Supplementary information for Spin Torque Nano-Oscillators with Tilted Magnetic Anisotropy

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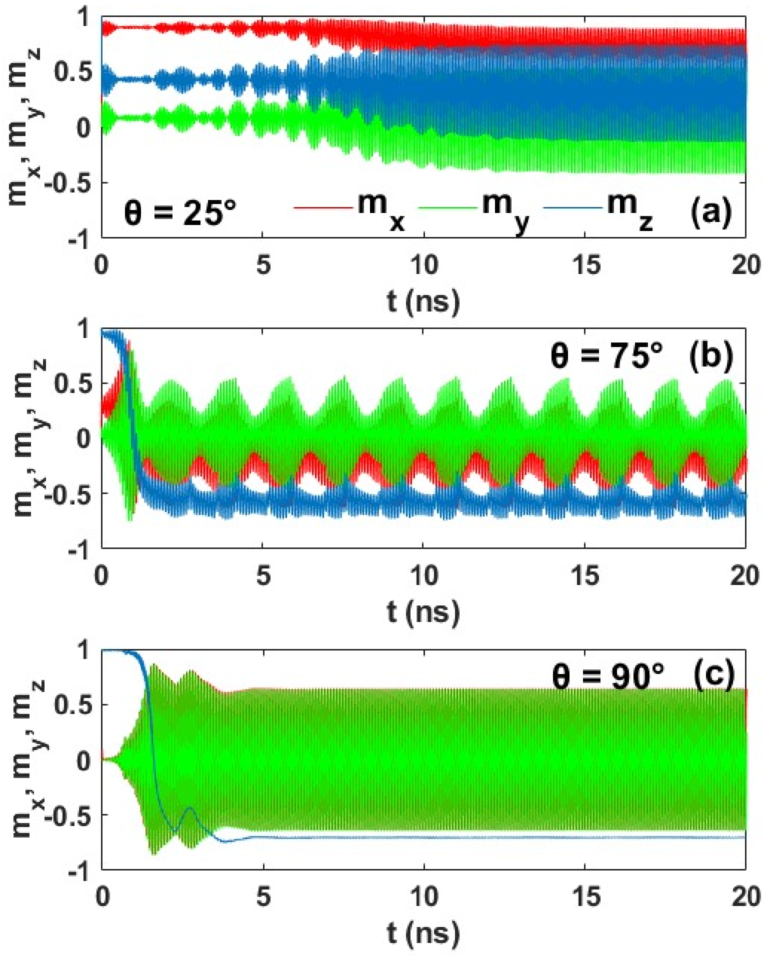

Figure S1: **Supplementary Fig. 1.** Simulated time evolution of the spatially averaged magnetization components m_x (red), m_y (green), and m_z (blue) at −20 mA for field angles of (a) 25°, (b) 75° and (c) 90°. These results correspond to the marked angles in Fig. 2(f). The steady-state precession over a 20 ns timespan illustrates the distinct dynamical regimes.

To quantitatively evaluate the mode coexistence and competition, we extracted the integrated power of each spectral peak by fitting the experimental spectra with Lorentzian functions. The results are summarized in Figure S2. At θ = 30° and 90°, nearly all integrated power is concentrated in a single mode, confirming single-mode operation. At θ = 45° and −14 mA, a single mode dominates, while at –20 mA, two peaks with comparable integrated intensities are observed, indicating balanced coexistence of two localized (bullet) modes. In contrast, at θ = 75°, the relative power distribution strongly depends on current: a single PSW dominates at lower current, while at higher current two modes appear with asymmetric power ratios, demonstrating current-dependent competition between the droplet-like PSW and the conventional propagating mode.

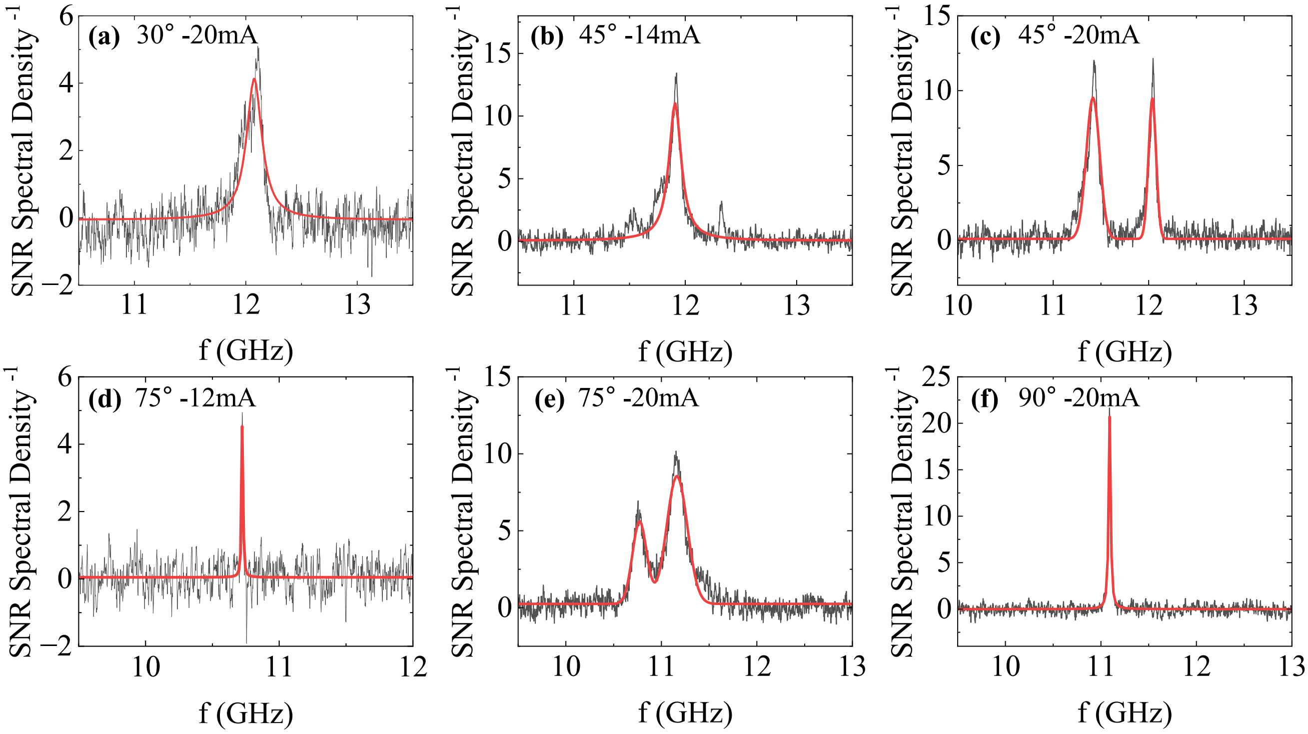

Figure S2: **Supplementary Fig. 2.** Microwave power spectra of the NC-STNO measured at different magnetic field angles and driving currents, corresponding to Fig. 3(a-d) in the main text. The black curves represent the experimental SNR spectral density, and the red curves are Lorentzian fits. (a) 30° at −20 mA; (b) 45° at −14 mA; (c) 45° at −20 mA; (d) 75° at −12 mA; (e) 75° at −20 mA; and (f) 90° at −20 mA.

To further clarify the physical origins of the magnetic modes shown in Fig. 4(a), we performed comparative micromagnetic simulations using three distinct configurations, all based on a dual-nanocontact (NC) geometry with an NC diameter of 100 nm and a center-to-center separation of 200 nm to investigate the spatial distribution of the m_z component. For the droplet mode, following Ref. [75], the [Co/Ni]/NiFe free layer was replaced with a pure PMA [Co/Ni] multilayer (M_s = 716 kA/m, K_u = 447 kJ/m^3, A_{ex} = 30 pJ/m) under the conditions of -20 mA and 0.4 T at a 90° field angle. For the propagating spin-wave (PSW) mode, based on Ref. [73], the free layer was replaced with NiFe (M_s = 700 kA/m, K_u = 0 kJ/m^3, A_{ex} = 10 pJ/m), with the applied conditions set to -20 mA and 0.7 T at a 70° field angle. In contrast, the droplet-like PSW mode was simulated using the same TMA magnetic parameters as described in the main text, under identical conditions of -20 mA and 0.4 T at a 90° field angle. These comparisons illustrate that while the droplet is highly localized and the conventional PSW is purely propagating, the droplet-like PSW in our TMA system uniquely maintains a localized droplet-core with extended propagation.

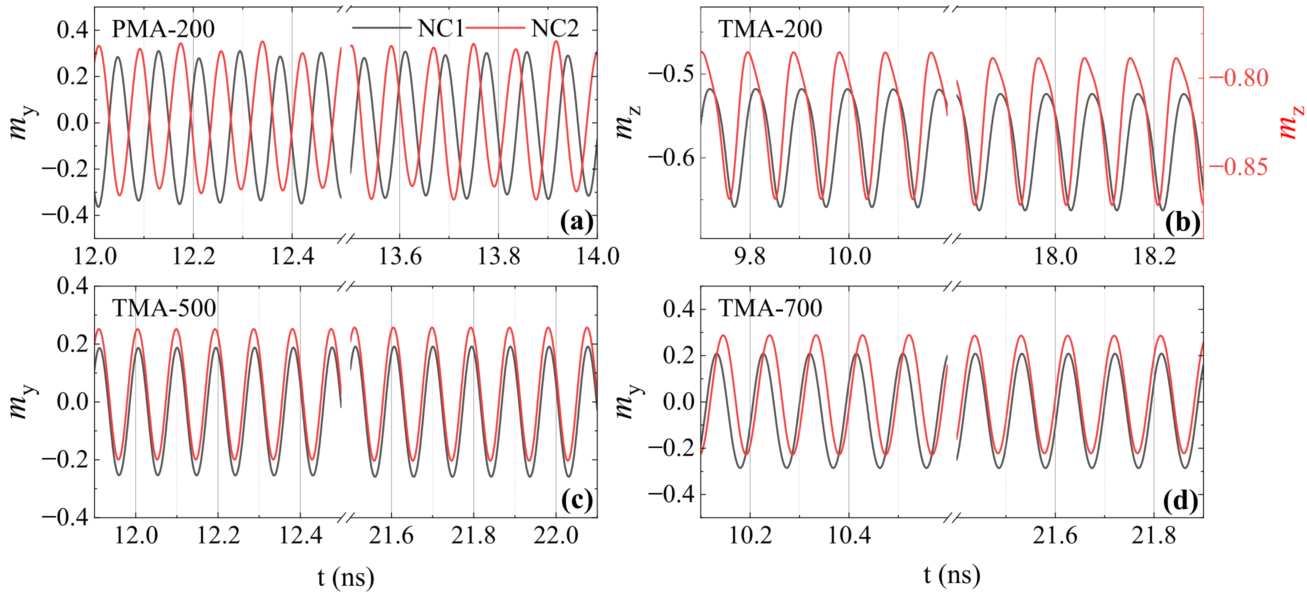

Figure S3: **Supplementary Fig. 3.** Time evolution of the m_y/m_z component for double-nanocontact devices under different configurations, corresponding to the spatial maps shown in Fig. 4 of the main text. For the all-PMA reference structure with a separation of 200 nm (PMA-200), the two nanocontacts exhibit distinct oscillation frequencies and a clear phase shift, indicating loss of synchronization. For the TMA free-layer devices with separations of 200 nm (TMA-200) and 500 nm (TMA-500), the two nanocontacts oscillate with stable phase coherence and identical frequencies, demonstrating sustained synchronization. At a larger separation of 700 nm (TMA-700), the phase coherence begins to deteriorate and the oscillation frequencies diverge, marking the approximate synchronization limit. These results illustrate the enhanced coupling capability of the droplet-like PSW mode in TMA systems compared to conventional PMA structures.