Supplementary Material for "Multi-dimensional memory in low-friction granular materials"

(Dated: May 22, 2025)

I. EXPERIMENTAL DETAILS

The full setup used is shown in Fig. S1a. Particles sit on a glass plate and are confined by an acrylic inner frame, which is controlled to produce shear by the t-slot rail outer frame. Actuators drive shear by pushing and pulling a slider attached to one of the belts so that all four pulleys rotate (SA) and by pushing and pulling one axle along diagonal so that all four axles move in and out (SB).

A. Coupling between strains in application of SB

When $S1$ is actuated via a linear actuator, it is intuitive that the walls will simply move together or apart with no change to the angles between them; $S2$ is unaffected. This is SA. In general one might also expect the reverse to be true, but because of the experimental setup driving $S2$ leads to some amount of $S1$ as well; this combination is defined as SB. To understand the origin of this coupling, consider the zoomed in schematic of one pulley and belt shown in Fig. S1b. Because the pulley is not free to rotate relative to the axle, the belt effectively rotates around the pulley, exposing new teeth on one side (in the schematic, this is the right side) and making new contact on the other (the left). The sliders that connect the inner frame to the belt are fixed in their connection to the belt so that they are forced to slide slightly as the belt changes its configuration.

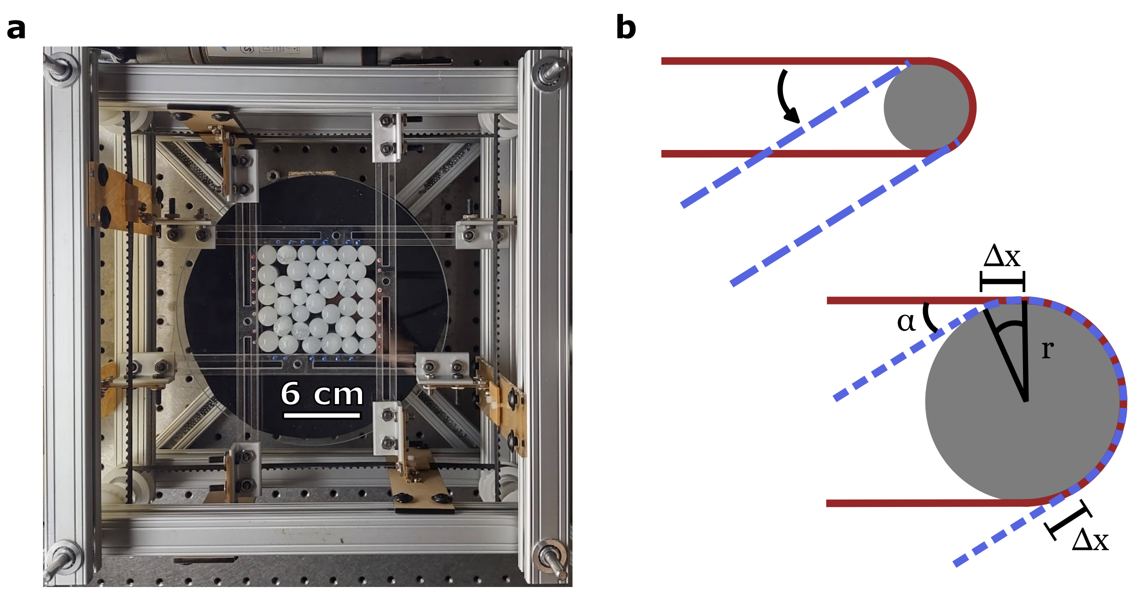

FIG. S1. Experimental setup. (a) T-slot rails are used as outer frame, with one axle at each corner. Inner frame (laser-cut acrylic) is connected to belts via laser-cut sliders, which are constrained to slide along the t-slot rails and are driven by the belts. Particles are visible in the inner cell, and red and blue dots used to fit the boundaries are visible on the inner frame. Light source (not directly visible) is to the left. (b) Schematic zoom-ins of one pulley and belt to illustrate the coupling of $S2$ and $S1$ when SB is applied.

With no hysteresis, this slide distance Δx can be calculated as the length of newly exposed belt (or, equivalently, belt newly in contact with the pulley), a function of the change in angle of the wall α (between 2.5° and 3° for a typical training cycle) and the radius r of the pulley (about 25 mm). This gives a sliding distance of

$$\Delta x = \frac{3^\circ}{360^\circ} \times 2\pi \times 25 \text{ mm} \sim 1.3 \text{ mm}.$$

Comparing this with the typical sliding distance for one training cycle $\Delta x_{train} \sim 2.3$ mm, we find that the magnitude of $S1$ when SB is applied should be about half that when SA is applied. In practice the ratio is closer to 0.3, likely due to friction preventing the sliders from moving freely.

This slide distance also gives rise to a compression effect because the slide occurs either "inward" or "outward" for all walls. If the original side length is ℓ, and the new side length is $\ell - 2\Delta x$ (where the factor of two comes from the fact that both walls move in by Δx), the fractional area A/A_0 is the given by

$$A/A_0 = \frac{(\ell - 2\Delta x)^2}{\ell^2} = 1 - \frac{4\Delta x}{\ell} + \mathcal{O}((\Delta x/\ell)^2).$$

Plugging in $\ell \sim 100$ mm and $\Delta x \sim 1.3$ mm, we find a fractional area of 1.05 and hence a linear compressive strain of $\sqrt{1.05} - 1 \sim 0.02$, or 2% as seen in Fig. 2d.

B. Variability and quantification of memory

The readout curves reported in the main text are averages of several experiments. Figure S2(a,b) shows individual experiment (partially transparent curves with circular markers) as well as the average (bold black line) for the two scenarios in the main text in which a memory is observed. This is quantified by Fig. S2(c,d), which shows the second derivative of Δ_{min} with respect to frame number (Δ''_{min}) divided by Δ_{min} in analogy with the "signal" or strength of memory defined by Paulsen et al [1]. Both experiments show peaks at the third frame, which corresponds to the amplitude of training (and the cusp in the left curves).

II. READOUT UNDER OTHER CONDITIONS

Figure S3a shows readout for packings that have not been trained at all. Readout is monotonically increasing, revealing the lack of training. We also demonstrate the ability to train values other than that shown in the main text. Figure S3b shows readout after training at a larger amplitude than in Fig. 3c in the main text ($\gamma_B^{train} = 7\%$ here instead of 5%); cusp appears when the readout amplitude reaches that larger value. Similar training is also possible with $S1$, as shown in Fig. S3(c,d) for $\gamma_B^{train} = 7\%$ (c) and $\gamma_A^{train} = 9\%$.

III. BOX MEMORY EFFECTS

As discussed in the main text, the box exhibits some memory of its own. Figure S4 shows the stroboscopic γ values introduced in Fig. 7 of the main text for (a) an experiment where there was box memory and (b) an experiment where there was no box memory at the training strain. Both correspond to cases where a memory was present in the readout signal at the training strain.

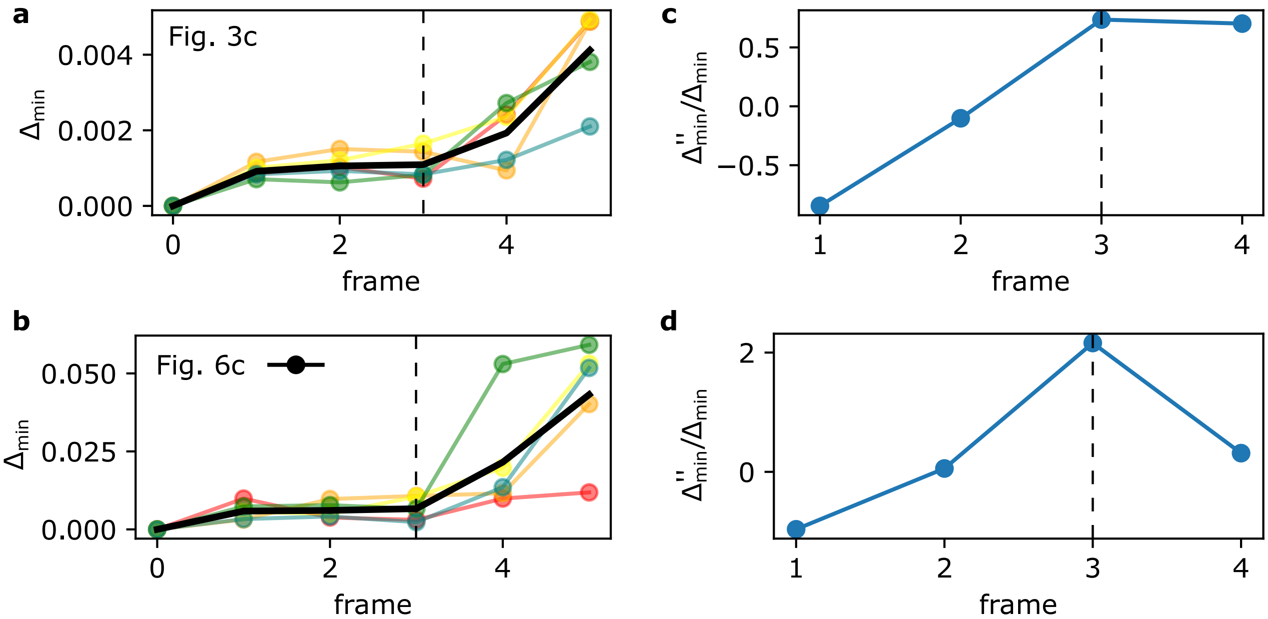

FIG. S2. (a) and (b) Individual experiments (transparent colored circles) averaged to obtain curves shown in main text Fig. 3c and Fig. 6c (”mixed” training), respectively. Bold black lines show the averaged values. (c) and (d) Corresponding value of $\Delta''_{min}/\Delta_{min}$, a measure of cusp strength. Because original measurements only include six data points, each $\Delta''_{min}/\Delta_{min}$ curve has only four values. In all plots, training amplitude is shown by a vertical dashed line.

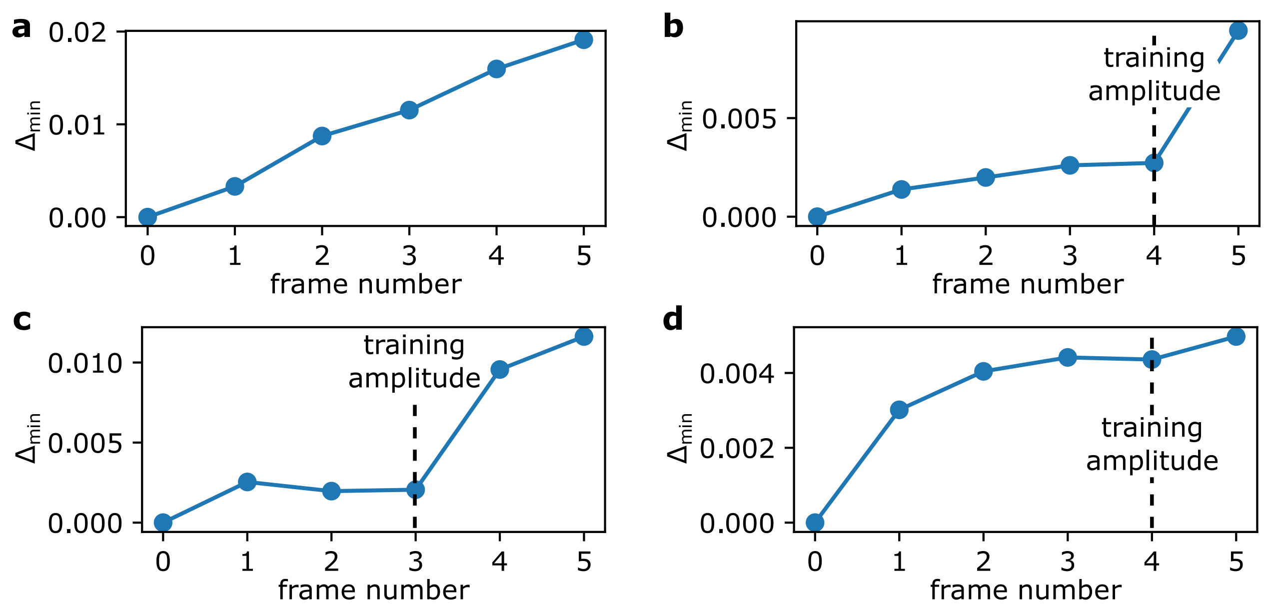

FIG. S3. (a) Readout via SB of untrained packings and (b) readout via SB of packings trained with $\gamma_B = 7\%$. (c) Readout via SA for packings trained with $\gamma_A = 7\%$ and (d) $\gamma_A = 9\%$. All sets of data are an average of five experiments under identical conditions.

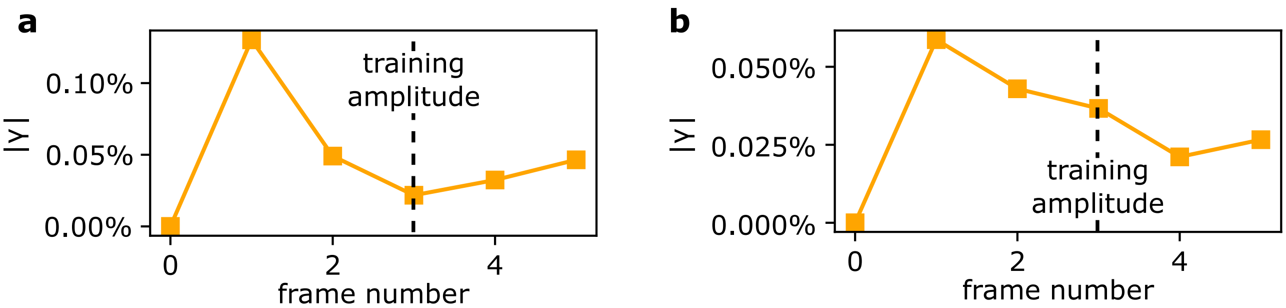

FIG. S4. Stroboscopic measured change in $S1$ strain values as in the main text Fig. 7b but with the absolute value taken to highlight a minimum at the training amplitude. (a) For the data in main text Fig. 3c and (b) for the experiments reported in Fig. S3c (trained and read out with SA).

[1] J. D. Paulsen, N. C. Keim and S. R. Nagel, *Physical review letters*, 2014, **113**, 068301.