Soft Matrix: Probing Local Mechanical Properties in Amorphous Solids

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SUPPLEMENTARY INFORMATION

Detecting plastic events

In Fig. 1 we show the evolution of the system's shear stress (σ), potential energy (U) as well as the Maximum displacement with applied strain γ, for 2D BMLJ. Here we have considered the displacement in Y since the shearing direction is along X. In 3D systems, we consider Z, the vorticity direction. Clearly the jumps in the σ and U is signed in term of kinks in the displacement. In order to differentiate the signal from the background noise, we compute the distribution of these displacements and is shown in Fig. 2. It is quite evident that we have a bimodal distribution and we take conservative estimate for the threshold to recognize the plastic event as 0.01. This threshold is robust for varying ages of the sample. Similar analysis are done for other models to obtain the appropriate thresholds.
Plastic events in subsystem with elastic background : We perform the above analysis in the soft matrix as well wherein only a subsystem is allowed to have plastic events. Even though the bimodality of the displacement distribution is coarsened (see Fig. 3), the threshold for the plastic event is quite evident and recognize the events accurately (see Fig. 4).

Choice of k value in the soft matrix

In the soft matrix method we allow the sub-system to relax completely without any constraint and the background is allowed relax affinely. This is achieved by addition of a spring potential with a sprint constant k. The choice of k is

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made such that (1.) no large displacement occur in the background but also (2.) do not alter the system's mechanical properties. In the Fig. 6 we show (for 2D BMLJ) the maximum displacement measured in the subsystem (left) and the full system (right), including the subsystem. For k=2 and k=5, we find that relative large displacement are occurring in the surround and not in the subsystem. But with the increase in the k, these events in the surrounding are curbed. In the Fig. 5 we show curvature of the PE profile (vs. γ) before the plastic event, the storage module (from the slope of σ vs γ before the plastic event) as well as the pressure difference $\Delta P = P_k - P_{k=0}$ for varying k values. These quantities are computed for the full system with soft matrix background. We find that for a range of k values (k¡=20), these measures do not change and are similar to bulk value (dashed line in Fig. 5). Hence $k = 10$ is a good estimate which satisfy both the criterion defined above. Similar protocols are followed for other models.

L_s dependence on the local yield threshold statistics

In Fig.8 we show the scaled plot of the normalized cumulative distribution function from soft as well as frozen matrix methods for two differently aged samples. The scaling helps in identifying the L_s beyond which minimal finite-size effects are observed. In soft matrix, poorly-aged samples shows a clear data collapse for all the L_s values. In the well-aged samples for $L_s \geq 10a$, minimal finite size effects are found. Unlike in soft matrix, the frozen matrix data show L_s dependence in both ages due to the rigid boundary imposed in the method. Going to even larger system size might reduce such effects. In Fig. 9 we show complete yield threshold statistics from frozen matrix method, for (a) poorly-aged and (b) well-aged samples. The lack of age dependence is evident in the range of threshold values obtained from our simulations but is highlighted in Fig. 9 (c), where we show that for $L_s < 10a$, F(X) do not differentiate between ages. In Fig. 9 (d) we show the small-X behavior of $F(X)$ and $P(X)$, which clearly illustrates the method's insensitivity to the samples' age.

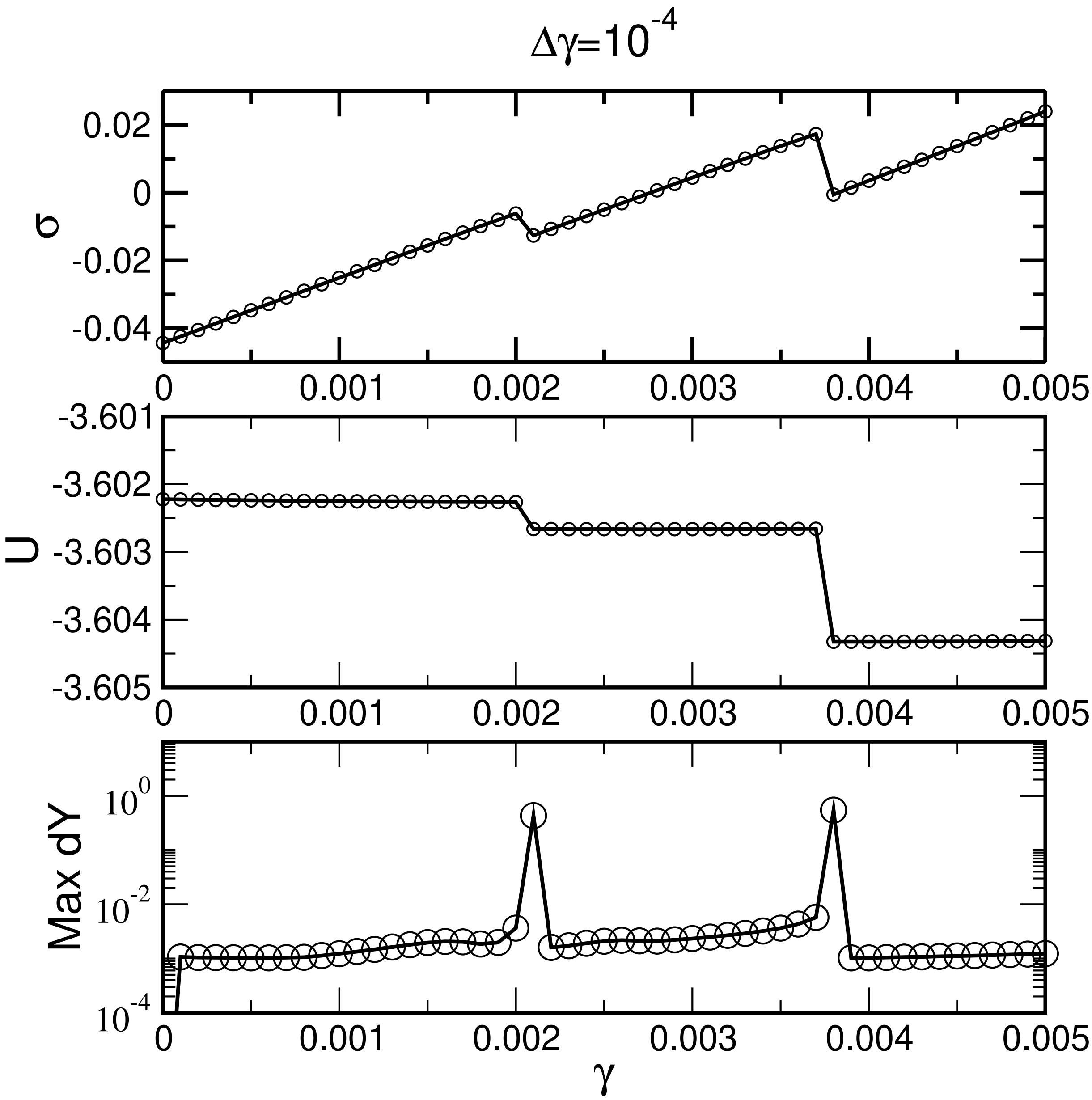

FIG. 1. **Evolution with strain.** Stress σ and potential energy U evolve with applied strain. Jumps in these quantities observed as the system show local yielding due to plastic events, hence non-affine displacement captured by maximum displacement measure. This data is for 2D BMLJ model and poorly-aged samples

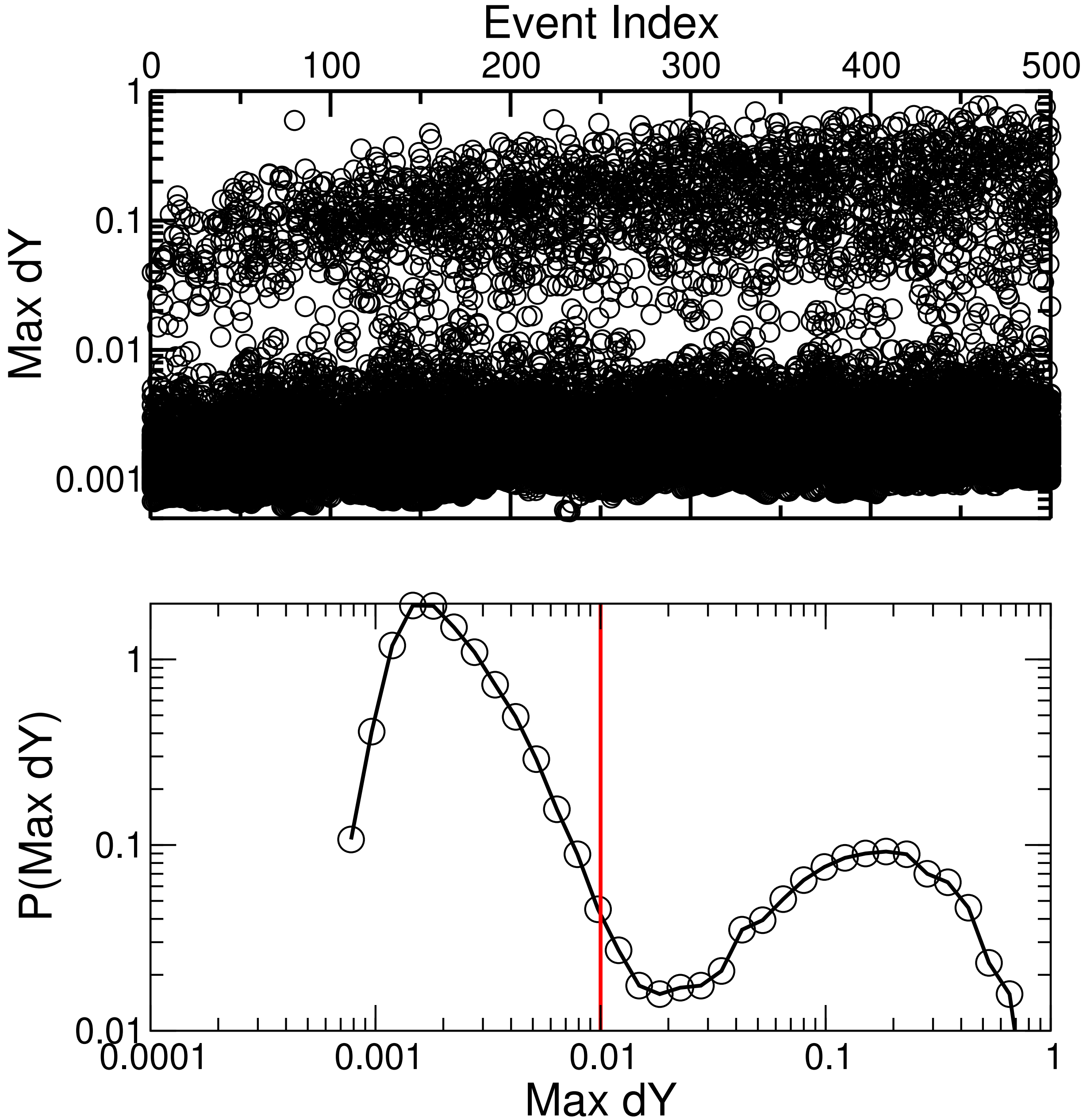

FIG. 2. **Maximum displacement distribution.** Top plot show the raw data of maximum displacement and bottom its distribution. This data is for 2D BMLJ model and poorly-aged samples

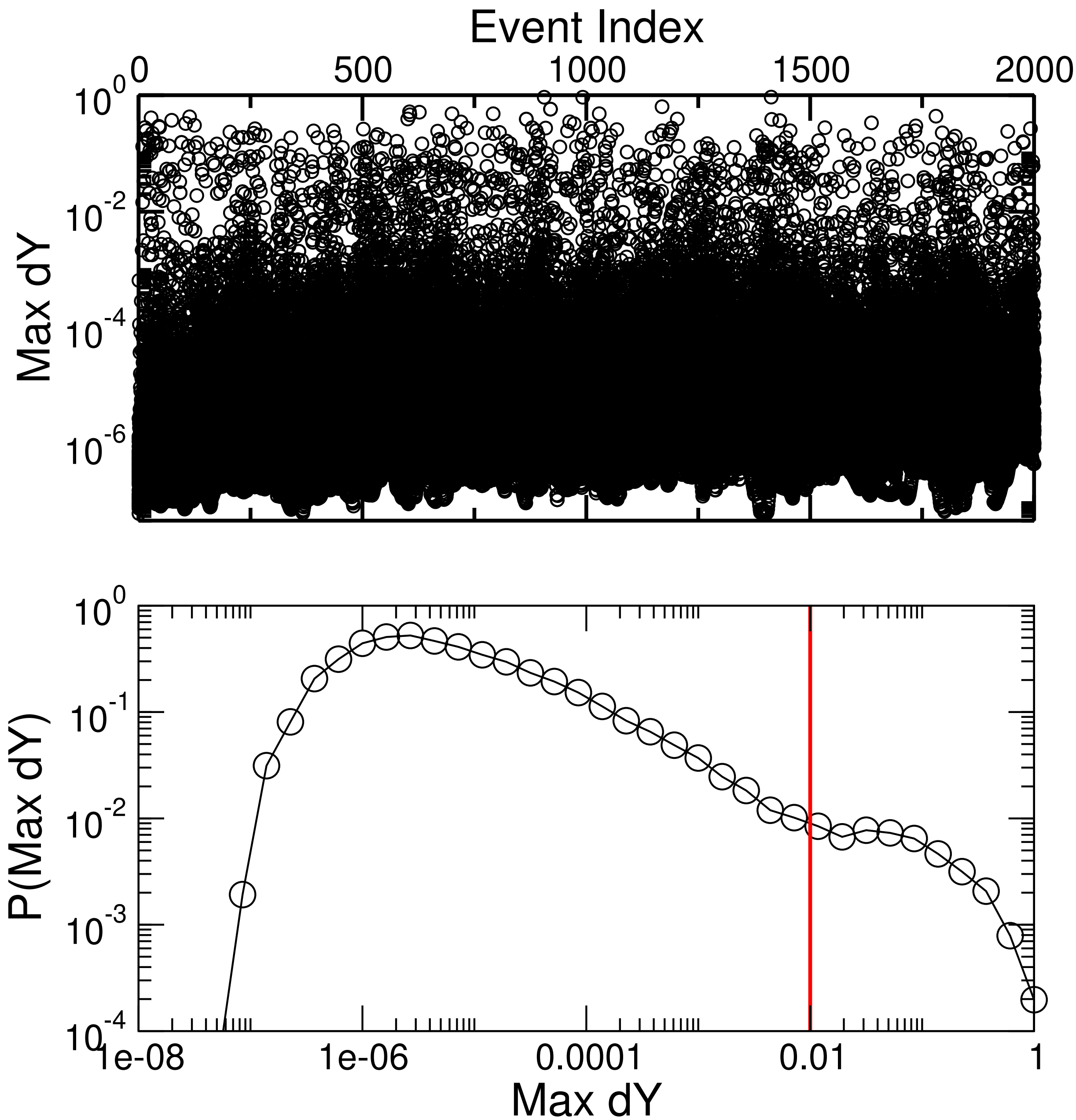

FIG. 3. **Maximum displacement distribution in the soft matrix.** Top plot show the raw data of maximum displacement and bottom its distribution, within the subsystem. This data is for 2D BMLJ model and poorly-aged samples with a subsystem size of $L_s = 16a$.

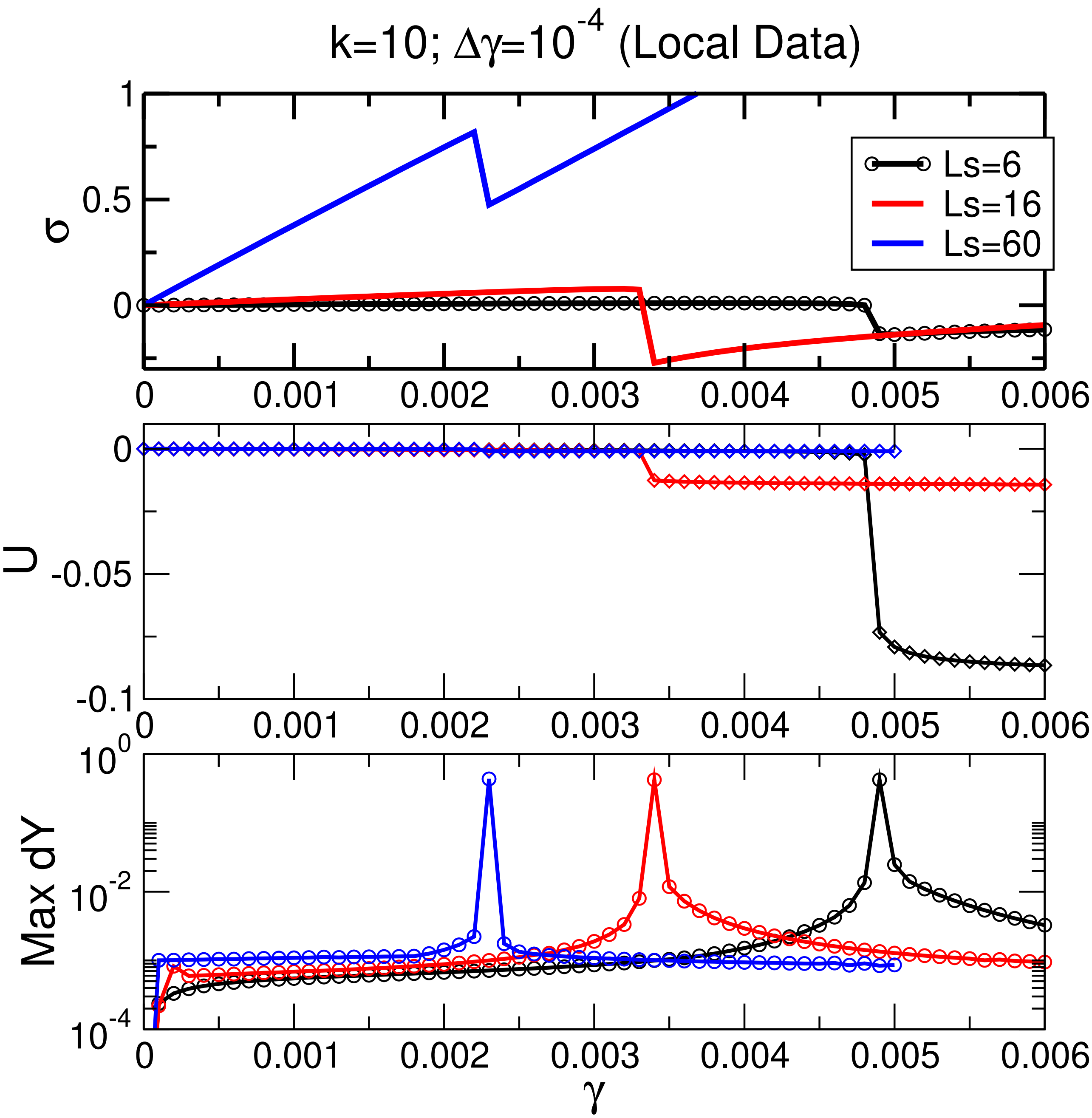

FIG. 4. **Evolution with strain in the sub-system.** Stress σ and potential energy U evolve with applied strain. Jumps in these quantities observed as the system show local yielding due to plastic events in the sub-system. This data is for 2D BMLJ model (poorly-aged) in the soft matrix of k=10 and $L_s = 16a$.

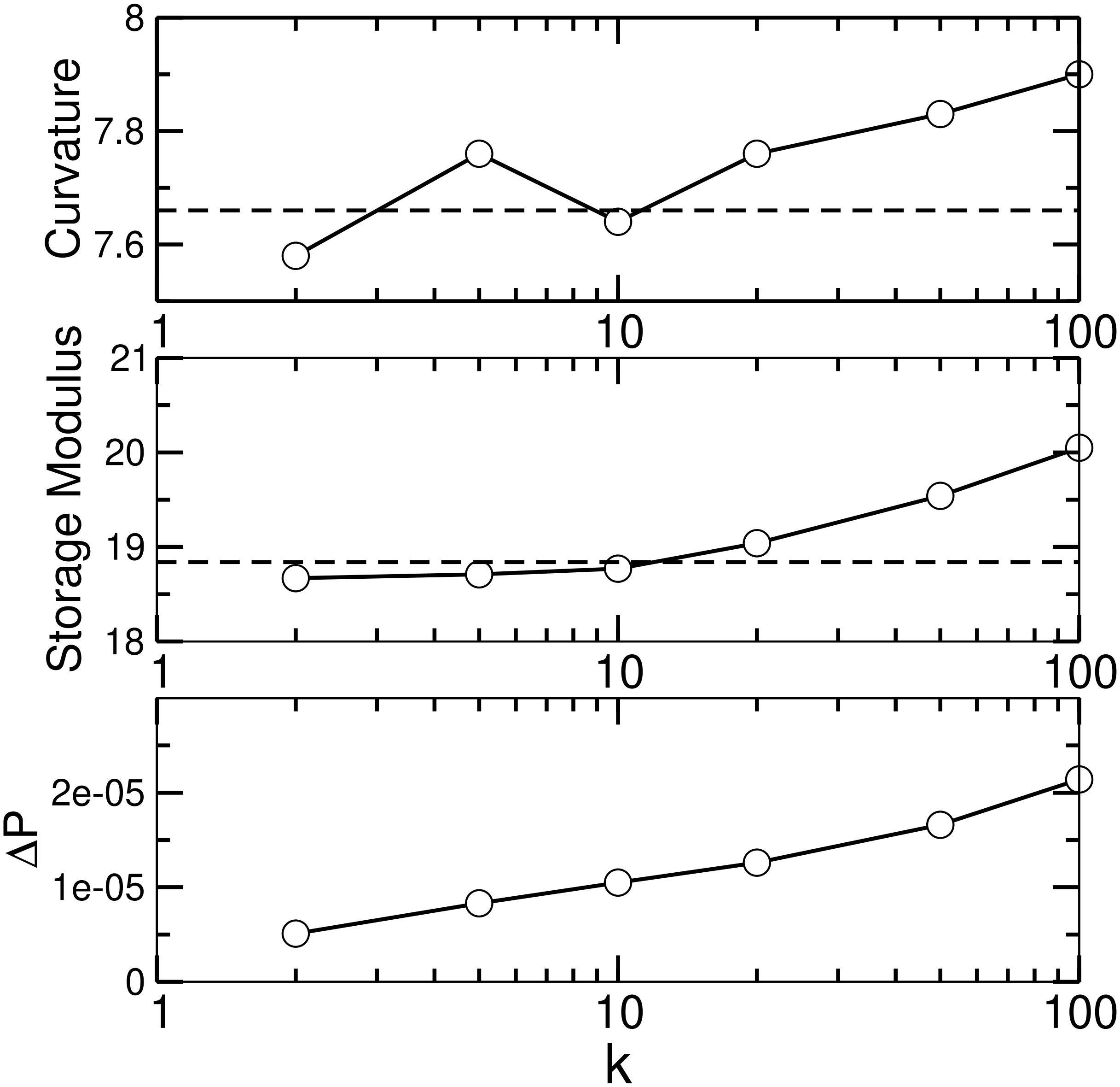

FIG. 5. **Various measures of the full system with the soft matrix background (2D BMLJ).** For $k \leq 20$, these measures do not vary substantially. The dotted line is the measure for unconstrained (without the soft matrix) full system. Here $\Delta P = P_k - P_{k=0}$ and is measure at small strain of $10\delta\gamma$

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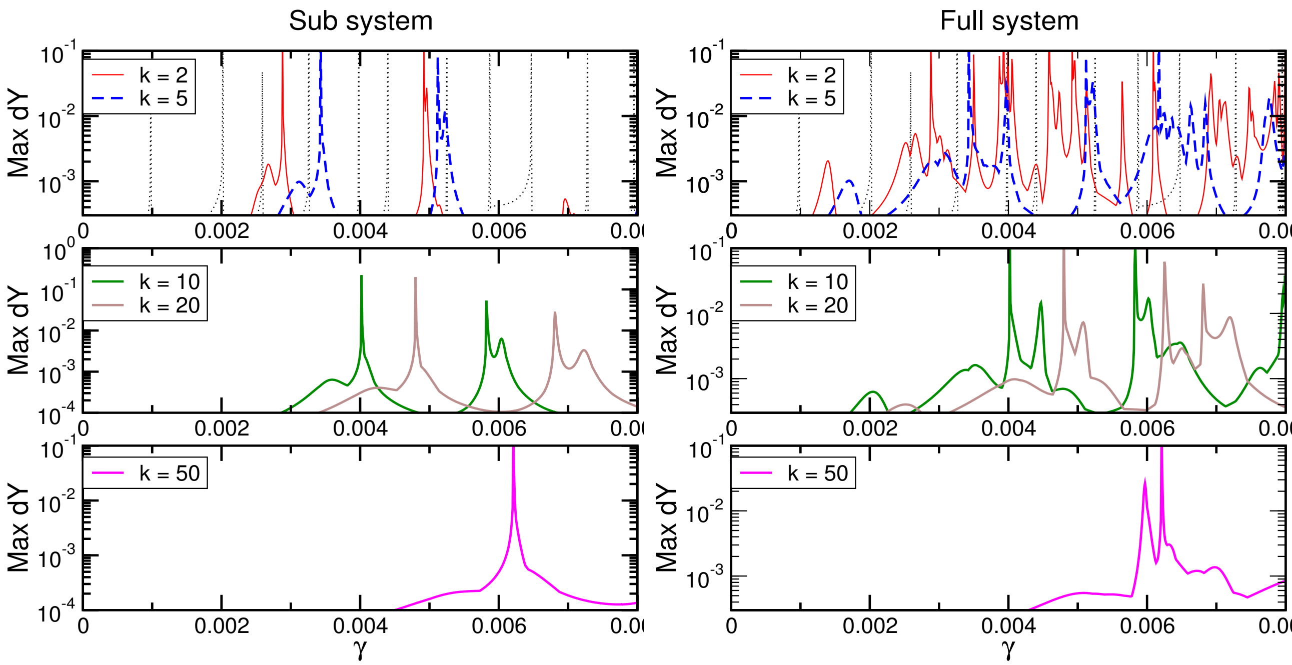

FIG. 6. **k dependence on events inside and outside the sub-system.** Maximum displacement measure discussed in the text is computed for various k values, in the subsystem (left) and the full system (right) as a function of strain. The plastic event is signalled by the jump in the measure. Increasing the value of k, events occur only within the subsystem. This is for the 2D BMLJ system.

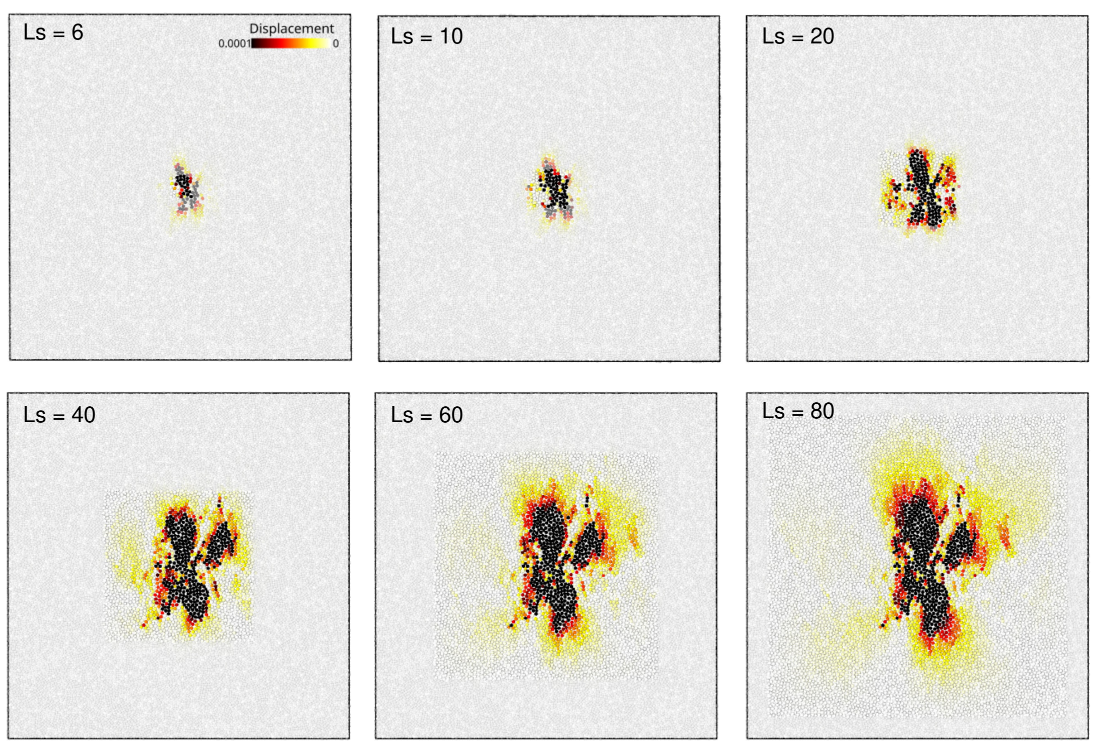

FIG. 7. **Displacement maps.** Obtained from 2D BMLJ simulation for fix $k = 10\epsilon/a^2$ and varying Ls (6a, 10a, 20a, 40a, 60a, 80a). The heatmap coloring scheme is based on the particle displacements (white indicates no displacement and black corresponds to the displacement of magnitude 0.0001). The region where soft matrix is imposed is made translucent to emphasize the sub-system.

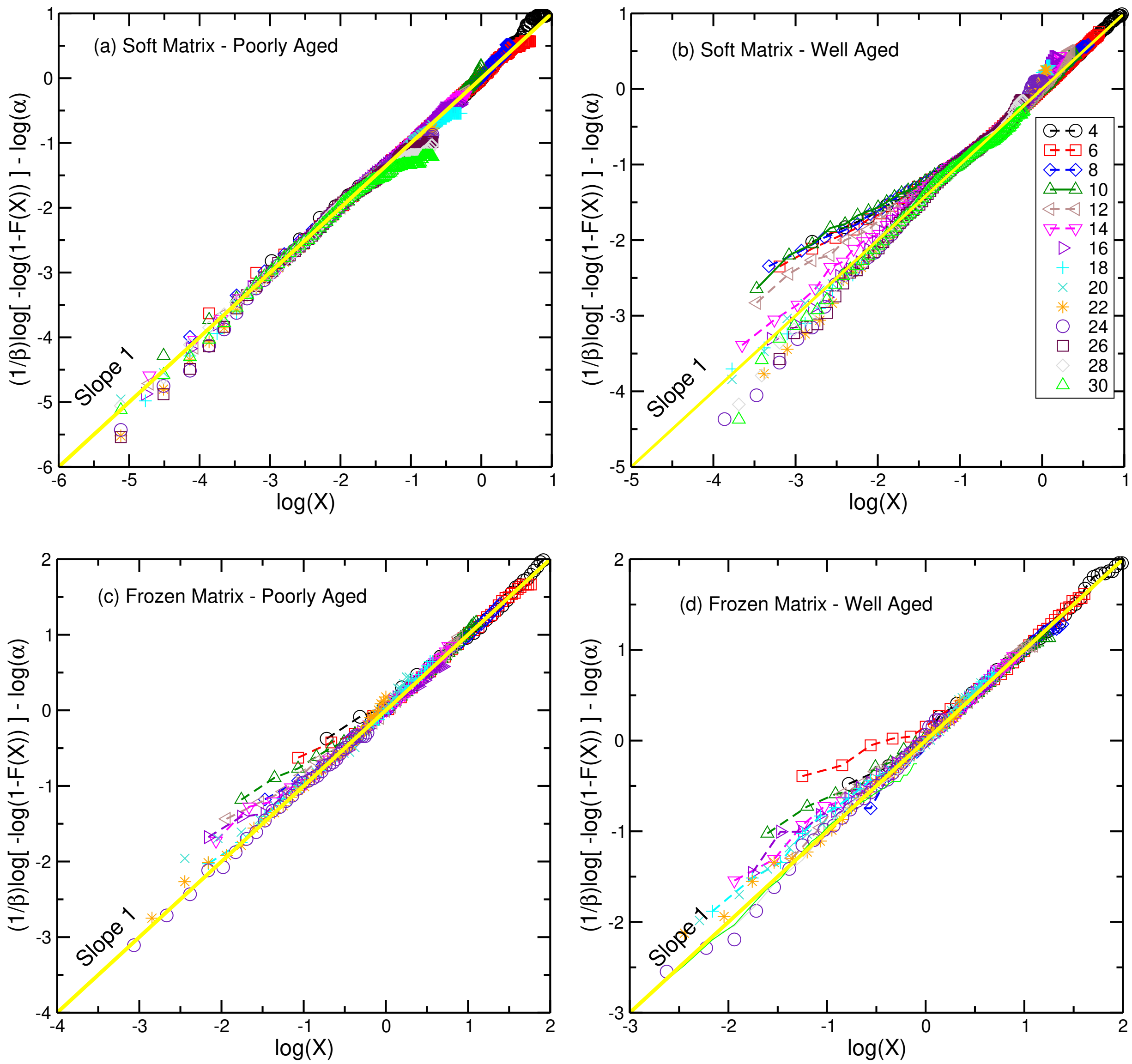

FIG. 8. **Scaled local yield stress statistics.** The cumulative distribution function $F(X)$ is transformed to assess the validity of Weibull scaling and identify the minimum sampling size L_s required for meaningful statistics. The transformation gives a linear relation and is highlighted by yellow solid line with slope 1. In (a) and (b) we show the scaled $F(X)$ from soft matrix for poorly-aged and well-aged samples respectively. In (c) and (d) we show the scaled $F(X)$ from frozen matrix for poorly-aged and well-aged samples respectively. In soft matrix, for the complete range of L_s varying from $4a$ and $40a$, poorly-aged samples shows a linear relation and in well-aged samples for $4a \leq L_s \leq 10a$ a finite probe size dependence is evident. In the same range, independent of age, the frozen matrix show a finite probe size depenence which is due to the rigid background.

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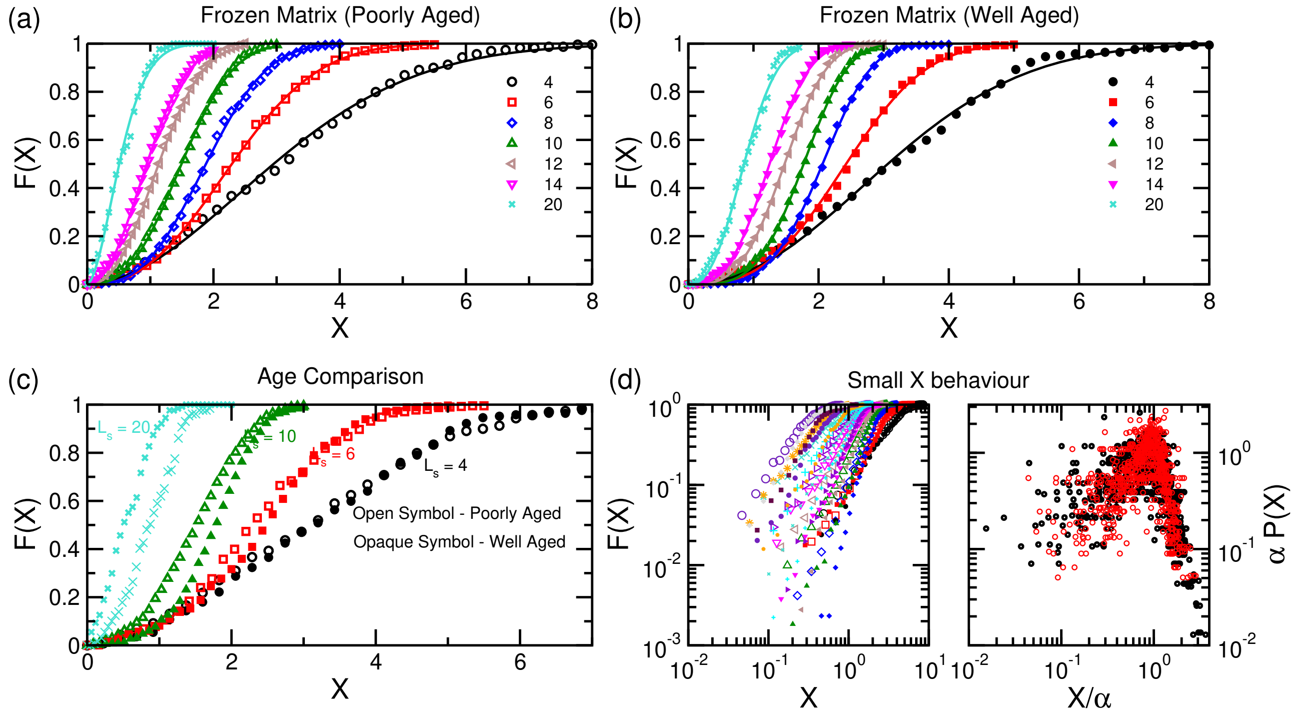

FIG. 9. **Local yield stress statistics from frozen matrix.** Cumulative local yield stress threshold distribution $F(X)$ for varying L_s for (a) Poorly-aged samples ($U_{init} = -3.6\epsilon$) (b.) Well-aged samples ($U_{init} = -3.78\epsilon$) . (c.) Comparison of F(X) across different ages, emphasizing the minimal variation observed for $L_s < 10a$ (d.) The Weibull fit parameters α and β. The scaling parameter (α) shows little variation with age, and its absolute values, linked to the average yield threshold, are notably high, approximately an order of magnitude greater than those of the soft matrix. The shape parameter (β) exhibits some age dependence, although the saturation value seems to converge to a similar value. Large system simulations are required to confirm the same

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