Supplementary Information

A.1: Criticality indicators:

Details on the different considered criticality criteria are available in the SI of the ref. [1]. All criticality related indicators have been calculated using Eq. 1:

$$Cr_{CAM} = \frac{1}{M_C} \sum_i x_i \, I_{i,j} M_i \frac{1}{Q_{th}} \qquad (1)$$

Where

M_C=molar mass of final product

M_C=molar mass of raw material

x_i=molar stoichiometry

$I_{i,j}$=Criticality indicator for considered raw material

Q_{th}=theoretical energy density

Results have been rescaled to 1-100 using according to the minimum and maximum values provided in the different studies to make them comparable as shown via Eq. 2 and Eq. 3. The Normalization depends on min or max orientations e.g. import reliance should be minimized, whilst eof of life return rate should be maximized:

$$Cre, up = \frac{max_{re} - min_{re}}{max_{base} - min_{base}} * \left(x - max_{based}\right) + max_{new} \qquad (2)$$

or

$$Cre, down = \frac{max_{re} - min_{re}}{max_{base} - min_{base}} * \left(x - min_{based}\right) + min_{new} \qquad (3)$$

All numeric results for the normalization are provided in the SI of the ref. [1].

A.2: Carbon Footprint:

The carbon footprint calculation is based on an LCA-approach to calculate global warming potentials (GWP_{100}) using ecoinvent database version 3.8. The impacts for the market mix of each material precursor are calculated using the Environmental Footprint (EF) 3.0 methodology as recommended by the EC [1]. Material precursors including all upstream processes and the impacts of material synthesis are considered here. Synthesis steps are assumed to take place in Europe using an average electricity mix (heat and electricity). Infrastructure and possible auxiliary inputs, are disregarding as there is no information available on specific requirements of all considered CAM, and the impact of auxiliary input for CAM synthesis is typically negligible. A simplified system boundary is provided in Figure A1.

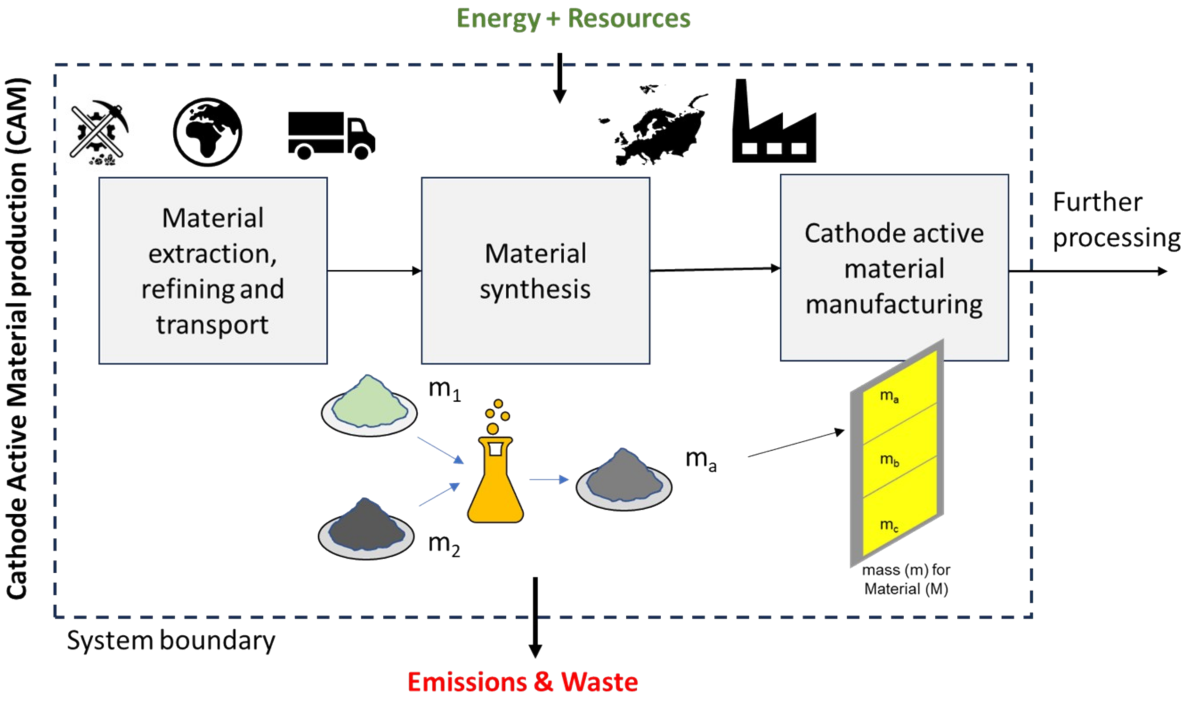

Figure A 1:Simple system boundary scheme for CAM production of different sodium ion batteries in frame of the LCA oriented assessment

A. 3: HFL-MCDM methodology:

The working principle of HFLTS is as follows:

Step 1: X is a set. HFS is a function which is a subset of [0, 1] and is represented as Eq. 4.

$$E = \{ < x, h_E\,(x) > \,|\, x \in X \} \tag{4}$$

A set of membership functions (n) is defined as $M = \{ \mu_1, \mu_2, \mu_3, \ldots, \mu_n \}$. The HFS, h_m, is associated with M and described as

$$h_M : M \rightarrow [0, 1] \tag{5}$$

$$h_M\,(x) = \bigcup_{\mu \in M} \{\mu\,(x)\} \tag{6}$$

A set of linguistic terms is defined as S, S = {S_0, S_1, …, S_g}. An H_s is a subset of the sequential linguistic terms of S.

The lower and upper bounds of HFLTS are H_s^- and H_s^+ respectively and is expressed as Eqs. 7 and 8:

$$H_S^- = \min\left(S_i\right) = s_j, s_i \in \; H_s \; and \; s_i \le s_j \forall_i \tag{7}$$

$$H_S^+ = \min\left(S_i\right) = s_j, s_i \in \; H_s \; and \; s_i \le s_j \forall_i \tag{8}$$

The function E_{GH} that transforms the words in HFLTS, H_S. E_{GH} utilises the linguistic terms set in S is an out-of-context grammar and S_{ll} is the expression generated by Eq. 9:

$$env\left(H_s\right) = \left[H_S^-, H_S^+\right], H_S^- \le \; H_S^+ \tag{9}$$

The key benefits of these techniques include allowing decision-makers to express their choices more easily, especially in situations involving uncertainty or hesitation, through the use of linguistic term sets. Additionally, the model offers a wide range of linguistic expression options due to its high flexibility. It also supports the natural adoption of expressions while preserving their uniqueness. For these reasons, this method is particularly valuable when multiple factors need to be considered [2–6].

A.4: HFL-TOPSIS:

This approach also shows similar characteristics to the proposed methodology. Thus, this method is used to validate the rank of the alternative based on the calculated weights of the criteria. This approach is a distance-based MCDM and calculates the alternatives from the ideal solution [7]. In this method the positive and negative ideal solutions are estimated depending on the criterion type. Then from the positive and negative ideal solutions, the distances (Euclidean distance) of each alternative are measured. By using these values, the relative closeness is identified and with the highest closeness value, the ranking has been done. In this work, the TOPSIS method is integrated with HFLTS to overcome the limitations of linguistic uncertainties. The scale is shown in Table A.3 [8]. The steps of this method are as follows [9].

Step 1: The positive ideal solution (PIS) and negative ideal solution (NIS) are estimated by using Eqs. 10 and 11.

$$A^* = \left(h_1^*, h_2^*, \ldots, h_n^*\right), where\; h_j^* = \; U_{i=1}^{m} h_{ij} \tag{10}$$

$$A^{-} = \left(h_{1}^{-}, h_{2}^{-}, \dots, h_{n}^{-}\right), where\ h_{j}^{-} = \sqcap_{i=1}^{m} h_{ij} \tag{11}$$

Step 2: The distance from the ideal solution of each value is estimated by using Eqs. 12 and 13. The hesitant normalised Euclidean distance is proposed for this purpose.

$$D_{i}^{*} = \sum_{j=1}^{n} \| h_{ij} - h_{j}^{*} \| * W_{j} \tag{12}$$

$$D_{i}^{-} = \sum_{j=1}^{n} \| h_{ij} - h_{j}^{-} \| * W_{j} \tag{13}$$

Where W_j is the crisp value of the weights of the criterion. It is calculated by using the HFL-AHP method.

Step 3: The relative closeness index is calculated and the rank of the alternatives are determined. The highest relative closeness value is considered as the best alternative solution. To calculate the closeness index Eq. 14 is considered.

$$C_{i} = \frac{D_{i}^{-}}{D_{i}^{*} + D_{i}^{-}} \tag{14}$$

Table A. 1: HFL-TOPSIS for ranking [8]

Linguistic Variable	Abb.	Fuzzy number
Highly unsatisfied	HU	(1,1,3)
Unsatisfied	U	(1,3,5)
Fair	F	(3,5,7)
Satisfied	S	(5,7,9)
Highly satisfied	HS	(7,9,9)

B. Results:

B. 1: Assessment without toxicity:

Table A.2: Crisp values from NEAT-Fuzzy PROMETHEE without toxicity

Alternatives		Crisp phi plus	Rank	Crisp phi plus	Crisp phi minus
NMC 111	A1	-0.4839	24	0.2414	0.7253
NMC 622	A2	-0.2727	19	0.3329	0.6056

NaMnO2	A3	0.0701	13	0.4479	0.3779
Na0.67Mn0.95Mg0.05O2	A4	-0.1095	16	0.3619	0.4715
NaMn0.5Fe0.5O2	A5	-0.3547	22	0.2374	0.5921
NaNi0.5Mn0.5O2	A6	-0.2467	18	0.3154	0.5618
Na0.9[Mn0.4Fe0.5Ti0.1]O2	A7	-0.3359	21	0.24	0.5758
NaMn0.33Fe0.33Ni0.33O2	A8	0.2305	9	0.5098	0.2793
Na0.6Fe0.11Mn0.66Ni0.22O2	A9	-0.3911	23	0.2313	0.6221
NaMn0.3Fe0.4Ni0.3O2	A10	0.0828	12	0.4531	0.3704
Na0.6Fe0.2Mn0.65Ni0.15O2	A11	0.4301	3	0.611	0.181
Na0.6Ni0.22Al0.11Mn0.66O2	A12	0.4918	2	0.6408	0.149
LiFePO4 (LFP)	A13	0.2185	11	0.4963	0.2777
Na3V2(PO4)3	A14	-0.7607	26	0.1058	0.8665
Na1.7O2Fe3(PO4)3	A15	-0.0239	15	0.3804	0.4044
Na2MnPO4F	A16	0.3675	5	0.5767	0.2093
Na2MnFe(CN)6	A17	0.3246	7	0.575	0.2504
Na0.61Fe[Fe(CN)6]0.94	A18	0.3607	6	0.5882	0.2274
Na0.81Fe[Fe(CN)6]0.79	A19	0.2751	8	0.5511	0.2761
Na2FeSiO4	A20	0.5732	1	0.7071	0.1339
Na2MnSiO4	A21	0.4047	4	0.5928	0.1883
NaFePO4	A22	0.0462	14	0.4047	0.3586
Na4MnV(PO4)3	A23	-0.5955	25	0.1463	0.7418
Na3MnTi(PO4)3	A24	-0.2201	17	0.2939	0.514
Na3MnTi(PO4)3	A25	0.2209	10	0.4954	0.2745
Na3MnZr(PO4)3	A26	-0.2999	20	0.2632	0.5631

B.2: Assessment with toxicity:

Table A.3: Crisp values from NEAT-Fuzzy PROMETHEE with toxicity

Alternatives		Crisp phi plus	Rank	Crisp phi plus	Crisp phi minus
NMC 111	A1	-0.7513	26	0.1071	0.8584
NMC 622	A2	-0.5759	24	0.1798	0.7557
NaMnO2	A3	0.0056	14	0.4089	0.4034
Na0.67Mn0.95Mg0.05O2	A4	-0.1387	19	0.34	0.4789

NaMn0.5Fe0.5O2	A5	-0.2088	21	0.3036	0.5124
NaNi0.5Mn0.5O2	A6	-0.055	16	0.4071	0.4618
Na0.9[Mn0.4Fe0.5Ti0.1]O2	A7	-0.1858	20	0.3086	0.4942
NaMn0.33Fe0.33Ni0.33O2	A8	0.0209	13	0.401	0.3802
Na0.6Fe0.11Mn0.66Ni0.22O2	A9	-0.3713	22	0.2363	0.6073
NaMn0.3Fe0.4Ni0.3O2	A10	0.2542	8	0.534	0.2799
Na0.6Fe0.2Mn0.65Ni0.15O2	A11	0.3575	5	0.5684	0.211
Na0.6Ni0.22Al0.11Mn0.66O2	A12	0.1742	10	0.4759	0.3018
LiFePO4 (LFP)	A13	-0.0029	15	0.3806	0.3833
Na3V2(PO4)3	A14	-0.7278	25	0.1221	0.8499
Na1.7O2Fe3(PO4)3	A15	0.0333	12	0.4028	0.3697
Na2MnPO4F	A16	0.3259	6	0.552	0.2262
Na2MnFe(CN)6	A17	0.449	4	0.6308	0.1819
Na0.61Fe[Fe(CN)6]0.94	A18	0.4948	1	0.6449	0.1501
Na0.81Fe[Fe(CN)6]0.79	A19	0.4615	3	0.6333	0.1719
Na2FeSiO4	A20	0.4696	2	0.6486	0.1791
Na2MnSiO4	A21	0.2805	7	0.5266	0.2462
NaFePO4	A22	0.2429	9	0.5009	0.2581
Na4MnV(PO4)3	A23	-0.5638	23	0.1581	0.7219
Na3MnTi(PO4)3	A24	-0.0879	18	0.3566	0.4445
Na3MnTi(PO4)3	A25	0.1623	11	0.4618	0.2996
Na3MnZr(PO4)3	A26	-0.0609	17		

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