hat{t})$$

then

$$\hat{G}(\hat{t}+1) = \int_0^{\hat{t}+1} \hat{g}du = (\int_0^1 + \int_1^{\hat{t}+1})\hat{g}du = \hat{G}(1) + \hat{G}(\hat{t})$$

$$\hat{K}(\hat{t}+1) = e^{S_m \hat{G}(\hat{t}+1)} = e^{S_m \hat{G}(1)} e^{S_m \hat{G}(\hat{t})} = \hat{K}(1)\hat{K}(\hat{t})$$

Therefore, for $\hat{t} = N + \hat{\xi}$, where $0 \le \hat{\xi} < 1$ and N is a non-negative integerit, is easy to get

$$\hat{g}(\hat{t}) = \hat{g}(\hat{\xi})$$

$$\hat{j}(\hat{t}) = \hat{j}(\hat{\xi})$$

$$\hat{G}(\hat{t}) = N\hat{G}(1) + \hat{G}(\hat{\xi})$$

$$\hat{K}(\hat{t}) = \hat{K}^N(1)\hat{K}(\hat{\xi})$$

Considering $d\hat{K} = S_m \hat{g}\hat{K}d\hat{t}$, we have

$$\hat{Q}(\hat{t}) = \frac{1}{\hat{K}(\hat{t})}\int_{\hat{k}(0)}^{\hat{K}(\hat{t})} \hat{j}\,d\hat{k} = \frac{1}{\hat{K}(\hat{t})}\int_0^{\hat{t}} S_m\hat{g}\hat{j}\hat{K}du = \frac{1}{\hat{K}(\hat{t})}\int_0^{\hat{t}} pdu$$

where $p = S_m\hat{g}\hat{j}\hat{K}$. Obviously

$p(\hat{t}) = \hat{K}^N(1)p(\hat{\xi})$

then

$$\hat{Q}(\hat{t}) = \frac{1}{\hat{K}^N(1)\hat{K}(\hat{\xi})}(\int_0^1 + \int_1^2 + ... + \int_{N-1}^{N} + \int_N^{N+\xi})pdu$$

$$= \frac{1}{\hat{K}^N(1)\hat{K}(\hat{\xi})}((1+\hat{K}(1) + \hat{K}^{N-1}(1))\int_0^1 pdu + \hat{K}^N(1)\int_0^{\xi} pdu)$$

$$= \hat{Q}(1)\frac{1-\hat{K}^{-N}(1)}{1-\hat{K}^{-1}(1)}\frac{1}{\hat{K}(\hat{\xi})} + \hat{Q}(\hat{\xi})$$

SI 3 : For *Limiting conditions*

As $S_m \rightarrow 0$

$$\hat{K}_{S_m \rightarrow 0} = e^{S_m\hat{G}}|_{S_m \rightarrow 0} = 1 + S_m\hat{G} = o(S_m^2)$$

$$\hat{Q}_{S_m \rightarrow 0} = S_m\frac{1}{\hat{K}_{S_m \rightarrow 0}}\int_0^{\hat{t}} \hat{j}\hat{g}\hat{K}_{S_m \rightarrow 0}d\hat{t} = S_m\int_0^{\hat{t}} \hat{j}\hat{g}d\hat{t}$$

$$\hat{Q}_{S_m \rightarrow 0}^{S}(\hat{t}) = \hat{Q}_{S_m \rightarrow 0}(1)\frac{1}{1-\hat{K}_{S_m \rightarrow 0}^{-1}(1)}\frac{1}{\hat{K}_{S_m \rightarrow 0}(\hat{t})} + \hat{Q}_{S_m \rightarrow 0}(\hat{t})$$

$$= \frac{\int_0^1 \hat{j}\hat{g}d\hat{t}\,1 + S_m\int_0^1 \hat{g}d\hat{t}}{\int_0^1 \hat{g}d\hat{t}\;1 + S_m\int_0^{\hat{t}} \hat{g}d\hat{t}} + S_m\int_0^{\hat{t}} \hat{j}\hat{g}d\hat{t}$$

$$= \frac{\int_0^1 \hat{j}\hat{g}d\hat{t}}{\int_0^1 \hat{g}d\hat{t}}$$

Considering the relationship $\hat{g}\hat{j} = \hat{g} - 1$ inherent to the lateral sliding mode TENG, we have

$$\hat{Q}_{S_m^{S} \to 0} = \frac{\int_0^1 \hat{j}\hat{g}\,d\hat{t}}{\int_0^1 \hat{g}\,d\hat{t}} = 1 - \frac{1}{\int_0^1 \hat{g}\,d\hat{t}} = 1 - \frac{1}{\hat{G}(1)}$$

where $\hat{G}(1) = \int_0^1 \hat{g}\,d\hat{t}$.