Supplementary Information

Isothermal models

The Freundlich equation :

$$Q_e = K_f C_e^{1/n} \quad (1)$$

where K_f is a Freundlich isotherm constant, C_e is an equilibrium dye concentration (mg/l) and 1\n represents the strength of adsorption.

The Langmuir isotherm:

$$Q_e = \frac{Q_m K_L C_e}{1 + K_L C_e} \quad (2)$$

where Q_e is the amount of dye adsorbed at equilibrium (mg/g), K_L is the Langmuir adsorption constant (mg/g), Q_m is the monolayer adsorption (mg/g) and C_e is the equilibrium dye concentration (mg/L). The separation factor, known as RL, determines the degree of adhesion between an adsorbent and an adsorbate and is calculated from

$$RL = \frac{1}{1 + K_L C_0} \quad (3)$$

where C_o is the initial dye concentration. A higher RL value (closer to 1) indicates a stronger, favorable attraction, while lower values (closer to 0) indicate a weaker, unfavorable interaction.

Langmuir-Freundlich isotherm:

$$Q_e = \frac{Q_m (K_{LF} C_e)^n}{1 + (K_{LF} C_e)^n} \quad (4)$$

Sipes isotherm is a flexible model that combines also Langmuir and Freundlich, allowing for diverse scenarios like heterogeneous or homogeneous, monolayer or multilayer, and predicts saturation.[31].

$$Q_e = \frac{Q_m K_S C_e^{1/n_s}}{1 + K_S C_e^{1/n_s}} \quad (5)$$

where K_S is the Sipes equilibrium constant

The Baudu isotherm model:

$$Q_e = \frac{Q_m b_0 C_e^{(1+x+y)}}{1 + b_0 C_e^{(1-x)}} \quad (6)$$

where Q_m is the Baudu maximum adsorption capacity (mg/g), b_0 is the equilibrium constant, *x* and *y* are the Baudu parameters.

Kinetics models

The Pseudo-first-order model (PFO):

$$q_t = q_e\,(1 - e^{(-k_1 t)}) \quad (7)$$

Where k_1 is the kinetic rate constant (min $^{-1}$),t is the time of the experiment (min), q_e is the equilibrium adsorption capacity (mg adsorbate/g adsorbent) and q_t represents the adsorption capacities (mg adsorbate / g adsorbent) at time t.

Pseudo-second-order model (PSO):

$$q_t = \frac{k_2 q_e^2 t}{1 + k_2 q_e t} \quad (8)$$

Where k_2 is the kinetic rate constant (min $^{-1}$).

Mixed-1,2-order model (MO):

$$q_t = q_e \frac{1 - e^{(-kt)}}{1 - f_2 e^{(-kt)}} \quad (9)$$

Here, k is the adsorption rate constant ($mg.g^{-1}.min^{-1}$)and f_2 is the dimensionless coefficient of mixed-1,2-order.

Avrami model:

$$q_t = q_e(1 - e^{(-k_{av}t)^{n_{av}}}) \quad (10)$$

Where n_{av} is the Avrami dimensionless number and k_{av} is the Avrami rate constant (min^{-1}).