Supplementary Information of

"Skyrmionic topology perspective on Lehmann clusters"

by

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1. Singularity-free director field of b=p dislocations

1.1 Geometrical construction of the director field

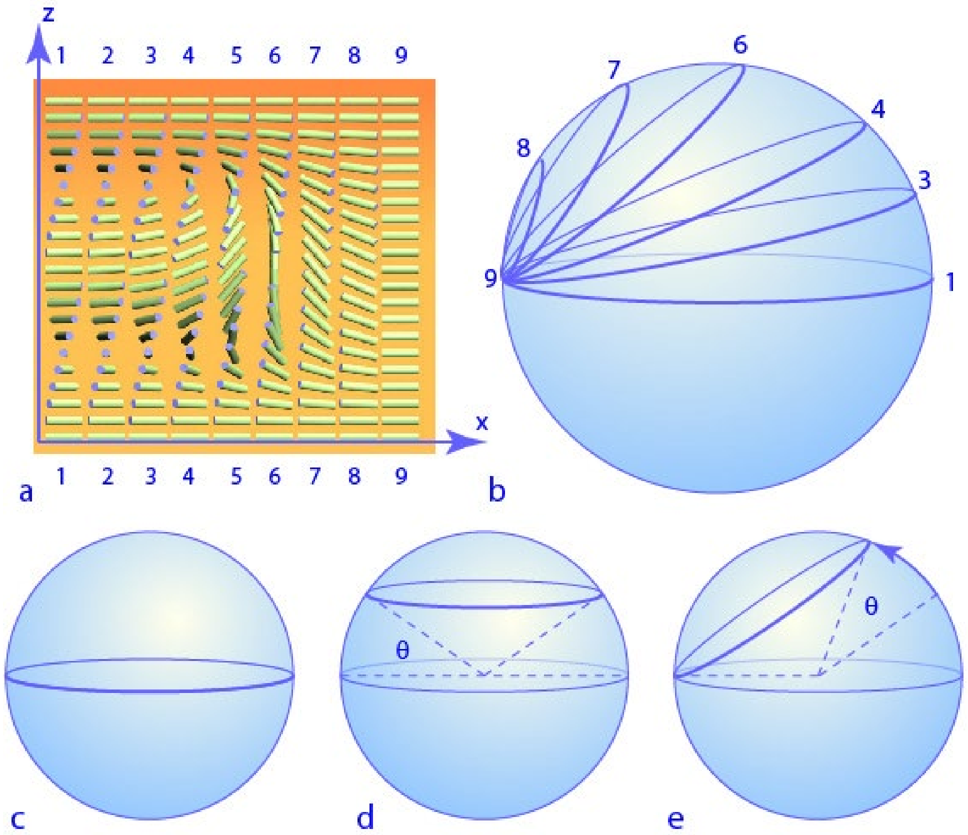

Figure 1: Geometrical construction of the singularity-free director field of the b=p dislocation orthogonal to the anchoring direction x. a) Perspective view of the director field. b) Mapping of the director field on the unit sphere. c) Mapping on the unit sphere of the the perfect helix $n=\{cos\phi, sin\phi, 0\}$. d) Mapping on the unit sphere of a conical helix $\boldsymbol{n}=\{cos\theta cos\phi, cos\theta sin\phi, sin\theta\}$. e) Mapping on the unit sphere of the field $\boldsymbol{n}'=R\boldsymbol{n}$, where R is the rotation matrix by θ around the axis y.

The singularity-free director field of the full-pitch *(b=p)* dislocation represented in Figure 1a has been constructed by a purely geometrical method which does not result from minimization of the distortion energy. Dislocation in Figure 1a is orthogonal to the x axis. The field in the first column of Figure 1a is that the perfect helix

$$n = \{cos\phi, sin\phi, 0\}$$

where $\phi=2\pi z$ is the azimuthal angle of the director in the (x,y) plane. Fields in the eight other columns of Figure 1a were obtained in two steps:

1°- First, the perfect helix is deformed into the conical helix shown in Figure 1d

$$\boldsymbol{n} = \{cos\theta cos\phi, cos\theta sin\phi, sin\theta\}$$

with

$$\theta = (tanh\ (x / \lambda) + 1)\pi / 4$$

The function $\theta(x) = (tanh(x/\lambda)+1)\pi/4$ was chosen in such a way that, as shown in the figure below, the angle θ varies from 0 to π/2 when *x* goes from -∞ to -∞. In terms of this qualitative geometrical model, the full pitch dislocation *(b=p)* can be seen as a wall of width λ which in our calculation was set to 0.5 for a good visibility.

2° Subsequently, the conical helix is tiled by the angle θ around the y axis.

When the dislocation has another orientation making the angle γ with anchoring direction, the analytical expression of the singularity-free director field becomes:

$$n_x(x,z,\mathrm{g}) = cos\,(\gamma)(cos\,(2\pi z)cos^2(\frac{1}{4}\pi(tanh\,(2x)+1)) + sin^2(\frac{1}{4}\pi(tanh\,(2x)+1))) + sin\,(\gamma)sin\,(2\pi z)cos\,(\frac{1}{4}\pi(tanh\,(2x)+1))$$

$$n_y(x,z,\mathrm{g}) = cos\,(\gamma)sin\,(2\pi z)cos\,(\frac{1}{4}\pi(tanh\,(2x)+1)) - sin\,(\gamma)(cos\,(2\pi z)cos^2(\frac{1}{4}\pi(tanh\,(2x)+1)) + sin^2(\frac{1}{4}\pi(tanh\,(2x+1)))$$

$$n_z(x,z,g) = sin\,(\frac{1}{4}\pi(tanh\,(2x)+1))cos\,(\frac{1}{4}\pi(tanh\,(2x)+1)) - cos\,(2\pi z)sin\,(\frac{1}{4}\pi(tanh\,(2x)+1))cos\,(\frac{1}{4}\pi(tanh\,(2x)+1))$$

1.2. Energy per unit length of the b=p dislocations

Knowing the analytical expression of the director field ***n****(x,z,γ),* the energy per unit length of the full-pitch dislocations has been calculated by numerical integration of the Frank energy density

$$F(\gamma) = \int_0^1 \int_{-2}^{2} \frac{1}{2}\{K_{11}\,(div\vec{n})^2 + K_{22}\,(\vec{n}\cdot rot\vec{n}+\pi)^2 + K_{22}\,(\vec{n}\times rot\vec{n})^2\}dxdz$$

in the approximation of isotropic elasticity $K_{11}=K_{22}=K_{33}$.

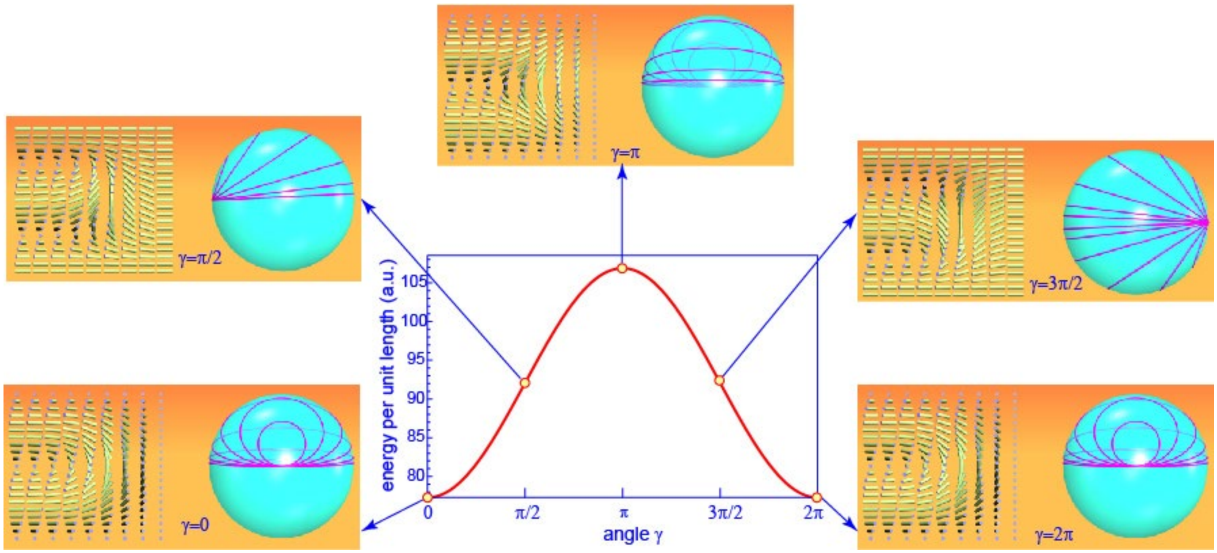

Figure 2: Variation of the energy per unit length of the full-pitch dislocations as a function of the angle γ between the dislocation and anchoring direction.

3. Materials

In our experiments we used mixtures of 5CB (4-Pentyl-4-cyanobiphenyl) from Sigla-Aldrich) with CB15 ((S)-4-Cyano-4´-(2-methylbutyl)biphenyl) from Synthon Chemicals.

4. Videos are quoted in the article:

1°- Video S1 - Generation of a Lehmann cluster.

2°- Video S2 - Generation and drifts of Lehmann cluster

3°- Video S3 - Drift of a Lehmann cluster

3°- Video S4 - Splitting of a Lehmann cluster