Supplemental Information for:

Microscopic Structural Study on the Growth History of Granular Heaps Prepared by the Raining Method

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Supplementary Note 1: Coarse-grain processing

In order to determine the appropriate coarse-graining scale for our structural analysis, we systematically tested multiple smoothing lengths, specifically 0.5d, 1d, and 2d, where d is the particle diameter. As shown in Fig. S1, the overall spatial trends of packing fraction ϕ remain consistent across all tested scales, although finer details vary slightly. This consistency confirms that the key conclusions of our work are robust with respect to the choice of coarse-graining scale. We ultimately selected a smoothing length of 1d in the present analysis, as it provides the best balance between preserving meaningful spatial variations and ensuring sufficient particle representation within each averaging cell for statistically reliable measurements.

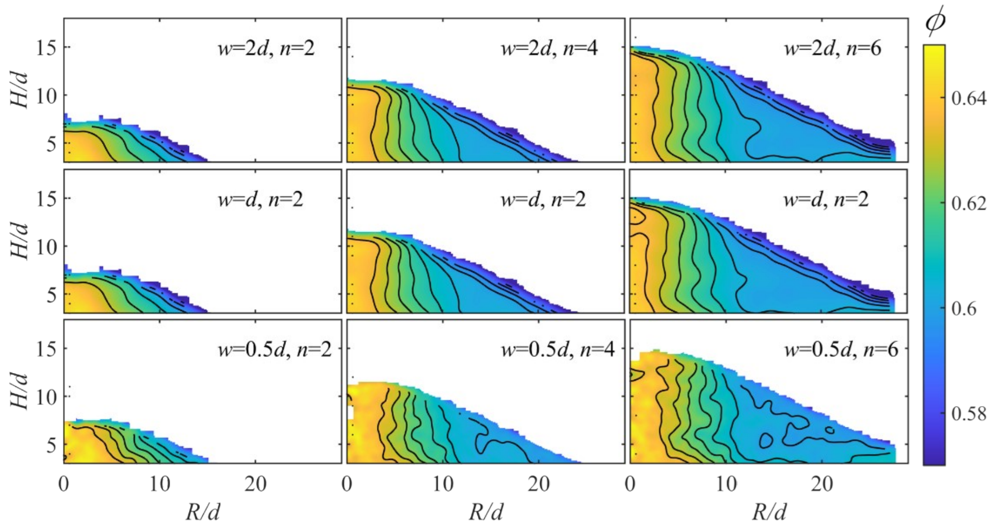

Fig. S1. Spatial distributions of ϕ as a function of radial distance R and height H for different deposition numbers n and various smoothing lengths.

Supplementary Note 2: Determination of particle contact

Ideally, the surface-to-surface distance between two particles in actual contact should be zero. However, due to experimental uncertainties, such as limited X-ray spatial resolution, particle non-sphericity, and image processing artifacts, contacts may be misidentified as small gaps or even overlaps. Consequently, the probability distribution of surface-to-surface distances Δr (Fig. S2a) shows a Gaussian core (red curve) arising from measurement uncertainties among true contacts, and a fat tail reflecting the contribution of nearby but non-contacting particles. To address this, we apply a well-established method that fits the entire distance distribution to separate these two contributions[1]. Specifically, we consider two particles to be in contact if their surface-to-surface distance is smaller than a threshold δ. In practice, we plot the average contact number Z versus δ (blue symbols in Fig. S2b), which corresponds to the cumulative behavior of the PDF of Δr in Fig. R1a. This cumulative curve is modeled by a combination of an error function representing true contacts (yellow curve) and a linear term for non-contacting neighbors (purple line). By fitting the experimental data with this composite model (red curve), we determine the appropriate threshold δ_c, below which particles are considered to be in contact.

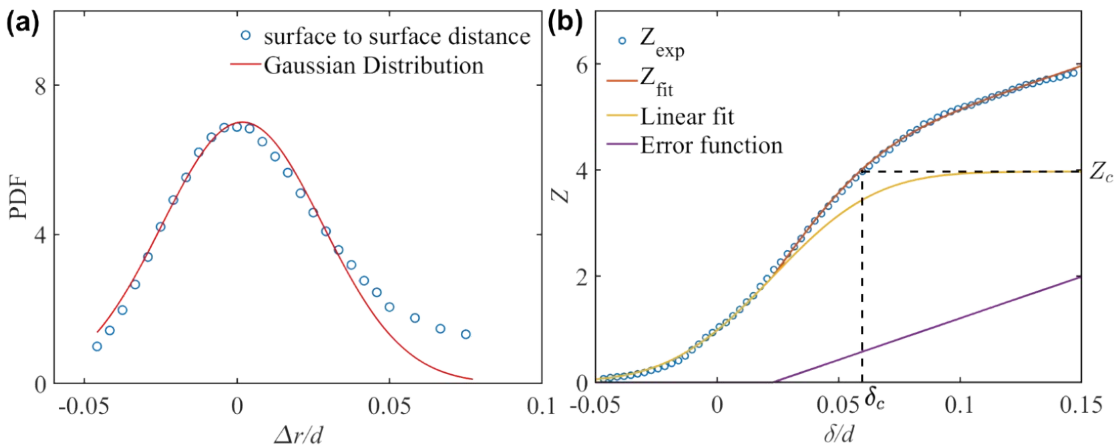

Fig. S2. a PDF of surface-to-surface distance Δr among neighboring particles. The Gaussian core results from experimental uncertainties on contacting particles, while the fat right tail stems from the neighboring but non-contacting particles. **b** Complementary error function fitting to yield the critical threshold δ_c of surface-to-surface distance and the corresponding global average contact number Z_c.

Supplementary References

1. Aste, T., Saadatfar, M. & Senden, T.J. Geometrical structure of disordered sphere packings. *Phys. Rev. E* **71**, 061302 (2005).