

## Appendix A

### Codebook: Resource Activation in Titration Curve Interpretation

Supplemental Material — Becker, Farheen, Salmon, & Mishulin

This codebook describes the cognitive resources identified in the analysis of twelve second-semester general chemistry students's think-aloud interviews during a titration curve preservative selection task. Resources were operationalized broadly, consistent with Hammer and Elby's (2003) account of the cognitive resources perspective as encompassing conceptual knowledge elements, activity-type epistemological resources, graphical forms (Rodriguez, Bain & Towns, 2020). Codes are organized by resource type rather than by activation pattern. Where a code is activated predominantly by point-based or rate-based students, this is noted in the definition. Section A covers conceptual resources including p-prims (diSessa, 1993), compiled knowledge elements, domain knowledge, and constraint resources; Section B covers activity-type epistemological resources (Hammer & Elby, 2003); Section C covers graphical forms in the Rodriguez, Bain & Towns (2020) registration-schema sense, with literature provenance noted for each form. For Section C, three forms (Slope as rate / Steepness as rate, Straight means constant rate, Curve means change) are established in the graphical forms literature (Rodriguez, Bain & Towns, 2020; Rodriguez & Jones, 2024); the remaining three forms (Smaller slope means more resistance to change, Flat = completion, Inflection = equivalence point) are extensions or domain-specific applications not previously identified in the literature.

*Resource Type categories: Conceptual resource = an intuitive or compiled knowledge element activated in response to task or perceptual features (diSessa, 1993; Hammer, 2000); includes p-prims, compiled elements, domain knowledge, constraint resources, and intuitive analogies. Epistemological resource: activity-type = a way of operating on knowledge (e.g., comparing, listing, excluding), classified as an epistemological resource by Hammer & Elby (2003). Graphical form = a learned registration-schema pairing in which a graphical feature activates a conceptual schema (Rodriguez, Bain & Towns, 2020).*

Code Name	Resource Type	Definition	Indicator Language	Participants
<b>Section A: Conceptual Resources (diSessa, 1993; Hammer, 2000; Stavy &amp; Tirosh, 2000)</b>				
<b>Stronger is better</b>	<i>Conceptual resource: intuitive rule (p-prim)</i>	Treating the most extreme value as most effective; more of a property (e.g., acid strength) produces a better outcome. Instance	<i>"I would go with red because it would make it more tart and it's a stronger acid"; "preservatives"</i>	P3, P7, P11

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		of the "More A-More B" intuitive rule (Stavy & Tirosh, 2000). Analogous to diSessa's (1993) phenomenological primitives: activated in response to task features, neither inherently correct nor incorrect. Activated primarily by point-based students.	<i>are usually very strong."</i>	
<b>Part-for-whole</b>	<i>Conceptual resource: compiled knowledge element</i>	Treating a single discrete value — typically the equivalence point — as representative of the preservative's overall character or system behavior. A compiled resource in the diSessa (1993) sense: a knowledge element that has become tightly linked through prior instruction. Activated primarily by point-based students.	Referencing the equivalence point as representing what the preservative "is" or "does"; treating endpoint pH as the preservative's defining property.	P3, P4, P5, P7, P9, P11
<b>Safety threshold</b>	<i>Conceptual resource:</i>	Applying a harm-based lower	<i>"I don't think it's edible"; "I</i>	P1, P4, P5, P6, P7, P9, P12

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	<i>constraint knowledge element</i>	bound that excludes options below a threshold of acceptable safety, regardless of how well they satisfy other criteria. Functions as a constraint resource that overrides other selection criteria when activated.	<i>didn't pick red because it has such a low pH, I think it could maybe be harmful."</i>	
<b>Resistance as opposition to pH change</b>	<i>Conceptual resource: intuitive knowledge element</i>	The knowledge element that titrant additions constitute perturbations to the system and that the magnitude of pH change reflects the degree to which the system opposes those perturbations; treats resistance as an intrinsic property of the system rather than as a graphical observation. Distinct from Smaller slope means more resistance to change (Section C), which is the graphical form	<i>"resisting pH change"; "resists perturbation"; "it definitely has the smallest change, which means it's resisting the pH the most."</i>	P2, P6, P8, P12

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		<p>through which this conceptual element is typically activated: the student perceives a flat curve region (registration) and assigns the resistance schema. Consistent with Hammer's (2000) account of intuitive resources as activated in response to perceptual features of a situation. Activated primarily by rate-based students.</p>		
<p><b>System as a dynamic entity</b></p>	<p><i>Conceptual resource: intuitive analogy (structural)</i></p>	<p>Conceptualizing the juice or food system as a stateful entity whose properties can drift over time without intervention and be maintained or restored. A structural analogy treating the chemical system as having a trajectory through state space that must be steered.</p>	<p>"as you add the NaOH, it shows how it would age over time... the shape of it would show how well it keeps its flavor, tartness over the period that it would be on the shelves" "over time it becomes less tart"; treating the system as something</p>	<p>P3, P10, P12</p>

Code Name	Resource Type	Definition	Indicator Language	Participants
		and aging on shelf analogy.	whose state changes over time unless actively maintained.	
<b>Lower pH means more tartness</b>	<i>Conceptual resource: domain knowledge element</i>	Domain knowledge linking pH to acidity, tartness, and microbial safety. Encompasses knowledge that lower pH produces more tart flavor; that the pH 6-8 range supports microbial activity and that lower pH conditions inhibit it. A compiled knowledge element from prior instruction. Invoked by both point-based and rate-based students, though point-based students tend to invoke it to justify equivalence-point comparisons while rate-based students invoke it to connect resistance to preservation function.	<i>"lower pH would make it more tart"; connecting pH to dual preservation goals; knowing that pH 6-8 supports microbial growth.</i>	P1, P2, P3, P4, P5, P6, P7, P9, P10, P11, P12

Code Name	Resource Type	Definition	Indicator Language	Participants
<b>Analogies (self-generated)</b>	<i>Conceptual resource: domain analogy</i>	Spontaneous mapping of the titration task onto a familiar physical or experiential domain, used to ground reasoning about curve features or preservation goals. Three distinct analogies appear in this dataset: (1) Digestive tract analogy (P6): maps the x-axis (volume of NaOH added) onto passage through the human digestive tract, with each volume increment representing a region at a different baseline pH; the preservative must maintain a stable pH across all regions, motivating resistance reasoning across the full curve. (2) Aging on shelf analogy (P12): maps the x-axis onto elapsed shelf	<i>"Your juice is going to interact with different acidities as it goes through your entire digestive tract, and so it has to be able to withstand each of those possible pHs... as NaOH is added to the preservative, it's going to be able to resist that pH change."</i>	P6 (digestive tract); P7 (orange juice); P12 (aging on shelf + lemon/vinegar)

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		<p>time, with curve shape representing how the preservative maintains tartness across the product's shelf life; motivates attention to the full curve trajectory. (3) Orange juice / lemon analogy (P7): maps tartness to acidity using a familiar acidic food as a reference, grounding the Lower pH means more tartness resource in everyday experience. All three are self-generated rather than interviewer-prompted.</p>		

**Section B: Epistemological Resources: Activity-Type (Hammer & Elby, 2003)**

<p><b>Comparing</b></p>	<p><i>Epistemological resource: activity-type</i></p>	<p>Evaluating curve features against task-relevant criteria to identify the most suitable preservative. Consistent with Hammer &amp; Elby's (2003) characterization of activities such as "listing" and "comparing" as a form of epistemological resource: ways of operating on knowledge that are activated in response to task demands. Comparing takes two forms in this dataset: qualitative comparison, in which students evaluate discrete values at specific locations (e.g., whether equivalence points fall within the pH 6-8 range), activated primarily by point-based students; and quantitative comparison, in which students</p>	<p><i>"the equivalence point for yellow is not between six and eight"</i> (P4, qualitative); <i>"yellow had the smallest change in pH from zero milliliters of NaOH to 50"</i> (P2, quantitative); <i>"changed pH the least amount when adding the same amount"</i> (P6, quantitative)</p>	<p>P2, P4, P6, P7, P9, P10, P11</p>
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		attend to differential change rather than absolute values (e.g., which curve shows the smallest total pH change across the titration interval), activated primarily by rate-based students.		
<b>Magnitude optimization</b>	<i>Epistemological resource: activity-type</i>	Selecting the option whose value is closest to a target magnitude rather than most extreme or simply within a safe range; optimization toward a specific reference point. Distinct from Stronger is better (Section A), which selects the most extreme value, and from Comparing, which evaluates against a threshold. In the task context, students reason that the ideal preservative should be as close to neutral as possible while still remaining acidic,	"you want the one which is closest to seven, a pH of seven, and yellow is the closest to seven" (P4); selecting for proximity to a target value rather than maximizing or thresholding.	P4

		and select accordingly. An activity-type resource in the Hammer & Elby (2003) sense.		
<b>Excluding</b>	<i>Epistemological resource: activity-type</i>	Eliminating options whose discrete values or properties fall in problematic ranges, reducing the candidate set through rule-based filtering. An activity-type resource (Hammer & Elby, 2003) used by both point-based and rate-based students, though the feature attended to differs: point-based students exclude based on equivalence point location; rate-based students exclude based on slope magnitude or curve behavior.	<i>"I didn't pick red because...";</i> ruling out curves based on pH 6-8 range criteria or based on steepness or resistance.	P1, P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, P12 (all participants)

**Section C: Graphical Forms (Rodriguez, Bain & Towns, 2020; Rodriguez & Jones, 2024)**

<p><b>Slope as rate / Steepness as rate</b></p>	<p><i>Graphical form: registration-schema pairing</i></p>	<p>Registration: slope magnitude or steepness of the curve. Schema: rate of pH change. Students perceive steeper regions as corresponding to faster or more intense change, and shallower regions as slower or more gradual change. Identified in the original graphical forms framework (Rodriguez, Bain &amp; Towns, 2020) and confirmed empirically across multiple chemistry and mathematics contexts (Rodriguez &amp; Jones, 2024), where it is among the most consistently observed forms. Activated primarily by rate-based students.</p>	<p>"Because the slope is the least steep"; "the yellow and green lines have the smallest slope"; reading steepness as rate of change.</p>	<p>P1, P2, P6, P8</p>
<p><b>Straight means constant rate</b></p>	<p><i>Graphical form: registration-schema pairing</i></p>	<p>Registration: linear or straight (slanted) curve region. Schema: constant, predictable, steady-state rate of change. Students interpret linearity as signaling manageable, stable</p>	<p>"you want something linear"; characterizing linear behavior as "constant" and "predictable" change.</p>	<p>P1, P8</p>

		<p>behavior over an interval. Identified in Rodriguez, Bain &amp; Towns (2020) and confirmed by Rodriguez &amp; Jones (2024), who name this form straight means constant rate (slanted line, rate is constant) and distinguish it from horizontal as constant value (flat line, y-value is constant). Activated primarily by rate-based students.</p>		
<p><b>Curve means change</b></p>	<p><i>Graphical form: registration-schema pairing</i></p>	<p>Registration: curved or nonlinear curve region, including regions where steepness itself is increasing or decreasing across an interval. Schema: the rate of change is itself changing; the system is accelerating or decelerating rather than changing at a constant pace. Covers two related applications: (a) identifying curvature globally as signaling a non-constant rate, and (b) tracking how steepness changes across an interval,</p>	<p><i>"more of an increasing and increasing rate type of curve" (global curvature); "your pH is changing very drastically at that rate" (accelerating steepness); curvature or increasing steepness associated with drastic and undesirable change.</i></p>	<p>P8</p>

		<p>corresponding to what is formally the second derivative. Identified in Rodriguez, Bain &amp; Towns (2020); Rodriguez &amp; Jones (2024) refine the name to curve means changing rate. Activated primarily by rate-based students.</p>		
<p><b>Smaller slope means more resistance to change</b></p>	<p><i>Graphical form: registration-schema pairing</i></p>	<p>Registration: flat or slowly-changing curve region. Schema: the system resists perturbation; the curve's flatness is evidence of buffering capacity. Not previously identified in the graphical forms literature. Distinct from Flat = completion (same registration, completion schema) and from Straight means constant rate (slanted line, constant rate schema). The key distinction is in the schema assigned: where Straight means constant rate treats linearity as indicating a steady process, Smaller slope</p>	<p><i>“it definitely has the smallest change, which means it's resisting the pH the most”</i></p>	<p>P1, P2, P6, P8</p>

		<p>means more resistance to change treats flatness specifically as evidence that the system is pushing back against imposed change. Connects directly to the conceptual resource Resistance as opposition to pH change (Section A), which is the knowledge element activated by this graphical form. Activated primarily by rate-based students. Exemplar language to be confirmed by authors.</p>		
<p><b>Flat = completion</b></p>	<p><i>Graphical form: registration-schema pairing</i></p>	<p>Registration: flat or slowly-changing curve region. Schema: reaction completion; no further significant change is occurring. Not previously identified in the graphical forms literature. Related to but distinct from Rodriguez &amp; Jones's (2024) plateau as leveling off (curve approaches a stable value) and</p>	<p>Describing a flat region as the reaction having "finished" or "stopped"; treating post-equivalence flatness as indicating no further activity.</p>	<p>P11, P12</p>

		<p>horizontal as constant value (y-value is constant); our form shares a horizontal or flat registration but assigns a completion schema rather than constancy or leveling off. Also distinct from Smaller slope means more resistance to change (above), which assigns the same registration a buffering schema rather than a completion schema.</p>		
<p><b>Inflection = equivalence point</b></p>	<p><i>Graphical form: registration-schema pairing</i></p>	<p>Registration: the steepest point on the curve (inflection point). Schema: neutralization; the point at which acid and base are in equal amounts. Not previously identified in the graphical forms literature; a chemistry-domain-specific form arising from prior instructional experience with acid-base titrations. Students treat the inflection point as marking the equivalence point</p>	<p>Using the point of greatest slope as the neutralization point; identifying the midpoint of the steepest rise as the equivalence point.</p>	<p>P1, P4, P5</p>

		or the moment of neutralization.		
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## References

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