On the Electrostatic Boundary Effect: Key Influencing Factors and Underlying Mechanisms in Patterned Triboelectric Sensors

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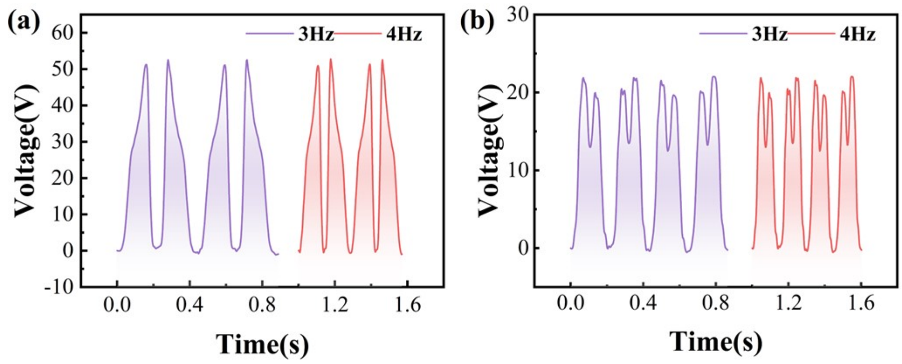

Fig.S1 Response curves of elliptical electrode and sine electrode at high driving frequencies

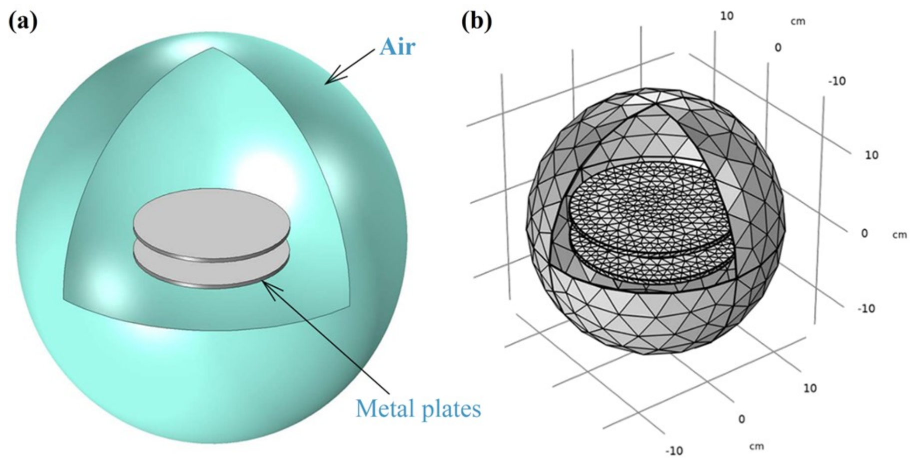

Fig.S2 Simulation modeling and meshing

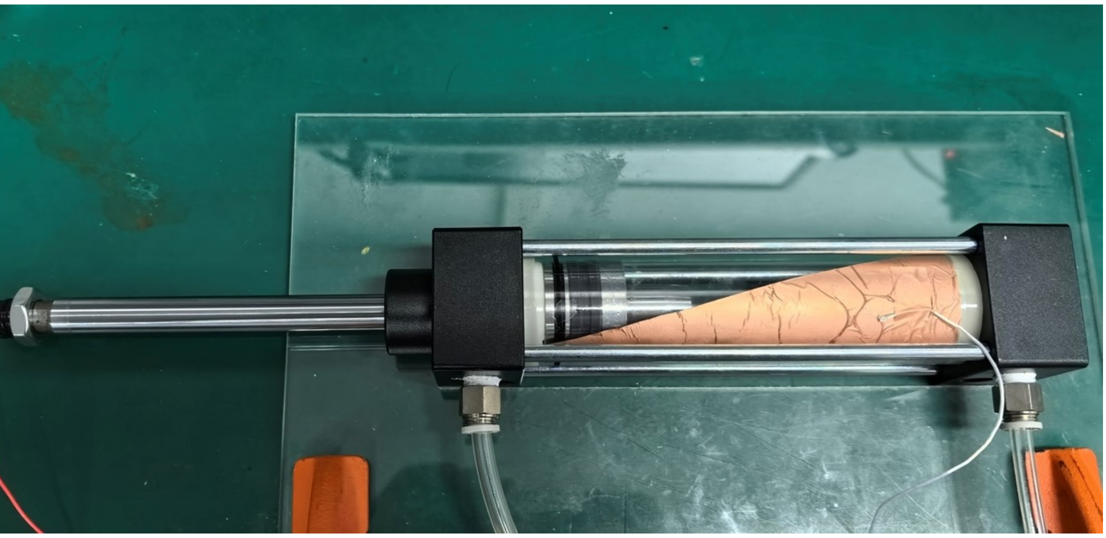

Fig.S3 Physical test diagram of the triangular electrode

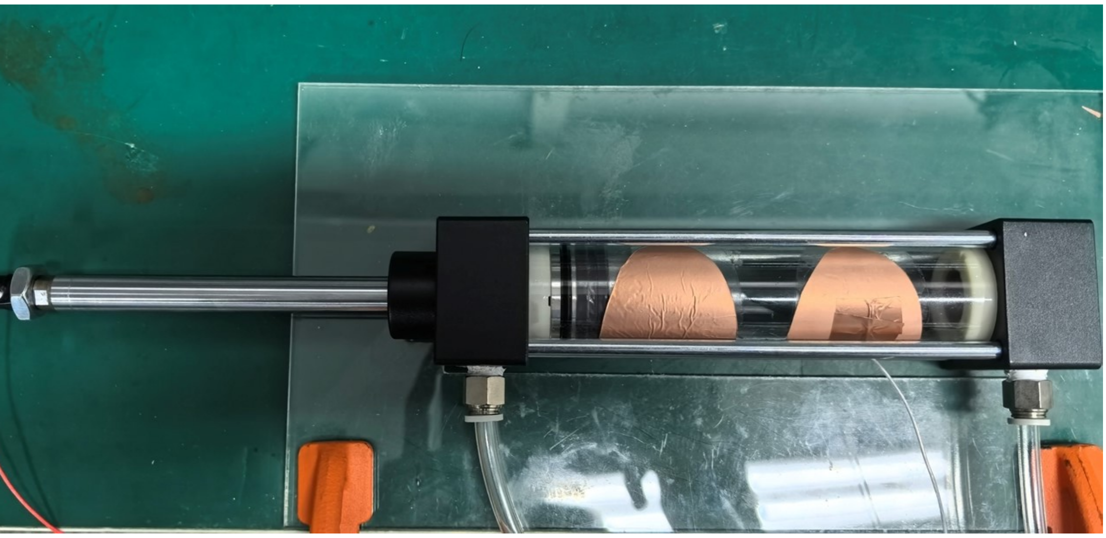

Fig.S4 Physical test diagram of the sine electrode

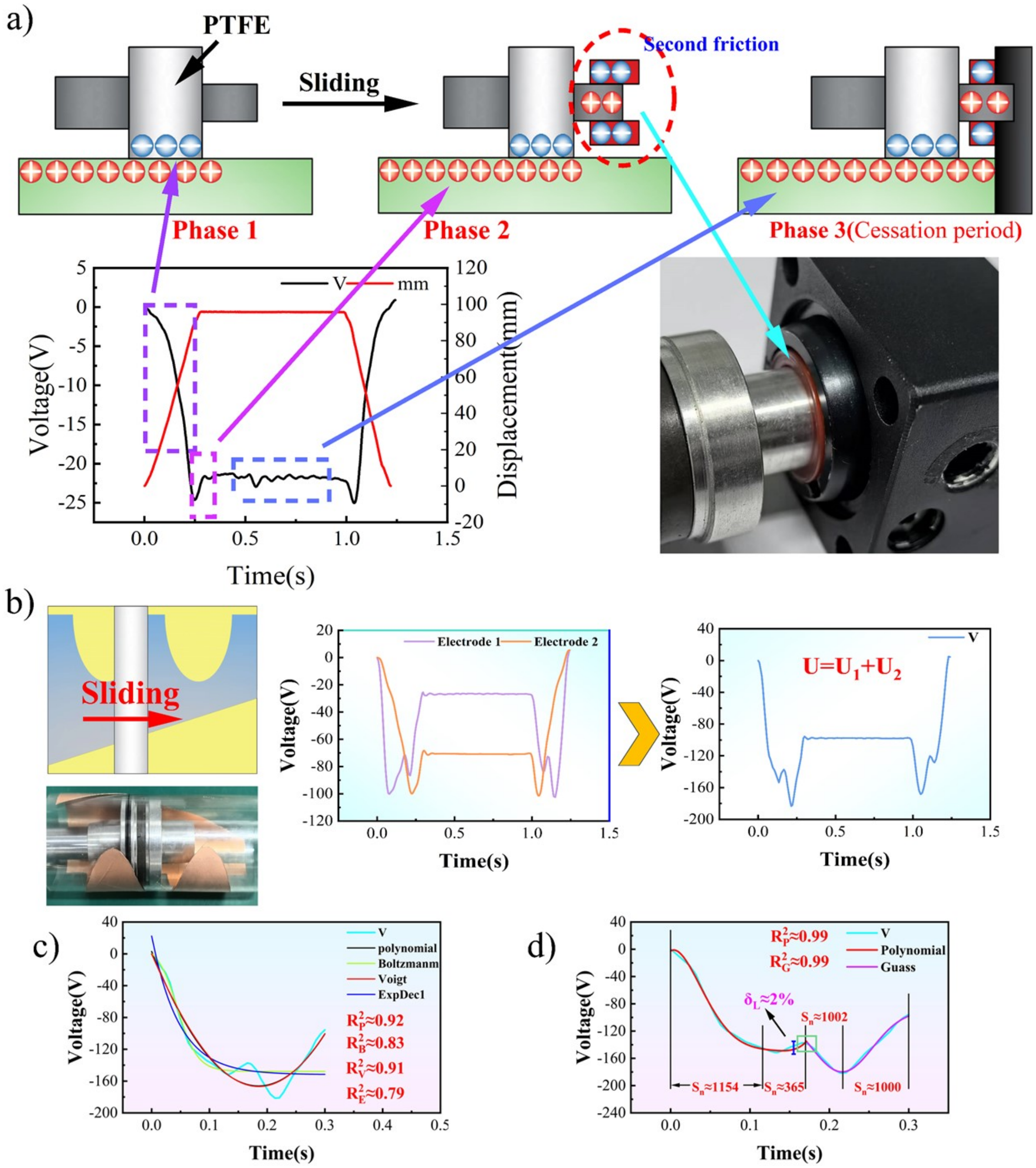

Fig.S5 Integrated pneumatic actuator displacement sensing test. (a) Working principle of triboelectric displacement sensor. (b) Dual-electrode test chart. (c) The response curve fits the result without segmentation. (d) The response curve fits the result with segmentation.

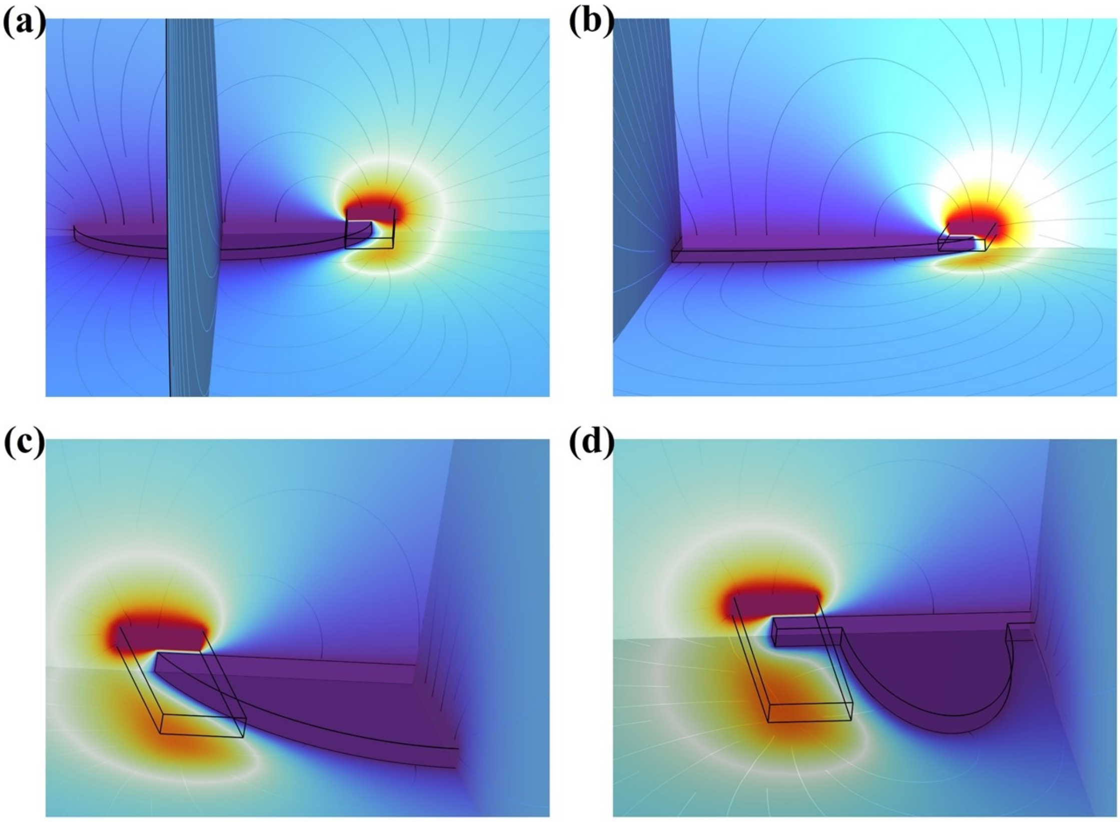

Fig.S6 3D display of the simulation model

Theoretical derivation part

In the electrostatic field case, where the electric field is independent of the magnetic field and time, Maxwell's system of equations simplifies to:

$$\nabla \cdot D = \rho \#(S1)$$

$$\nabla \times E = 0 \#(S2)$$

ρ: free charge; D: potential shift vector; E: spinless field.

It is evident that, due to the absence of rotation in the electrostatic field, the introduction of a scalar potential is a viable method of describing the electrostatic field. From the perspective of the work accomplished by the electric field force, the integral of the electrostatic field remains unaffected by the path selected, and is solely contingent upon the positions of the initial and terminal points.

$$\varphi_P = \int_P^Q E \cdot dl \#(S3)$$

Q: zero potential point; P: potential function.

When confronted with pragmatic challenges, the maximum ground surface is commonly regarded as the pivotal reference point for the potential. Conversely, within theoretical inquiries, if a charge is allocated within a circumscribed domain, then infinity can be designated as the reference point for the potential, thereby facilitating the expression of the potential at a specific field point P as follows:

$$\varphi_P = \int_P^{\infty} E \cdot dl \#(S4)$$

$$E = -\nabla \varphi \#(S5)$$

The introduction of the potential serves to facilitate a reduction in the complexity of the calculation of the vector field, whereby it is transformed into a scalar problem. In the context of a homogeneous, isotropic medium, it can be demonstrated that $D = \varepsilon E$. This relationship subsequently gives rise to the following equation:

$$\nabla^2 \varphi = -\frac{\rho}{\varepsilon} \#(S6)$$

It is evident that the issue of determining the electric field for a specified charge distribution can be reduced to the task of solving the Laplace equation for the electric potential.

The analysis of edge field effects (boundary effects) in triboelectric sensors necessitates the consideration of non-orthogonal slanted boundaries. The observed slanted geometries are attributed to the presence of friction surface structures or contact-separation modes. The boundaries are modelled as conducting wedges with electric potential V and aperture angle α. The analysis is centred on the distortion of electric fields in the proximity of wedge vertices. Triboelectric sensor electrodes frequently demonstrate macroscopic scale advantages or periodicity along specific directions (e.g. length). Assuming infinite extension along the z-axis (perpendicular to the analysis planes), the potential distribution can be reduced to a two-dimensional electrostatic problem. This approach facilitates the solution to Laplace's equation through the separation of variables. The method of separation of variables is utilised to solve for potential φ(ρ,∅) and electric field E(ρ,∅) in polar coordinates. The objective of this study is to provide a quantitative characterisation of electric field enhancement in triboelectric boundary effects. Setting φ(ρ,∅) = R(ρ)Φ(∅), we obtain:

$$\rho^2 \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} - v^2 R = 0 \#(S7)$$

$$\frac{d^2 \Phi}{d\emptyset^2} + v^2 \Phi = 0 \#(S8)$$

where: v is a positive real number or zero, and its general solution is of the form

$$\varphi(\rho,\emptyset) = \left(A_0 + B_0 ln\rho\right)\left(C_0 + D_0 \emptyset\right) + \sum_{V \neq 0} \left(A_v \rho^v + B_v \rho^{-v}\right)\left(C_v \cos v\emptyset + D_v s \sin v\emptyset\right) \#(S9)$$

Considering the boundary conditions: $\varphi|_{\rho \to 0} =$ finite value, we get $B_0 = B_v = 0$; $\Phi(0) = V$, which gives $A_0 C_0 = V$, $C_v = 0$; The coefficients in Equation (S9) (A_0, B_0, Av, Bv, Cv, Dv) are integral constants determined by specific boundary conditions, reflecting the contribution of fields of different frequency components to the total potential; and then using $\Phi(2\pi - \alpha) = 0$ ($D_v \neq 0$), thus:

$$v = \frac{n\pi}{(2\pi - \alpha)}, n = 1,2,3,... \#(S10)$$

Finally, the solution is of the form:

$$\varphi(\rho,\emptyset) = V + \sum_{n=1}^{\infty} A_n \rho^{\frac{n\pi}{2\pi - \alpha}} \sin \frac{n\pi}{2n - \alpha} \emptyset \#(S11)$$

Where: A_n is the amount to be quantified.

In the vicinity of the splitting angle, $\rho \to 0$, thereby signifying that the contribution to the summation is predominantly derived from n = 1.

$$\varphi(\rho,\emptyset) \approx V + A_1\rho^{\frac{1}{2-\frac{\alpha}{\pi}}}\sin\frac{1}{2-\frac{\alpha}{\pi}}\emptyset \#(S12)$$

The corresponding electric field strength is:

$$E_\rho = -\frac{\partial\varphi}{\partial\rho} \approx -A_1\frac{1}{2-\frac{\alpha}{\pi}}\rho^{-\frac{1+\frac{\alpha}{\pi}}{2-\frac{\alpha}{\pi}}}\sin\frac{1}{2-\frac{\alpha}{\pi}}\emptyset \#(S13)$$

$$E_\emptyset = -\frac{1\partial\varphi}{\rho\partial\rho} \approx -A_1\frac{1}{2-\frac{\alpha}{\pi}}\rho^{-\frac{1+\frac{\alpha}{\pi}}{2-\frac{\alpha}{\pi}}}\cos\frac{1}{2-\frac{\alpha}{\pi}}\emptyset \#(S14)$$

The charge density is:

$$\rho_S = \varepsilon_0 E_n = \begin{cases} -\varepsilon_0 A_1\frac{1}{2-\frac{\alpha}{\pi}}\rho^{-\frac{1+\frac{\alpha}{\pi}}{2-\frac{\alpha}{\pi}}},\ \emptyset = 0 \\ -\varepsilon_0 A_1\frac{1}{2-\frac{\alpha}{\pi}}\rho^{-\frac{1+\frac{\alpha}{\pi}}{2-\frac{\alpha}{\pi}}}, \emptyset = 2\pi - \alpha \end{cases} \#(S15)$$

In the limit as $\alpha \ll 1$, $\rho_S \propto \rho^{-\frac{1}{2}}$. This indicates that the charge density and the electric field strength are significantly elevated in proximity to the cleavage tip.

The method of images has been demonstrated to be an effective solution to electrostatic boundary value problems. Within the domain of triboelectric sensors, a pivotal element of research concerns the analysis of the impact specific boundaries, notably those constituted by conductor surfaces, exert upon the fields of tribocharge. The following hypothesis is to be considered: what would be the outcome of a tribocharge occurring in the proximity of an infinite grounded conductor? The interaction of this charge's field with the conductor gives rise to the induction of surface charge distributions. It has been demonstrated that the total spatial field combines contributions from both the original tribocharge and induced surface charges. The fundamental principle of the method is as follows: The complex induced charge distribution is to be replaced by fictitious image charges outside the boundary. It is demonstrated that the aforementioned substitution fulfils the conditions that must be met by conductors, whilst concomitantly ensuring that equivalent external forces are maintained. It has been demonstrated that image charges can be effective in satisfying surface boundary conditions, albeit with a qualitative outcome. These models facilitate the transformation of complex boundary problems, particularly those involving conductor-dielectric interfaces, into simplified systems of point charges within homogeneous media.

The analysis of triboelectricity frequently entails comparatively elementary charge distributions (surface charge layers or point charges). However, the presence of nearby conductive or dielectric boundaries (e.g. sensor electrodes) has been shown to have a significant impact on the resulting electric fields. The development of effective solutions for such complex boundary problems represents a significant challenge. The method of images provides a powerful simplification tool. The model effectively addresses boundary-induced field distortions, including edge field enhancement. This approach provides a theoretical foundation for the design and performance prediction of triboelectric sensors.

The analysis of electric fields in triboelectric sensors frequently employs the method of images. This method is employed to address conductor boundary effects in the vicinity of tribocharge sources, which are modelled as point charges Q. It is imperative to consider the following hypothesis: that of a point charge Q situated above an infinite grounded conductor plane. The charge is located at a height *h* in a homogeneous dielectric medium (ε). The medium may take the form of either air or packaging material. The method involves the placement of a fictitious image charge, designated Q', with a magnitude of -Q, at a point z* that corresponds to -h*. This position is reflected by Q symmetrically below the conductor plane. The removal of the conductor plane, with the subsequent filling of all space with the original dielectric, serves to simplify analysis. The potential at any point P above the plane is equivalent to the sum of the potentials from Q and Q'. A Cartesian coordinate system facilitates quantitative analysis: The conductor plane is defined by the xy-plane; charge Q lies on the positive z-axis; and the potential reference is given by the equation: $\varphi|_{\infty} = 0$.

$$\varphi = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{L} - \frac{1}{L'}\right),\ z \geq 0 \#(S16)$$

$$L = \sqrt{x^2 + y^2 + (z - h)^2} \#(S17)$$

$$L' = \sqrt{x^2 + y^2 + (z + h)^2} \#(S18)$$

Where L and L' are the distances of the point charge Q and its mirror charge Q' to the field point P, respectively.
The fixed solution problem after using the mirror method:

$$\begin{cases} \nabla^2 \varphi = -\frac{Q}{\varepsilon}\delta(z - h), z > 0 \\ \varphi|_{z=0} = \frac{Q}{4\pi\varepsilon\sqrt{x^2 + y^2 + h^2}} - \frac{Q}{4\pi\varepsilon\sqrt{x^2 + y^2 + h^2}} = 0 \end{cases} \#(S19)$$

It has been demonstrated that the initial problem and the problem of the fixed solution when employing the mirror method are precisely equivalent, in accordance with the established uniqueness theorem.
The electric field strength at any point is:

$$E_n = -\frac{Q}{2\pi\varepsilon(x^2 + y^2 + h^2)^{\frac{3}{2}}} e_z \#(S20)$$

The free charge density is:

$$\rho_S = \varepsilon E_n = -\frac{Q}{2\pi(x^2 + y^2 + h^2)^{\frac{3}{2}}} \#(S21)$$

The total induced charge is:

$$Q' = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{Q}{2\pi(x^2 + y^2 + h^2)^{\frac{3}{2}}} dxdy = -Q \#(S22)$$

Coulomb force between Q and the plane of the conductor:

$$F = \frac{Q(-Q)}{4\pi\varepsilon(2h)^2} e_z = -\frac{Q^2}{16\pi\varepsilon h^2} e_z \#(S23)$$

The calculation of the field strength at any point in the angular domain can be achieved by extending this to infinite conductor planes intersecting at α = π/N.