

## Supplementary Information

### Tuning the magnetic state and topological transition of monolayer Kagome $\text{Co}_3\text{Pb}_3\text{SSe}$ with large magnetic Anisotropy

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The contribution of the orbit affects the magnetic anisotropy through spin-orbit coupling, the MAE having the same spin coupling upward (uu) or downward (dd) is expressed as

$$MAE_{uu/dd} = E_z - E_x = \xi^2 \sum_{ou} \frac{|\langle o|L_x|u\rangle|^2 - |\langle o|L_z|u\rangle|^2}{E_u - E_o} \quad (\text{S1})$$

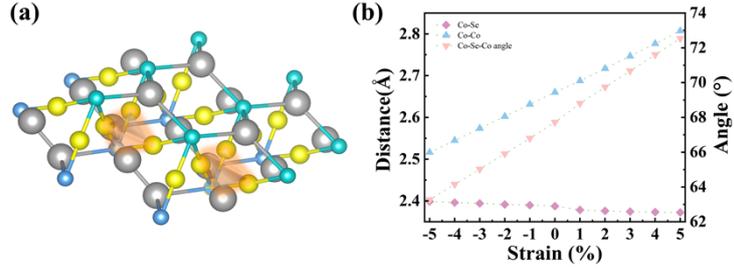
where  $\langle o|$  ( $|u\rangle$ ) donates the occupied(unoccupied) states,  $\xi^2$  represents the spin-orbit coupling constant.  $L_x$  and  $L_z$  are the x-component and z-component of angular momentum operators. When having opposite spin (ud) coupling, the MAE is expressed as

$$MAE_{ud} = E_z - E_x = \xi^2 \sum_{ou} \frac{|\langle o|L_z|u\rangle|^2 - |\langle o|L_x|u\rangle|^2}{E_u - E_o} \quad (\text{S2})$$

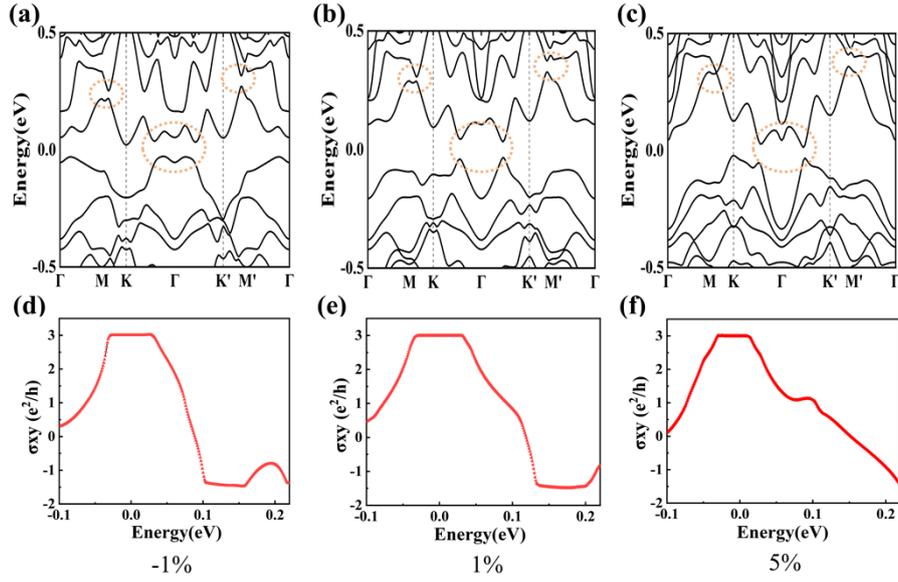
The orbital angular momentum with  $d_{z^2}, d_{xy}, d_{x^2-y^2}, d_{xz}, d_{yz}$  and  $p_x, p_y, p_z$  as the basis group are respectively expressed as

$$L_z = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2i & 0 & 0 \\ 0 & -2i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & i & 0 \end{bmatrix} \quad L_x = \begin{bmatrix} 0 & 0 & 0 & 0 & \sqrt{3}i \\ 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & i & 0 \\ 0 & i & 0 & 0 & 0 \\ -\sqrt{3}i & 0 & -i & 0 & 0 \end{bmatrix}$$

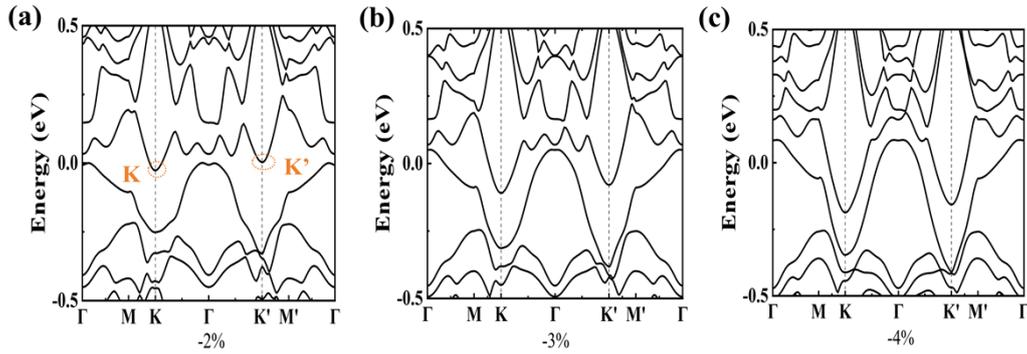
$$L_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2i \\ 0 & 2i & 0 \end{bmatrix} \quad L_x = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (\text{S3})$$



**Fig. S1.** (a) the monolayer  $\text{Co}_3\text{Pb}_3\text{SSe}$  structure considering magnetic interactions, with yellow representing three Co atoms forming a triangle; bond length and bond Angle as functions of applied biaxial strain.



**Fig. S2.** Energy band structures with SOC of  $\text{Co}_3\text{Pb}_3\text{SSe}$  monolayer under different strains.(a)-1%, (b)1%, (c)5%. (d)-(f) The corresponding anomalous Hall conductance,respectively.



**Fig. S3.** Energy band structures with SOC of  $\text{Co}_3\text{Pb}_3\text{SSe}$  monolayer under different strains.(a)-2%, (b)-3%, (c)-4%. All are metallic state.