

Supplementary information

Nickel BNW magnetisation reversal

Simulations were performed for Ni BNWs, saturation magnetisation of $M_{\text{sat}} = 9.4 \times 10^5 \text{ A m}$ and an exchange stiffness of $A_{\text{ex}} = 15 \text{ pJ}$ were used. The remaining parameters were held the same as in the main text for FeCo. The results are shown in figure S1. From the remanent state, in the first set of images (at 0 ns), an external magnetic field of 18 mT is applied.

In qualitative terms, the same demagnetisation process occurs at roughly the same field (18 mT), with a "good" chirality DW being nucleated at the wide segment meeting an opposing vorticity at the diameter modulation, and a magnetic charge compensation from the vortex/skyrmion tube texture appearing in the MFM image. However, slight differences are observed like the lack of ripple structure in the skyrmion tubes' shape due to a smaller magnetostatic energy.

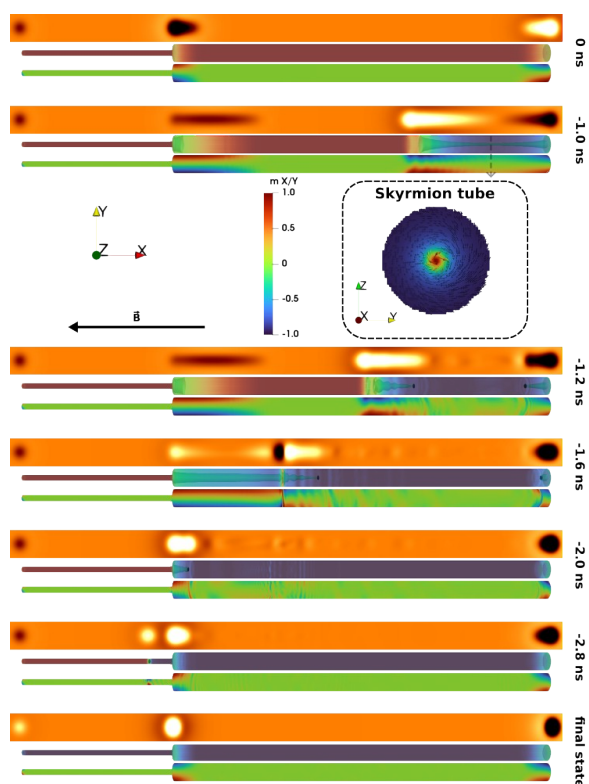


Figure S1. Sets of three images for different stages of the magnetisation reversal for a Ni BNW. The nucleated DW at the wide NW is of a "good" chirality case. The top image plots the calculated MFM image using simulation data. The middle image shows the X component of magnetisation in a transparent NW surface to show the isosurface $X=0$ in green and the green spheres representing a Bloch Point defined as the intersection of the isosurfaces $X=0$, $Y=0$ and $Z=0$. The bottom image plots the Z component of the magnetisation in the NW surface. The inset shows a cross-sections of the skyrmion tube. The colour bar represents the X component of magnetisation in the middle images and the insets, and the Z component of magnetisation in the bottom images.

Dependence on segment length

We have performed simulations for shorter segments. In particular, the wide segments length was reduced to $1.5\ \mu\text{m}$ and the narrow segment to $500\ \text{nm}$. In these BNWs, the skyrmion tube cannot reach its critical length, preventing tubes break and the nucleation of the Bloch point DW. Instead, the two expanding skyrmion tubes (in the wide end and at the modulation) meet close to the centre of the wide segment, stabilising each other. This results in the two-step hysteresis loop shown in figure S1a) with the intermediate states containing either a skyrmion tube along the whole length of the wide segment (figure S2b) or two skyrmion tubes with opposing vorticities separated by a complex DW texture (figure S2c), depending on whether the vortices at the segments end have the same or opposite vorticities, respectively.

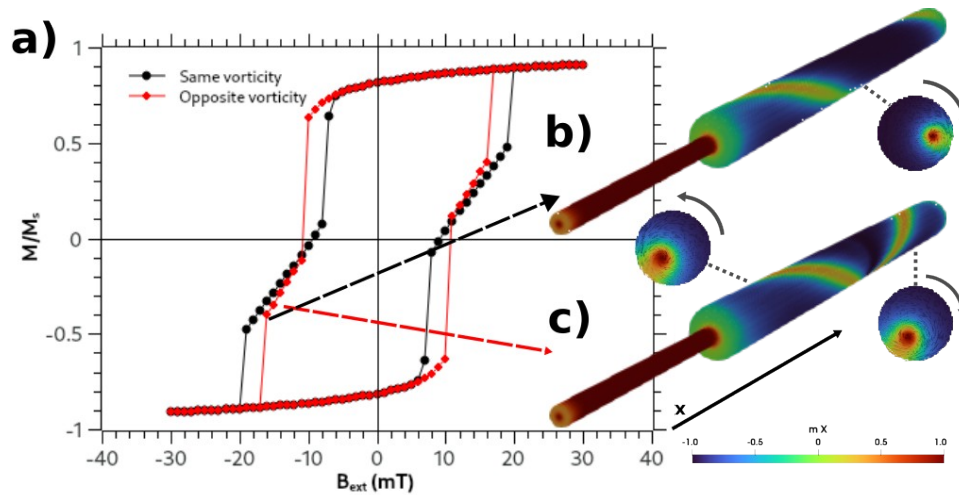


Figure S2. a) Hysteresis loop of a Ni BNW. **b)** Magnetisation state containing a skyrmion tube along the whole length of the wide segment. **c)** Magnetisation state containing two skyrmion tubes with the same polarity and opposite vorticities.