

Supplementary Data

Table S1. The kinetic model parameters for the adsorption of CV onto the SAC

Kinetic models	Equation	Parameters	Values
Pseudo first Order	$q_t = q_e (1 - e^{-k_1 t})$	k_1 (min ⁻¹)	0.10
		q_e (mg/g)	98.49
		R^2	0.99
Pseudo second Order	$q_t = \frac{q_e^2 k_2 t}{1 + q_e k_2 t}$	k_2 (min ⁻¹)	0.002
		q_e (mg/g)	102.79
		R^2	0.99
Mixed 1,2 order	$q_t = q_e \frac{(1 - \exp(-kt))}{1 - f_2 \exp(-kt)}$	k (mg/g min)	0.03
		q_e (mg/g)	100.27
		f_2	0.83
		R^2	0.99
Avrami	$q_t = q_e [1 - \exp(-k_{av} t)^{n_{av}}]$	q_e (mg/g)	98.49
		k_{av} (min ⁻¹)	0.67
		n_{av}	0.16
		R^2	0.99
Intraparticle diffusion	$q_t = K_{ip} \sqrt{t} + C_{ip}$	k_{ip} (mg/g min ^{1/2}),	1.04
		c_{ip} (mg/g)	82.20
		R^2	0.54

Table S2 lists the thermodynamic variables for CV adsorption onto a synthetic adsorbent.

Material	T(K)	ΔG° (kJ/mol)	ΔH° (kJ/mol)	ΔS° (kJ/mol. K)
Synthesized SAC	298	-510.44	-0.755	-223.59
	308	-2645.56		
	318	-5275.33		
	328	-7943.96		

Table S3. CV nonlinear adsorption isotherm models utilizing the produced adsorbent

Isotherm models	Expression	Adjustable model parameters*	Values	R^2
Two-parameters isotherm				
Langmuir	$q_e = q_{max} \left(\frac{k_L C_e}{1 + k_L C_e} \right)$	q_{max}	201.473	0.998
		k_L	0.005	
Freundlich	$q_e = k_F (C_e)^{1/n}$	k_f	3.253	0.989
		n_f	1.56	
Dubinin	$q_e = q_{max} e^{-k_D \epsilon^2}$	q_{max}	112.77	0.951

		k_D	3205.78	
		ϵ	0.01249	
Three-parameters isotherm				
Langmuir-Freundlich	$q_e = \frac{q_{max}(k_{LF} C_e)^{MLF}}{1 + (k_{LF} C_e)^{MLF}}$	q_{max}	182.91	0.998
		k_{LF}	0.00614	
		M_{LF}	1.0761	
Sips	$q_e = \frac{(q_{max}k_s C_e)^{1/n}}{(1 + k_s C_e)^{1/n}}$	q_{max}	201.248	0.998
		k_S	0.005	
		n	4.44E+44	
Redlich-Peterson	$q_e = \frac{k_{RP} C_e}{1 + a_R C_e^{\beta_{RP}}}$	k_{RP}	0.87	0.999
		a_R	6.655	
		β_{RP}	1.314	
Kahn	$q_e = \frac{q_{max} b_k C_e}{(1 + b_k C_e)^{a_k}}$	q_m	1082.47	0.998
		b_k	0.0008	
		a_k	3.69	
Four-parameters isotherm				
Baudu	$q_e = \frac{q_{max} b_0 C_e^{1+X+Y}}{1 + b_0 C_e^{1+X}}$	q_{max}	173.612	0.998
		b_o	0.00549	
		X	0.02003	
		Y	2.0768E-14	
Five-parameters isotherm				
Fritz-Schlunder	$q_e = \frac{q_{max_{FSS}} k_1 C_e^{m_1}}{1 + k_2 C_e^{m_2}}$	q_{mFSS}	183.93	0.998
		k_1	1.136	
		k_2	3.74E-05	
		m_1	0.878	
		m_2	1.705	

Table S4 summarizes the calculated error functions for the nonlinear adsorption isotherm models.

Error function	Langmuir r	Freundlich h	DR	Langmuir - Freundlich h	Sips	RP	Khan	Baudu	FS
SSE/ERRSQ	40.797	228.625	999.280	36.421	41.165	28.176	55.367	40.218	22.049
X²	1.455	10.081	45.436	2.176	1.458	1.843	2.234	1.572	0.855
R²	0.998	0.990	0.954	0.998	0.998	0.999	0.997	0.998	0.999
Adjusted R²	0.995	0.973	0.885	0.995	0.994	0.996	0.992	0.993	0.995
MAE	1.629	4.358	8.533	1.518	1.617	1.290	1.850	1.671	1.257
MAPE /ARE	8.050	29.434	49.591	11.132	8.047	9.487	10.460	8.799	6.566
RMSE	2.020	4.781	9.996	1.908	2.029	1.679	2.353	2.005	1.485
RMSE_2	2.258	5.346	11.176	2.281	2.425	2.006	2.812	2.589	2.100
NRMSE	0.048	0.114	0.239	0.046	0.048	0.040	0.056	0.048	0.035
HYBRID	10.062	36.793	61.989	15.903	11.496	13.553	14.943	14.665	13.132
HYBRID_2	18.191	126.018	567.955	31.090	20.833	26.329	31.920	26.204	17.091
HYBRID_3	1.455	10.081	45.436	2.176	1.458	1.843	2.234	1.572	0.855
MPSD	13.040	56.142	73.533	21.090	13.971	19.043	19.754	16.356	13.363
MPSD_2	0.136	2.522	4.326	0.311	0.137	0.254	0.273	0.161	0.089
SAE/EABS	16.286	43.581	85.328	15.179	16.165	12.905	18.503	16.708	12.567
RMS	11.663	50.215	65.770	17.645	11.689	15.933	16.528	12.669	9.449
NSD	0.117	0.502	0.658	0.176	0.117	0.159	0.165	0.127	0.094
ARE_2	1.360	25.215	43.257	3.114	1.366	2.539	2.732	1.605	0.893
ARE_3	3.688	15.879	20.798	5.580	3.696	5.038	5.227	4.006	2.988

Table S5. Response surface BBD and results for adsorption efficiency of CV.

Run	Actual variables				Yield %		
	A	B	C	D	Experimental	Predicted	Residue
1	500	10	0.125	6.5	0	-1.49	1.49
2	252.5	245	0.125	6.5	82	81.9	0.1
3	500	245	0.125	10	51.37	52.56	-1.19

4	252.5	245	0.05	3	56.39	55.67	0.7183
5	5	245	0.125	3	74.19	74.38	-0.1929
6	252.5	245	0.2	3	67.69	67.37	0.3183
7	252.5	480	0.2	6.5	94.5	94.88	-0.3763
8	252.5	245	0.2	10	96.5	95.93	0.5683
9	5	10	0.125	6.5	50.16	49.51	0.6517
10	252.5	480	0.125	10	96.5	96.31	0.1862
11	500	245	0.2	6.5	48.77	48.6	0.1746
12	252.5	10	0.125	3	16.35	16.44	-0.0921
13	252.5	480	0.05	6.5	82.5	83.33	-0.8263
14	500	245	0.05	6.5	38.27	38.05	0.2246
15	500	245	0.125	3	17.46	18.8	-1.34
16	5	480	0.125	6.5	100	100.21	-0.205
17	252.5	245	0.125	6.5	81.5	81.9	-0.4
18	5	245	0.05	6.5	89.5	89.58	-0.0804
19	252.5	480	0.125	3	67.19	66.6	0.5862
20	252.5	245	0.125	6.5	82	81.9	0.1
21	252.5	245	0.125	6.5	82	81.9	0.1
22	5	245	0.2	6.5	100	100.13	-0.1304
23	252.5	245	0.125	6.5	82	81.9	0.1
24	500	480	0.125	6.5	48.77	48.14	0.635
25	252.5	10	0.05	6.5	33.16	34.16	-1
26	252.5	245	0.05	10	87.5	86.53	0.9683
27	252.5	10	0.2	6.5	43.16	43.71	-0.5546
28	252.5	10	0.125	10	45.66	46.15	-0.4921
29	5	245	0.125	10	100	100.04	-0.0429

Table S6. Statistical Evaluation of Model Variance.

Source	SS	df	MS	F-value	p-value	
Model	21862.86	14	1561.63	1936.89	< 0.0001	significant
A-Initial concentration	7967.57	1	7967.57	9882.15	< 0.0001	
B-Contact time	7548.58	1	7548.58	9362.47	< 0.0001	
C-Adsorbent dose	333.91	1	333.91	414.14	< 0.0001	
D-pH	2648.05	1	2648.05	3284.37	< 0.0001	
AB	0.2862	1	0.2862	0.355	0.5608	
AC	0	1	0	0	1	

AD	16.4	1	16.4	20.34	0.0005	
BC	1	1	1	1.24	0.2842	
BD	0	1	0	0	1	
CD	1.32	1	1.32	1.64	0.2211	
A ²	1248.15	1	1248.15	1548.08	< 0.0001	
B ²	2326.65	1	2326.65	2885.74	< 0.0001	
C ²	7.28	1	7.28	9.03	0.0094	
D ²	281.09	1	281.09	348.64	< 0.0001	
Residual	11.29	14	0.8063			
Lack of Fit	11.09	10	1.11	22.18	0.0045	significant
Pure Error	0.2	4	0.05			
Cor Total	21874.15	28				

Table S7. Statistical summary of different models of adsorption of CV onto SAC.

Source	Std. Dev.	Sequential p-value	Press	Adj R ²	Pred R ²	Remark
Linear	11.86	< 0.0001	4668.3	0.8199	0.7866	
2FI	13.66	1	8316.88	0.7613	0.6198	
Quadratic	0.8979	< 0.0001	64.18	0.999	0.9971	Suggested
Cubic	1.11	0.8831	1028.78	0.9984	0.953	Aliased

Table S8. Sequential model sum of squares

Source	Sum of squares	df	Mean squares	F-value	P-value	Remark
Mean vs. Total	1.27E+05	1	1.27E+05			
Linear vs. Mean	18498.11	4	4624.53	32.88	< 0.0001	
2FI vs. Linear	19.01	6	3.17	0.017	1	
Quadratic vs. 2FI	3345.75	4	836.44	1037.43	< 0.0001	Suggested
Cubic vs. Quadratic	3.95	8	0.4932	0.403	0.8831	Aliased
Residual	7.34	6	1.22			
Total	1.48E+05	29	5115.24			

Table S9. Cost analysis of synthesized composite materials.

Material	Quantity purchased (g or mL)	Purchase price in USD	cost of purchase (USD/g or mL)	Quantity used (g or mL)	Quantity cost of use (USD)
Frankincense	200	1.5	0.0075	50	0.375
sulfuric acid	500	610	1.22	1	1.22
Equipment	Time (min)	Max. power (kW)	Unit cost of power	Cost	
Calcination	30	2	0.48	0.24	
		The total yield cost for 28.5 g =1.835 USD.	Cost of total yield = 0.06438 USD/g		