

**Mechanisms of Strength, Thermal Stability, and Doping Effects in Metal-Doped
Bilayer Borophene for Energy Storage Applications Using a DFT-Trained Deep-
Learning Potential**

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Table S1. DFT simulation parameters used in Quantum ESPRESSO for bilayer borophene.

Parameter type	Setting or value
Calculation type	NPT or SCF
Pressure (Kbar)	0
Temperature (K)	50~1000 K
Number of ionic steps	200
time step size (fs)	1
Kinetic energy cutoff (Ry) for wavefunctions	40
Kinetic energy cutoff (Ry) for charge density and potential	320
Convergence criterion of self-consistent calculation, (Ry)	2×10^{-5}
Pseudopotential (version)	SSSP efficiency (V1.30)
K-point setting level using Materials Cloud	Normal

Table S2. Parameter settings for DeePMD DLP training

Parameter	Value
model descriptor	se_a
rcut, Å	6
rsmth, Å	5.8
embedding neural network	{25, 50, 100}
fitting neural network	{240, 240, 240}
DPGEN models	4
training step for label process/final training	1*10 ⁶ /2.5*10 ⁶
p_e^{start} (starting energy weight)	1
p_e^{limit} (final energy weight)	1
p_f^{start} (starting force weight)	1000
p_f^{limit} (final force weight)	1

Table S3. Atomic structures employed in the DFT calculations at 50 K and 300 K. Since reference b does not provide explicit structure names, the borophene configurations shown in the corresponding figure are adopted.

Structure	Structural Type	Atoms
β 12 borophene ^a		15
x3 bigophone	Single-layer borophene	20
striped borophene ^a		16
Borophene(Fig.1a) ^b		32
Borophene(Fig.1c) ^b		16
Borophene(Fig.1e) ^b		16
Borophene(Fig.1f) ^b		8
Borophene(Fig.5i) ^b		30
Borophene(Fig.5j) ^b		60
Borophene(Fig.5k) ^b	Bilayer Borophene	60
Borophene(Fig.5m) ^b		62
Borophene(Fig.5n) ^b		14
Borophene(Fig.5o) ^b		28
Borophene(Fig.5p) ^b		28
Borophene(Fig.5s) ^b		56
Borophene(Fig.5t) ^b		16

a: Peng, Bo, et al. Stability and strength of atomically thin borophene from first principles calculations. *Materials Research Letters*, 2017, 5.6: 399-407.

b: Gao, Nan, et al. Structure and stability of bilayer borophene: The roles of hexagonal holes and interlayer bonding. *FlatChem*, 2018, 7: 48-54.

Table S4. Structures used in the DFT calculations. The simulations were performed at 300 K.

Structure	Structural Type	Atoms
p6m2 borophene ^a		64
p6m2 borophene ^a		64
p6m2 borophene ^a		16
p6m2 borophene ^a	Bilayer Borophene	16
p6mmm borophene ^a		18
p6m2 borophene ^a		16
p6mmm borophene ^a		18

^aGao, Nan, et al. Density functional theory study of bilayer borophene-based anode material for rechargeable lithium ion batteries. *Langmuir*, 2023, 39.29: 10270-10279.

Table S5. Structures used in the DFT calculations. The simulations were performed at five different temperatures: 300 K, 400 K, 500 K, 600 K and 1000 K.

Structure	Structural Type	Atoms
p6m2 borophene (Al doping)	Metal-doped bilayer	64
p6m2 borophene (Ru doping)		64
p6mmm borophene (Al doping)		72
p6mmm borophene (Ru doping)		72

Table S6. Structures used in the DFT calculations. The simulations were performed at six different temperatures: 50 K, 300 K, 400 K, 500 K, 600 K and 1000 K.

Formula	Structure	Atoms
p6m2 borophene ^c	Bilayer Borophene	16
p6mmm borophene ^c		18

Table S7. Configuration-averaged nearest-neighbor bond lengths for B–B, B–Al, and B–Ru pairs in pristine and doped p6m2 and p6mmm bilayers. Values are obtained from relaxed structures by averaging over all first-neighbor pairs. $\Delta L(\text{B-X})$ denotes the bond-length mismatch, defined as the difference between the dopant–boron bond length (B–Al or B–Ru) and the average B–B bond length in the same configuration.

Configuration	Average bond length (Å)			$\Delta L(\text{B-X})^a$ (Å)
	B-B	B-Al	B-Ru	
p6m2 (pristine)	1.7384	—	—	—
p6m2-Al(0.8 at.%)	1.7340	2.1566	—	+0.4226
p6m2-Al(1.6 at.%)	1.7324	2.1560	—	+0.4236
p6m2-Ru(0.8 at.%)	1.7333	—	2.1642	+0.4309
p6m2-Ru(1.6 at.%)	1.7329	—	2.1622	+0.4293
p6mmm (pristine)	1.8399	—	—	—
p6mmm-Al(1.05 at.%)	1.8002	2.1553	—	+0.3551
p6mmm-Al(2.1 at.%)	1.7929	2.1562	—	+0.3633
p6mmm-Ru(1.05 at.%)	1.8017	—	2.1649	+0.3632
p6mmm-Ru(2.1 at.%)	1.7957	—	2.1629	+0.3672

Table S8. Interlayer bond dissociation energies for pristine and doped p6m2 and p6mmm bilayers, evaluated from the energy difference between the intact bilayer and two isolated single layers separated by a distance much larger than the DLP cutoff. For each configuration, the total energy difference ΔE between the bound bilayer and the non-interacting monolayers is divided by the number of CN7-mediated interlayer bonds N_{bond} to obtain the per-bond dissociation energy $E_{\text{diss}}^{(\text{per CN7})}$. The table lists the total number of interlayer bonds, the numbers of B–X and B–B interlayer bonds, and the corresponding $E_{\text{diss}}^{(\text{per CN7})}$ values.

Configuration	interlayer bonds	B-X(Al/Ru) bonds	B-B bonds	$E_{\text{diss}}^{(\text{per CN7})}$ (eV/bonds)
pristine_p6m2	1260	0	1260	0.94
p6m2-Al(0.8 at.%)	1260	90	1170	0.89
p6m2-Al(1.6 at.%)	1260	180	1080	0.89
p6m2-Ru(0.8 at.%)	1260	90	1170	0.84
p6m2-Ru(1.6 at.%)	1260	180	1080	0.80
pristine_p6mmm	4322	0	4322	1.29
p6mmm-Al(1.05 at.%)	4322	144	4178	1.28
p6mmm-Al(2.1 at.%)	4322	288	4034	1.23

p6mmm-Ru(1.05 at.%)	4322	144	4178	1.03
p6mmm-Ru(2.1 at.%)	4322	288	4034	0.73

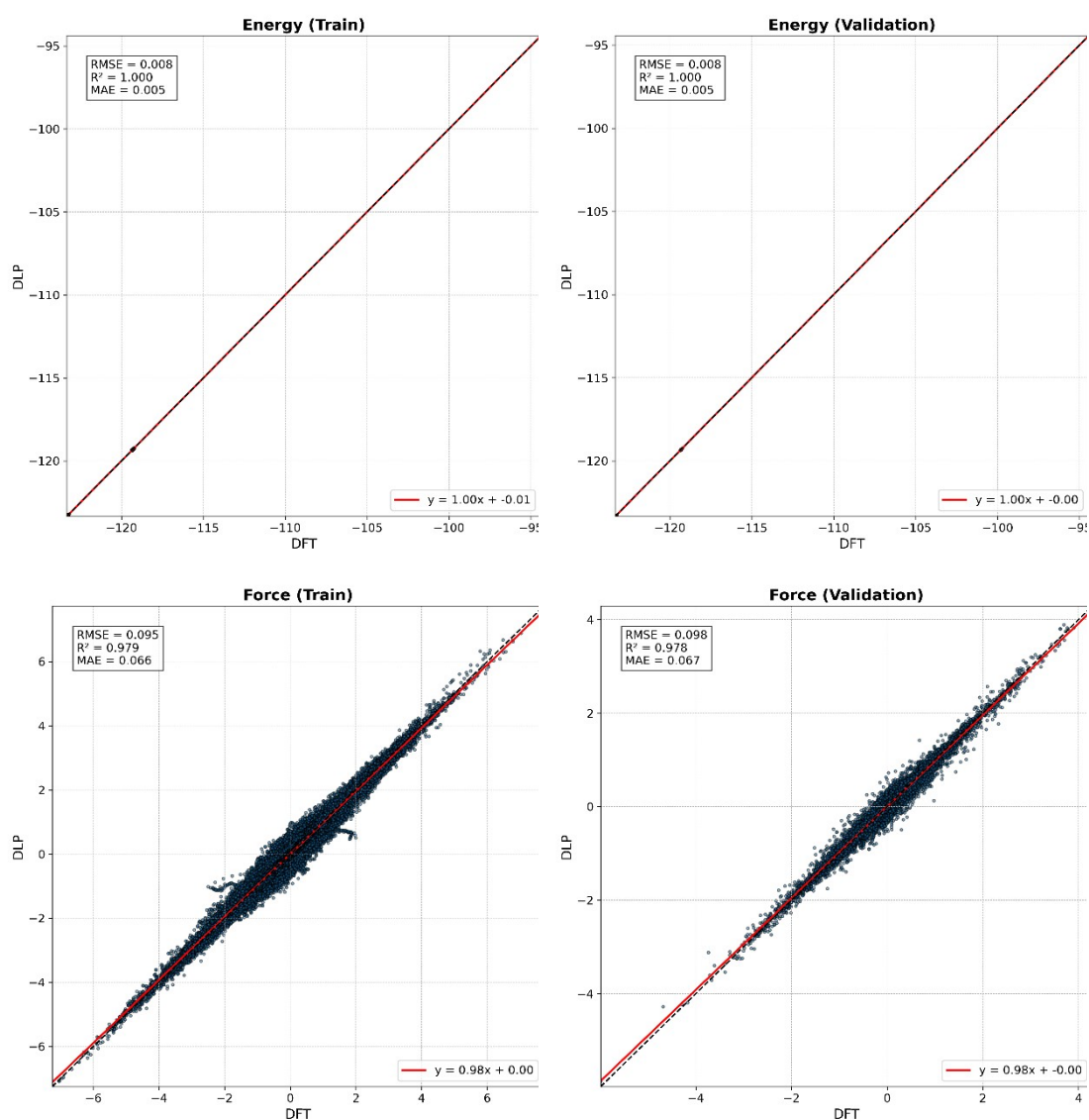
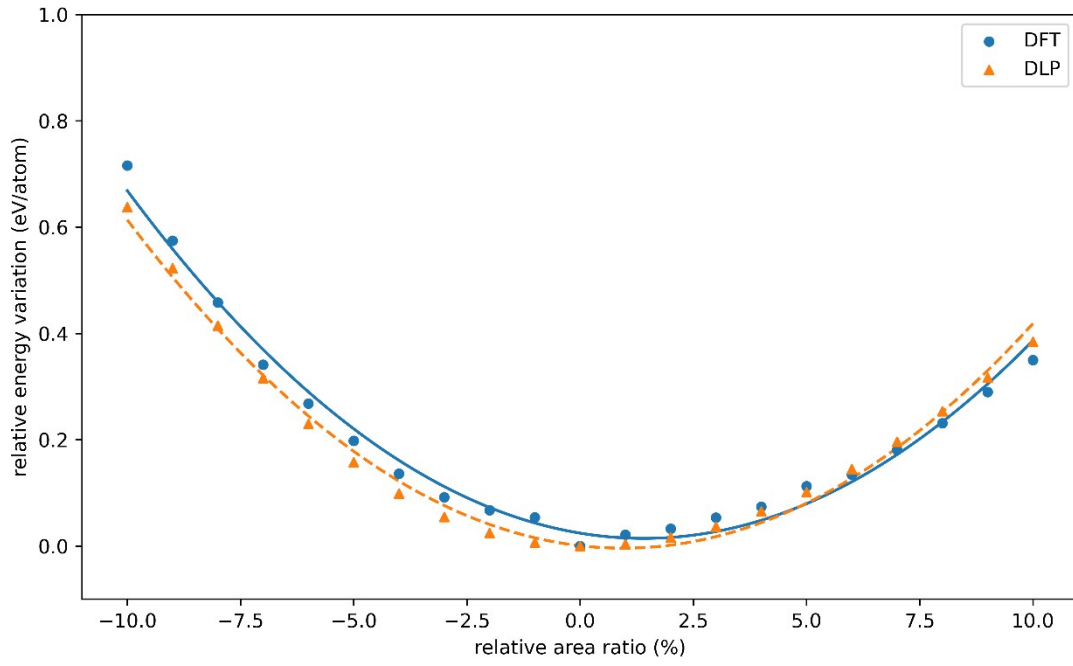
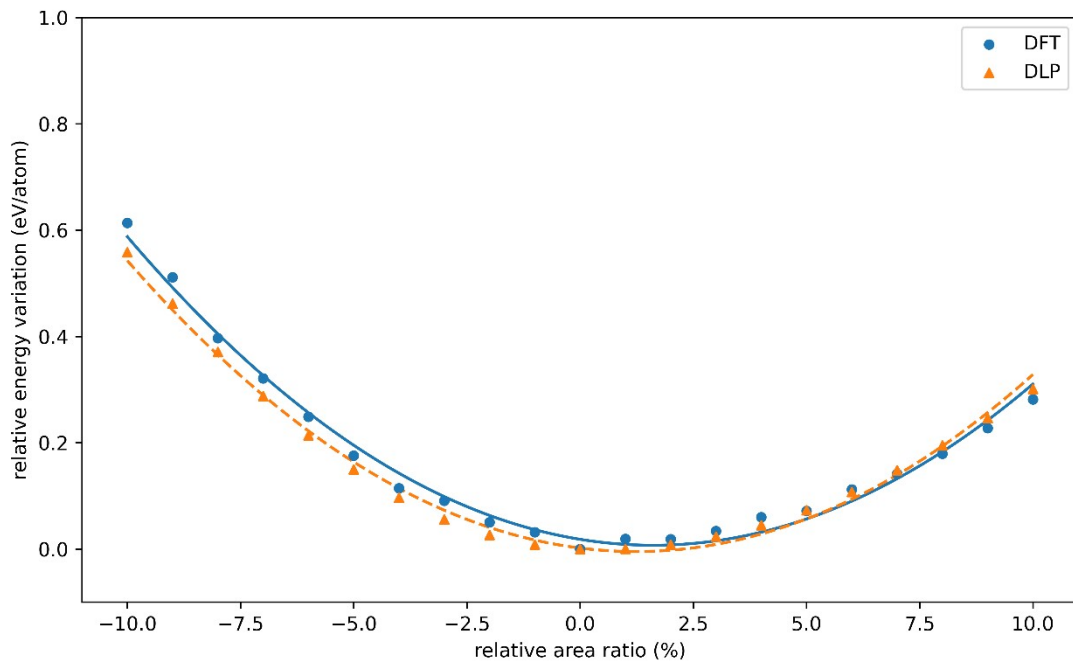


Figure S1. Correlation between DLP and DFT predictions for system energies and atomic forces. Results are shown for the training set (95%) and validation set (5%). Black lines represent the ideal $y = x$ correlation, while red lines indicate linear regression fits. Energy is in eV/atom and force in eV/Å.



(a)



(b)

Figure S2. Energy-area curves for bilayer borophene structures calculated using DLP and DFT: (a)p6m2-Al(0.8 at.%) and (b)p6m2-Ru(0.8 at.%). The relaxed configurations at 0% strain serve as reference states. Structures were uniformly scaled in both x and y directions.