

## Supplementary Material:

### Colloidal Rod Dynamics under Large Amplitude Oscillatory Extensional Flow

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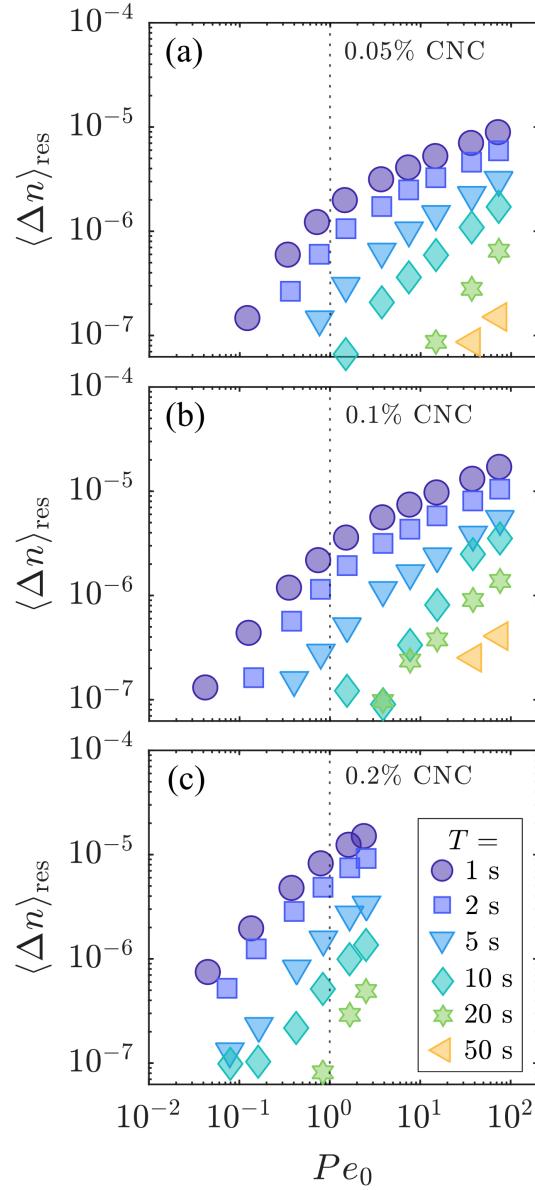
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## 1 Residual Birefringence

Fig. S1 displays the residual birefringence  $\langle \Delta n \rangle_{\text{res}}$  as a function of  $Pe_0$  for different pulsation periods ( $1 \leq T \leq 50$  s) and various CNC concentrations. As the period decreases, *i.e.*, as the pulsation frequency increases, the onset of detectable residual birefringence  $\langle \Delta n \rangle_{\text{res}}$  shifts to lower  $Pe_0$  values.



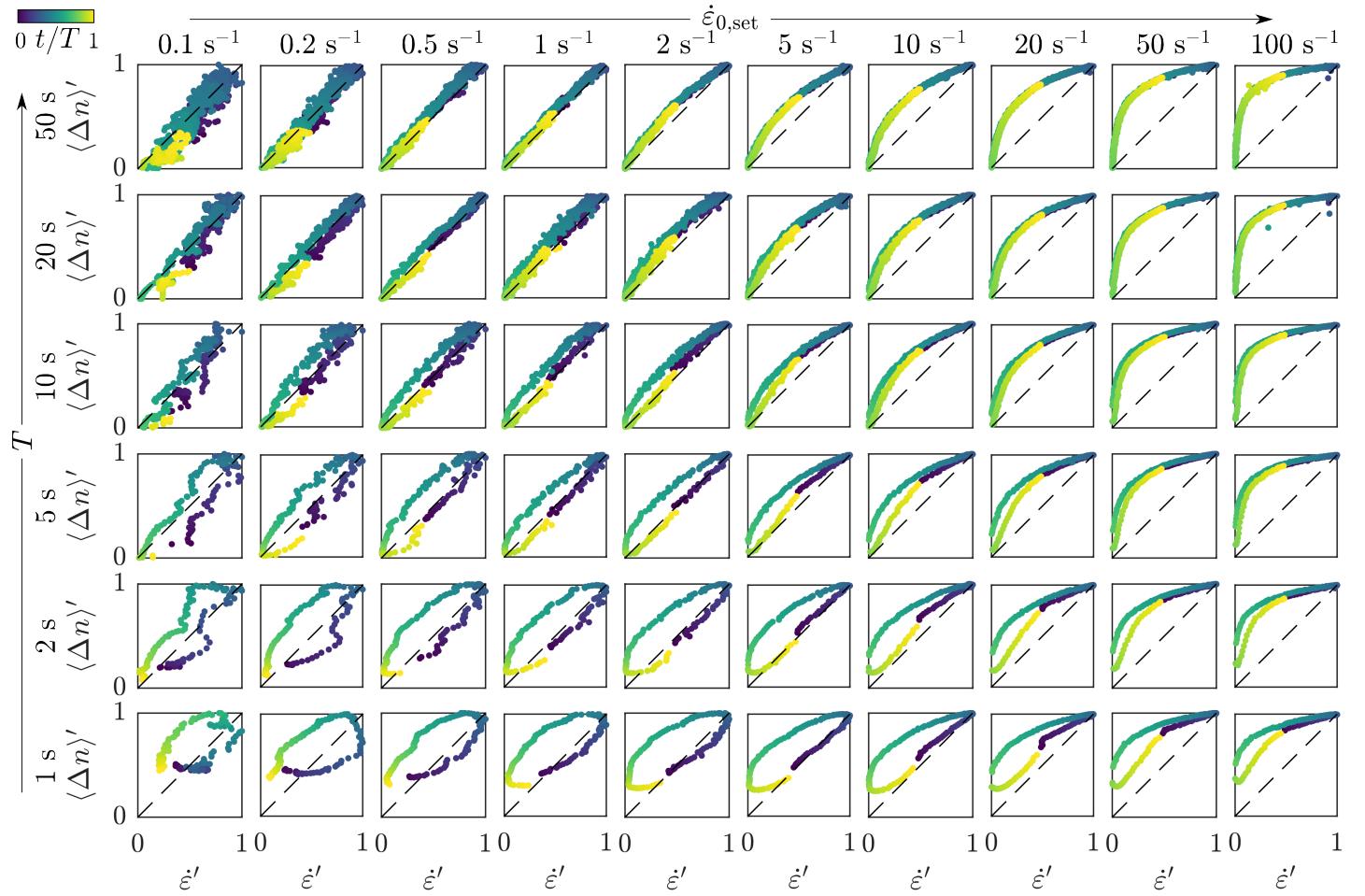
**Fig. S 1:** Residual birefringence  $\langle \Delta n \rangle_{\text{res}}$  during pulsatile LAOE. Data is shown as a function of  $Pe_0$  for the three samples (a-c) and various periods  $T$ .

## 2 Lissajous Plots

Fig. S2 displays the experimental Lissajous plots for a 0.1 % CNC sample over a broad range of  $T$  and  $\dot{\varepsilon}_{0,\text{set}}$ . We can highlight the following general trends.

At relatively high  $T$  (e.g.,  $T = 50$  s), there is a lack of hysteretic behavior as well as residual birefringence, indicating that the CNCs are in a flow-independent regime. However, at  $\dot{\varepsilon}_{0,\text{set}} \gtrsim 2 \text{ s}^{-1}$ , the birefringence becomes progressively non-linear.

As  $T$  decreases and approaches values comparable to  $1/\overline{D}_r$ , evidence of flow-history dependence in the CNCs emerges, manifested as hysteresis. At even smaller  $T$ , residual birefringence also appears e.g., see  $\dot{\varepsilon}_{0,\text{set}} = 5 \text{ s}^{-1}$  as  $T$  decreases). It appears that hysteresis provides a more sensitive measure for detecting the onset of flow-history dependence than residual birefringence. This is likely because the residual birefringence is limited by the detection threshold of the experimental setup.



**Fig. S 2:** Experimental Lissajous curves of the normalized average birefringence  $\langle \Delta n \rangle' = \langle \Delta n \rangle(t)/\langle \Delta n \rangle_{\text{max}}$  as a function of the normalized extension rate  $\dot{\varepsilon}' = \dot{\varepsilon}(t)/\dot{\varepsilon}_{\text{max}}$  for various  $T$  and  $\dot{\varepsilon}_{0,\text{set}}$ . Data is shown for the 0.1 % CNC sample. Black dashed lines correspond to slopes of 1.