## **Supporting Information**

Density Functional Theory Studies on Tuning TaxTi<sub>(1-</sub>

x)S2 For Insoluble Li2S2-Li2S Conversion in Lithium-

## Sulfur Batteries











$$\label{eq:sigma} \begin{split} \mbox{Figure S1. Energy band structure and PDOS of (a)} $Ta_{0.06}Ti_{0.94}S_2(b)Ta_{0.13}Ti_{0.87}S_2(c)Ta_{0.19}Ti_{0.81}S_2 $$ (d)$Ta_{0.25}Ti_{0.75}S_2(e)$Ta_{0.31}Ti_{0.69}S_2(f)$Ta_{0.44}Ti_{0.56}S_2(g)$Ta_{0.5}Ti_{0.5}S_2(h)$Ta_{0.56}Ti_{0.44}S_2 $$ (i)$Ta_{0.63}Ti_{0.37}S_2(j)$Ta_{0.69}Ti_{0.31}S_2(k)$Ta_{0.75}Ti_{0.25}S_2(l)$Ta_{0.82}Ti_{0.18}S_2(m)$Ta_{0.88}Ti_{0.12}S_2$ and $$ (n)$Ta_{0.94}Ti_{0.06}S_2$, respectively. \end{split}$$



Figure S2. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on TiS<sub>2</sub>. The grey, yellow brown and purple atom is Ti, S<sub>TiS<sub>2</sub></sub>, S<sub>LiPSs</sub>, and Li respectively.



Figure S3. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.06</sub>Ti<sub>0.94</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti, Ta,S<sub>Ta<sub>0.06</sub>Ti<sub>0.94</sub>S<sub>2</sub>, S<sub>LiPSs</sub>, and Li respectively.</sub>



Figure S4. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.13</sub>Ti<sub>0.87</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti, Ta,S<sub>Ta<sub>0.13</sub>Ti<sub>0.87</sub>S<sub>2</sub>, S<sub>LiPSs</sub>, and Li respectively.</sub>



Figure S5. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.19</sub>Ti<sub>0.81</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti, Ta,S<sub>Ta<sub>0.19</sub>Ti<sub>0.81</sub>S<sub>2</sub>, S<sub>LiPSs</sub>, and Li respectively.</sub>



Figure S6. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.25</sub>Ti<sub>0.75</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti,



Figure S7. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.31</sub>Ti<sub>0.69</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti, Ta,S<sub>Ta<sub>0.31</sub>Ti<sub>0.69</sub>S<sub>2</sub>, S<sub>LiPSs</sub>, and Li respectively.</sub>



Figure S8. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.44</sub>Ti<sub>0.56</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti,



Figure S9. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.5</sub>Ti<sub>0.5</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti, Ta,S<sub>Ta<sub>0.5</sub>Ti<sub>0.5</sub>S<sub>2</sub>, S<sub>LiPSs</sub>, and Li respectively.</sub>



Figure S10. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.56</sub>Ti<sub>0.44</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti, Ta,S<sub>Ta<sub>0.56</sub>Ti<sub>0.44</sub>S<sub>2</sub>, S<sub>LiPSs</sub>, and Li respectively.</sub>



Figure S11. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.63</sub>Ti<sub>0.37</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti, Ta,S<sub>Ta<sub>0.63</sub>Ti<sub>0.37</sub>S<sub>2</sub>, S<sub>LiPSs</sub>, and Li respectively.</sub>



Figure S12. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.69</sub>Ti<sub>0.31</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti, Ta,S<sub>Ta<sub>0.69</sub>Ti<sub>0.31</sub>S<sub>2</sub>, S<sub>LiPSs</sub>, and Li respectively.</sub>



Figure S13. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.75</sub>Ti<sub>0.25</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti, Ta,S<sub>Ta<sub>0.75</sub>Ti<sub>0.25</sub>S<sub>2</sub>, S<sub>LiPSs</sub>, and Li respectively.</sub>



Figure S14. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.82</sub>Ti<sub>0.18</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti,



Figure S15. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.88</sub>Ti<sub>0.12</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti, Ta,S<sub>Ta<sub>0.88</sub>Ti<sub>0.12</sub>S<sub>2</sub>, S<sub>LiPSs</sub>, and Li respectively.</sub>



Figure S16. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.94</sub>Ti<sub>0.06</sub>S<sub>2</sub>. The grey, green, yellow brown and purple atom is Ti, Ta,S<sub>Ta<sub>0.94</sub>Ti<sub>0.06</sub>S<sub>2</sub>, S<sub>LiPSs</sub>, and Li respectively.</sub>



Figure S17. The top and side views of the most optimized geometric structures of (a)LiS (b)Li<sub>2</sub>S (c)Li<sub>2</sub>S<sub>2</sub>(d)Li<sub>3</sub>S<sub>2</sub> on TaS<sub>2</sub>. The green, yellow brown and purple atom is Ta,S<sub>TaS<sub>2</sub></sub>, S<sub>LiPSs</sub>, and Li respectively.

$X) = 2(0 - 1 - 1)^{1/2}$							
E <sub>a</sub> (eV)	LiS	Li <sub>2</sub> S	Li <sub>2</sub> S <sub>2</sub>	Li <sub>3</sub> S <sub>2</sub>			
TiS <sub>2</sub>	-2.51	-3.36	-2.03	-2.75			
Ta <sub>0.06</sub> Ti <sub>0.94</sub> S <sub>2</sub>	-2.41	-3.25	-1.92	-2.69			
Ta <sub>0.13</sub> Ti <sub>0.87</sub> S <sub>2</sub>	-2.41	3.12	-1.94	-2.64			
Ta <sub>0.19</sub> Ti <sub>0.81</sub> S <sub>2</sub>	-2.37	-3.05	-1.90	-2.59			
Ta <sub>0.25</sub> Ti <sub>0.75</sub> S <sub>2</sub>	-2.29	-3.04	-1.87	-2.56			
Ta <sub>0.31</sub> Ti <sub>0.69</sub> S <sub>2</sub>	-2.29	-2.99	-1.82	-2.53			
Ta <sub>0.38</sub> Ti <sub>0.62</sub> S <sub>2</sub>	-2.40	-3.14	-1.87	-2.58			
Ta <sub>0.44</sub> Ti <sub>0.56</sub> S <sub>2</sub>	-2.31	-3.00	-1.86	-2.51			
Ta <sub>0.5</sub> Ti <sub>0.5</sub> S <sub>2</sub>	-2.25	-2.94	-1.77	-2.57			
Ta <sub>0.56</sub> Ti <sub>0.44</sub> S <sub>2</sub>	-2.32	-3.01	-1.98	-2.85			
Ta <sub>0.63</sub> Ti <sub>0.37</sub> S <sub>2</sub>	-2.32	-3.05	-1.95	-2.60			
Ta <sub>0.69</sub> Ti <sub>0.31</sub> S <sub>2</sub>	-2.34	-3.04	-1.88	-2.53			
Ta <sub>0.75</sub> Ti <sub>0.25</sub> S <sub>2</sub>	-2.38	-3.17	-1.89	-2.59			
Ta <sub>0.82</sub> Ti <sub>0.18</sub> S <sub>2</sub>	-2.31	-2.93	-1.79	-2.48			
Ta <sub>0.88</sub> Ti <sub>0.12</sub> S <sub>2</sub>	-2.38	-3.07	-1.90	-2.55			
Ta <sub>0.94</sub> Ti <sub>0.06</sub> S <sub>2</sub>	-2.32	-2.99	-1.82	-2.48			
TaS <sub>2</sub>	-2.42	-2.96	-1.83	-2.48			

**Table S1**. The calculated adsorption energy (Ea) of LiS, Li2S, Li2S2 and Li3S2 on TaxTi(1-<br/>X)S2( $0 \le X \le 1$ ).

The closest distance(Å)	d <sub>LiS</sub>	$d_{Li_2S}$	$\mathbf{d}_{\mathrm{Li}_2\mathrm{S}_2}$	$d_{Li_3S_2}$
TiS <sub>2</sub>	2.46	2.42	2.57	2.53
Ta <sub>0.06</sub> Ti <sub>0.94</sub> S <sub>2</sub>	2.45	2.46	2.58	2.54
Ta <sub>0.13</sub> Ti <sub>0.87</sub> S <sub>2</sub>	2.44	2.45	2.57	2.54
Ta <sub>0.19</sub> Ti <sub>0.81</sub> S <sub>2</sub>	2.45	2.42	2.56	2.53
Ta <sub>0.25</sub> Ti <sub>0.75</sub> S <sub>2</sub>	2.44	2.45	2.57	2.54
Ta <sub>0.31</sub> Ti <sub>0.69</sub> S <sub>2</sub>	2.44	2.41	2.58	2.53
Ta <sub>0.38</sub> Ti <sub>0.62</sub> S <sub>2</sub>	2.45	2.41	2.56	2.53
Ta <sub>0.44</sub> Ti <sub>0.56</sub> S <sub>2</sub>	2.45	2.45	2.55	2.53
$Ta_{0.5}Ti_{0.5}S_2$	2.46	2.42	2.56	2.54
$Ta_{0.56}Ti_{0.44}S_2$	2.45	2.44	2.56	2.48
$Ta_{0.63}Ti_{0.37}S_2$	2.46	2.43	2.59	2.53
Ta <sub>0.69</sub> Ti <sub>0.31</sub> S <sub>2</sub>	2.45	2.41	2.55	2.52
Ta <sub>0.75</sub> Ti <sub>0.25</sub> S <sub>2</sub>	2.46	2.41	2.57	2.53
Ta <sub>0.82</sub> Ti <sub>0.18</sub> S <sub>2</sub>	2.46	2.41	2.55	2.52
$Ta_{0.88}Ti_{0.12}S_2$	2.46	2.41	2.54	2.52
Ta <sub>0.94</sub> Ti <sub>0.06</sub> S <sub>2</sub>	2.43	2.41	2.53	2.49
TaS <sub>2</sub>	2.45	2.42	2.47	2.49

Table S2. The closest distance of the Li atom to  $S_{Ta_XTi_{(1-X)}S_2}$  when LiS, Li<sub>2</sub>S, Li<sub>2</sub>S<sub>2</sub> and Li<sub>3</sub>S<sub>2</sub> on $Ta_XTi_{(1-X)}S_2(0 \le X \le 1)$ .

$\Delta e_{Ta_{X}Ti_{(1-X)}S_{2}} =$				
	$\mathbf{Q}_{LiS}$	$\mathbf{Q}_{Li_2S}$	$\mathbf{Q}_{Li_2S_2}$	$\mathbf{Q}_{\mathbf{Li}_3\mathbf{S}_2}$
$-\Delta e_{\text{LiPSs}}$				
TiS <sub>2</sub>	-0.54	-1.12	-1.01	-1.09
Ta0.06Ti0.94S2	-0.48	-1.21	-0.98	-1.09
Ta0.13Ti0.87S2	-0.53	-1.15	-0.98	-1.08
Ta0.19Ti0.81S2	-0.53	-1.12	-1.03	-1.07
Ta0.25Ti0.75S2	-0.50	-1.18	-0.95	-1.04
Ta0.31Ti0.69S2	-0.54	-1.21	-0.92	-1.04
Ta0.38Ti0.62S2	-0.46	-1.09	-0.97	-1.06
Ta0.44Ti0.56S2	-0.50	-1.17	-0.94	-1.03
Ta0.5Ti0.5S2	-0.50	-1.15	-0.93	-1.02
Ta0.56Ti0.44S2	-0.52	-1.12	-0.92	-1.45
Ta0.63Ti0.37S2	-0.48	-1.15	-0.90	-1.02
Ta0.69Ti0.31S2	-0.51	-1.14	-0.93	-1.02
Ta0.75Ti0.25S2	-0.46	-1.10	-0.92	-1.01
Ta0.82Ti0.18S2	-0.47	-1.16	-0.87	-0.99
Ta0.88Ti0.12S2	-0.50	-1.15	-0.90	-1.00
Ta0.94Ti0.06S2	-0.50	-1.13	-0.87	-0.98
TaS <sub>2</sub>	-0.47	-1.11	-0.83	-0.99

**Table S3.** Bader charge numbers indicate the number of electrons transferred from LiS, Li<sub>2</sub>S,Li<sub>2</sub>S<sub>2</sub> and Li<sub>3</sub>S<sub>2</sub> to TaxTi(1-x)S<sub>2</sub>( $0 \le X \le 1$ ). Here, Bader charge difference means the difference of<br/>charge value between adsorbed case and free-standing case.

$\Delta e_{Ta_{X}Ti_{(1-X)}S_{2}}$	Li	S	Li	₂S	Li <sub>2</sub>	S <sub>2</sub>	Li <sub>3</sub>	S₂
=-∆e <sub>LIPSs</sub>	∆e <sub>Li</sub>	Δe <sub>s</sub>	Δe <sub>Li</sub>	Δe <sub>s</sub>	∆e <sub>Li</sub>	∆e <sub>s</sub>	∆e <sub>Li</sub>	∆e <sub>s</sub>
TiS <sub>2</sub>	-0.87	0.33	-1.72	0.60	-1.73	0.72	-2.62	1.53
Ta0.06Ti0.94S2	-0.87	0.39	-1.74	0.53	-1.74	0.76	-2.63	1.54
Ta0.13Ti0.87S2	-0.87	0.34	-1.75	0.60	-1.74	0.76	-2.63	1.55
Ta0.19Ti0.81S2	-0.87	0.34	-1.74	0.62	-1.76	0.73	-2.62	1.55
Ta0.25Ti0.75S2	-0.88	0.38	-1.74	0.56	-1.74	0.79	-2.62	1.58
Ta0.31Ti0.69S2	-0.87	0.33	-1.74	0.53	-1.74	0.82	-2.63	1.59
Ta0.38Ti0.62S2	-0.87	0.41	-1.74	0.65	-1.74	0.77	-2.63	1.57
Ta0.44 Ti0.56 S2	-0.87	0.37	-1.74	0.57	-1.74	0.80	-2.62	1.59
Ta0.5Ti0.5S2	-0.88	0.38	-1.74	0.60	-1.74	0.81	-2.62	1.60
Ta0.56Ti0.44S2	-0.87	0.35	-1.74	0.62	-1.75	0.83	-2.61	1.16
Ta0.63Ti0.37S2	-0.87	0.39	-1.73	0.58	-1.74	0.84	-2.62	1.60
Ta0.69Ti0.31S2	-0.87	0.36	-1.74	0.60	-1.74	0.81	-2.62	1.60
Ta0.75Ti0.25S2	-0.87	0.41	-1.73	0.63	-1.74	0.82	-2.62	1.61
Ta0.82Ti0.18S2	-0.87	0.40	-1.74	0.57	-1.74	0.87	-2.62	1.63
Ta0.88Ti0.12S2	-0.87	0.37	-1.74	0.59	-1.74	0.84	-2.62	1.62
Ta0.94 Ti0.06 S2	-0.87	0.37	-1.73	0.60	-1.74	0.87	-2.62	1.64
TaS <sub>2</sub>	-0.87	0.40	-1.74	0.63	-1.74	0.91	-2.62	1.63

**Table S4.** Bader charge numbers indicate the number of electrons transferred from Li and S ofLiS, Li2S, Li2S2 and Li3S2 to TaxTi(1-x)S2(0 $\leq$ X $\leq$ 1). Here, Bader charge difference means thedifference of charge value between adsorbed case and free-standing case.

ΔWF(eV)		$\Delta WF_{Li_2S}$	$\Delta WF_{Li_2S_2}$	$\Delta WF_{Li_3S_2}$
TiS <sub>2</sub>	-0.27	-0.60	-0.19	-0.84
Ta <sub>0.06</sub> Ti <sub>0.94</sub> S <sub>2</sub>	-0.30	-0.57	-0.19	-0.84
Ta <sub>0.13</sub> Ti <sub>0.87</sub> S <sub>2</sub>	-0.27	-0.54	-0.19	-0.84
Ta <sub>0.19</sub> Ti <sub>0.81</sub> S <sub>2</sub>	-0.27	-0.54	-0.16	-0.82
Ta <sub>0.25</sub> Ti <sub>0.75</sub> S <sub>2</sub>	-0.27	-0.54	-0.16	-0.82
Ta <sub>0.31</sub> Ti <sub>0.69</sub> S <sub>2</sub>	-0.24	-0.49	-0.14	-0.82
Ta <sub>0.38</sub> Ti <sub>0.62</sub> S <sub>2</sub>	-0.14	-0.38	-0.08	-0.71
Ta <sub>0.44</sub> Ti <sub>0.56</sub> S <sub>2</sub>	-0.19	-0.41	-0.11	-0.79
Ta <sub>0.5</sub> Ti <sub>0.5</sub> S <sub>2</sub>	-0.22	-0.46	-0.11	-0.79
Ta <sub>0.56</sub> Ti <sub>0.44</sub> S <sub>2</sub>	-0.22	-0.41	-0.14	-0.19
Ta <sub>0.63</sub> Ti <sub>0.37</sub> S <sub>2</sub>	-0.14	-0.44	-0.11	-0.79
Ta <sub>0.69</sub> Ti <sub>0.31</sub> S <sub>2</sub>	-0.22	-0.47	-0.11	-0.76
Ta <sub>0.75</sub> Ti <sub>0.25</sub> S <sub>2</sub>	-0.19	-0.44	-0.08	-0.73
Ta <sub>0.82</sub> Ti <sub>0.18</sub> S <sub>2</sub>	-0.16	-0.46	-0.08	-0.73
Ta <sub>0.88</sub> Ti <sub>0.12</sub> S <sub>2</sub>	-0.14	-0.44	-0.05	-0.73
Ta <sub>0.94</sub> Ti <sub>0.06</sub> S <sub>2</sub>	-0.14	-0.41	-0.05	-0.73
TaS <sub>2</sub>	-0.16	-0.90	0	-0.68

 $\label{eq:s5} \begin{array}{l} \mbox{Table S5}. \ The \ difference \ of \ Work \ function(\ \Delta WF), \ when \ LiS, \ Li_2S, \ Li_2S_2 \ and \ Li_3S_2 \ absorb \ on \\ Ta_XTi_{(1-X)}S_2(0 \le X \le 1). \end{array}$ 





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 $\begin{array}{l} \label{eq:s18.coh} Figure \ S18. \ COHP \ diagram \ of \ the \ Li-S_{Ta_{X}Ti_{(1-X)}S_{2}} & (0 \leq X \leq 1) \ bond \ in(a) TiS_{2}, (b) Ta_{0.06}Ti_{0.94}S_{2}, \\ (c) Ta_{0.13}Ti_{0.87}S_{2}, (d) Ta_{0.19}Ti_{0.81}S_{2}, (e) Ta_{0.25}Ti_{0.75}S_{2}, (f) Ta_{0.31}Ti_{0.69}S_{2}, (g) Ta_{0.38}Ti_{0.62}S_{2}, (h) Ta_{0.44}Ti_{0.56}S_{2}, \\ (i) Ta_{0.5}Ti_{0.5}S_{2}, (j) Ta_{0.56}Ti_{0.44}S_{2}, (k) Ta_{0.63}Ti_{0.37}S_{2}, (l) Ta_{0.69}Ti_{0.31}S_{2}, (m) Ta_{0.75}Ti_{0.25}S_{2}, (n) Ta_{0.82}Ti_{0.18}S_{2}, \\ & (o) Ta_{0.88}Ti_{0.12}S_{2}, (p) Ta_{0.94}Ti_{0.06}S_{2}, \ and \ (q) TaS_{2}, \ respectively. \end{array}$ 



Figure S19. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on TiS<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S20. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.06</sub>Ti<sub>0.94</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S21. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.13</sub>Ti<sub>0.87</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S22. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.19</sub>Ti<sub>0.81</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S23. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.25</sub>Ti<sub>0.75</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S24.Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.31</sub>Ti<sub>0.69</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S25.Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.38</sub>Ti<sub>0.62</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S26. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.44</sub>Ti<sub>0.56</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S27. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.5</sub>Ti<sub>0.5</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S28. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.56</sub>Ti<sub>0.44</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S29. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.63</sub>Ti<sub>0.37</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S30. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.69</sub>Ti<sub>0.31</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S31. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.75</sub>Ti<sub>0.25</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S32. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.82</sub>Ti<sub>0.18</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S33. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.88</sub>Ti<sub>0.12</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S34. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on Ta<sub>0.94</sub>Ti<sub>0.06</sub>S<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S35. Side views of the Plane-averaged charge density difference between (a) LiS, (b) Li<sub>2</sub>S, (c) Li<sub>2</sub>S<sub>2</sub>, (d) Li<sub>3</sub>S<sub>2</sub> on TaS<sub>2</sub>. blue and pink represent charge depletion and gain, respectively.



Figure S36. (a) Total energy change of Ta<sub>0.38</sub>Ti<sub>0.62</sub>S<sub>2</sub> in AIMD simulations at lower and higher concentration Li<sup>+</sup>, (b) top and side views of Ta<sub>0.38</sub>Ti<sub>0.62</sub>S<sub>2</sub> forming Li<sub>3</sub>S<sub>2</sub> at 0.8 ps in lower concentration of Li<sup>+</sup>. Top and side views of the final structure in (c) lower and (d) higher concentration Li<sup>+</sup>.



Figure S37. The leave-one-out cross-validation of  $Ta_X Ti_{(1-X)}S_2 MSE$  and RMSE.

 Table S6. The correlation (Pearson correlation coefficient) between a single parameter and the Gibbs free energy of Pathways 1.

Pearse	on							
correl	ation WF	Δ	WF <sub>Li<sub>2</sub>S</sub>	$\Delta WF_{Li_2S_2}$	∆d-p	$\Delta X$	Ea <sub>Li<sub>2</sub>S</sub>	$Ea_{Li_2S_2}$
coeffi	cient							
ΔG1-]	P1 -0.5	7 -0.2	21	0.42	0.32	-0.57	0.81	0.42
	Pearson							
	correlation	$Q_{\text{Li}_2S}$	$Q_{\text{Li}_2 S_2}$	$d_{Li_2S} \\$	$d_{Li_2S_2} \\$	ICOHP <sub>Li2</sub> S	ICOHP <sub>Li2S</sub>	2
	coefficient							

0.09

-0.46

-0.45

-0.06

 $\Delta G1-P1$ 

-0.09

0.65