Supplementary Information (SI) for Soft Matter $\vec{\mu}(h, \rho)$ — This journal is © The Royal Society of Chemistry 2025