Supporting Information for

Giant thermal Hall effect in Topological Magnon Insulator Cr_3Se_4 monolayer

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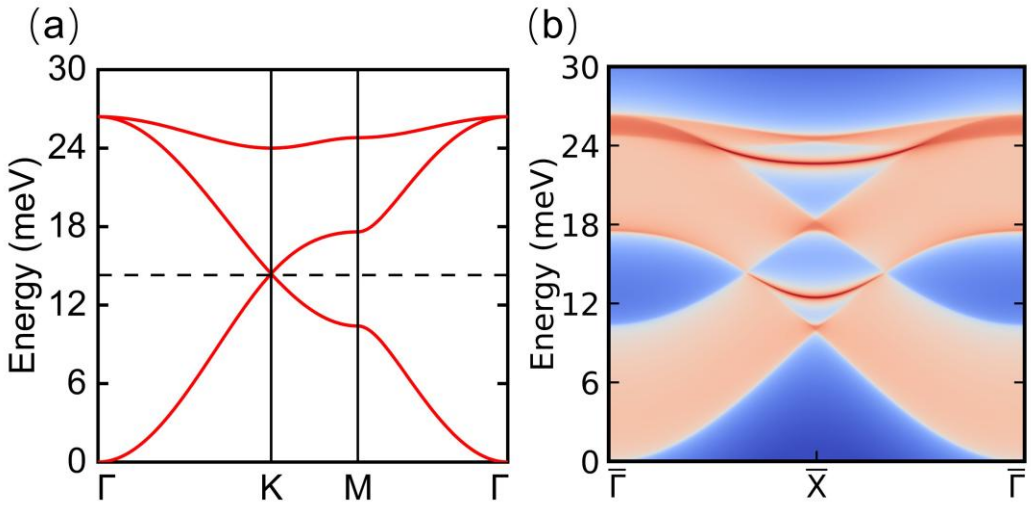

Fig. S1. (a) Magnonic band structure and (b) edge state of the Heisenberg-DM model with nearest and next-nearest neighbor DMI $D_1 = 0$ and $D_2 = 0$.

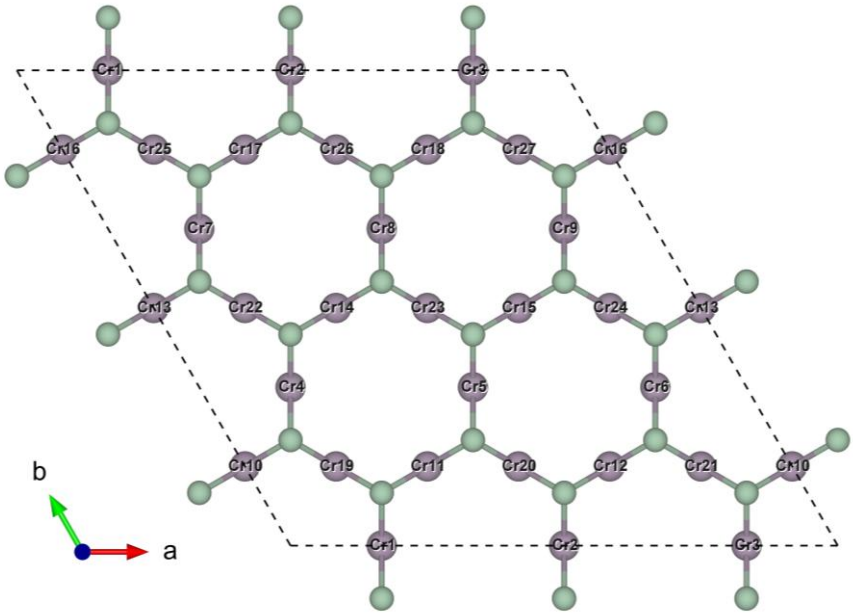

Fig. S2. Top view of the crystal structure of Cr_3Se_4 monolayer. Each spin site is labeled with a number. The unit cell is marked with a dashed black line.

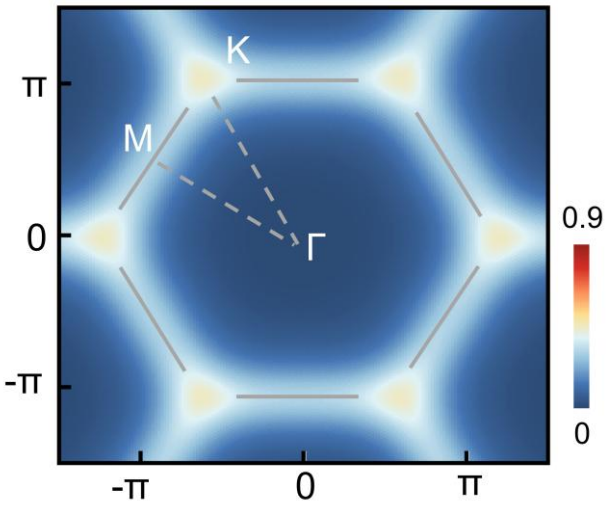

Fig. S3. K-space distribution of the Berry curvature within the nontrivial magnonic band gap of Cr_3Se_4 monolayer.

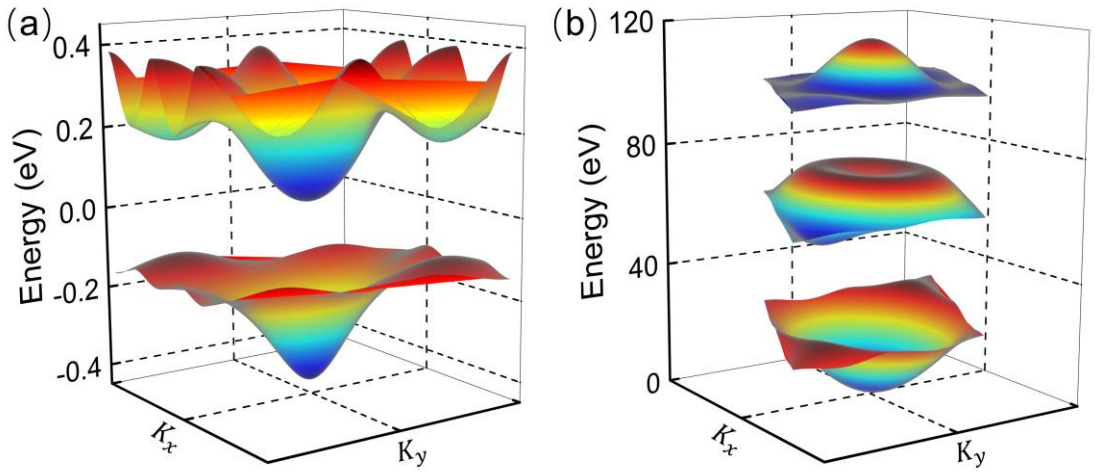

Fig. S4. Band structures of the entire Brillouin zone for Cr_3Se_4 monolayer in (a) electronic and (b) magnonic systems.

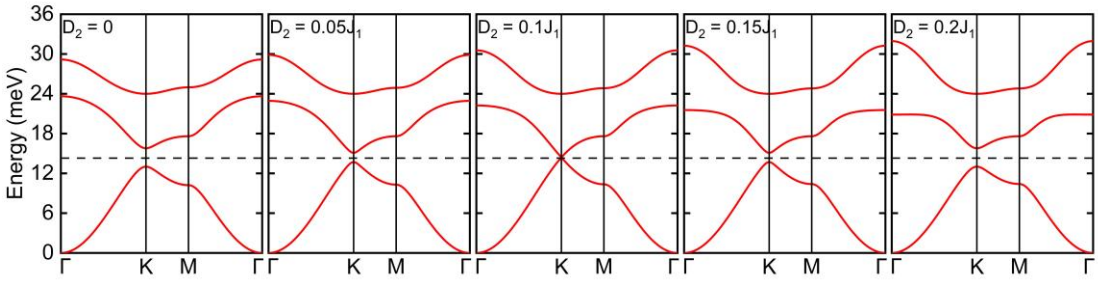

Fig. S5. Magnonic band structures of the Heisenberg-DM model with different NNN DMI D_2, where the NN DMI is set to $D_1 = 0.2J_1$.

Note S1: Four-state methodology (4SM)

The magnetic exchange interaction J between two neighboring spins can be represented as

$$J_{ij} = \begin{pmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix},$$

and one can define $J_{ij} = (J_{xx} + J_{yy} + J_{zz})/3$ and $D_{ij}^{z} = (J_{xy} - J_{yx})/2$. To calculate J_{xx}, we consider four magnetic states by setting the magnetic moments of two nearest neighboring magnetic atoms, while keeping the magnetic moments of other magnetic atoms as (0, 0, S), and $J_{xx} = \frac{E_1+E_3-E_2-E_4}{4S^2}$. Here E_1, E_2 , E_3 , E_4 are the energy of four states, which are defined as follows:

State 1 : $S_1 = (S, 0, 0)$, $S_2 = (S, 0, 0)$,

State 2 : $S_1 = (S, 0, 0)$, $S_2 = (-S, 0, 0)$,

State 3 : $S_1 = (-S, 0, 0)$, $S_2 = (-S, 0, 0)$,

State 4 : $S_1 = (-S, 0, 0)$, $S_2 = (S, 0, 0)$.

This process provided all interaction parameters.

Table S1: Parameters of the Cr_3Se_4 monolayer calculated by 4SM (meV). J_{xx}, J_{yy}, J_{zz}, J_1, and D_{1z} represent the nearest-neighbor exchange parameters. J_2 and D_{2z} represent the next nearest-neighbor exchange parameters, and A_z represents the single-ion anisotropy.

strain	J_{xx}	J_{yy}	J_{zz}	J_1	D_{1z}	J_2	D_{2z}	A_z
-2%	-6.935	-6.932	-5.445	-6.437	-0.433	1.245	-2.472	-0.277
0	-8.739	-8.734	-7.917	-8.463	-0.651	0.726	-2.503	- 0.302
2%	-9.684	-9.675	-9.513	-9.624	-0.877	0.437	-2.440	-0.382