

## Supplementary Information

### **A non-van der Waals platform for deep-subwavelength twist-polaritonics based on $\beta$ -**

#### **Ga<sub>2</sub>O<sub>3</sub> nanoflakes**

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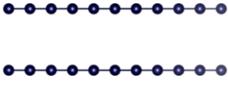
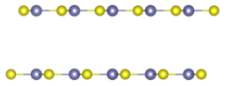
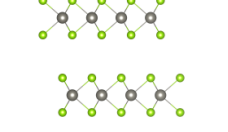
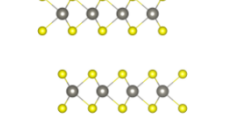
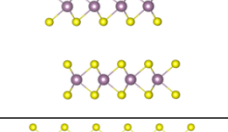
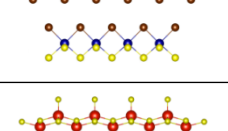
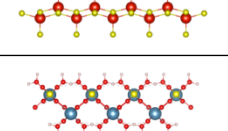
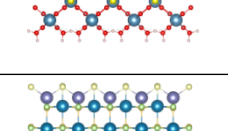
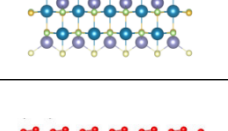
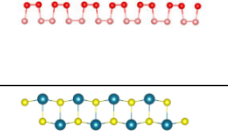
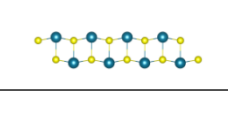
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**Table S1. Interlayer binding energy of representative 2D materials**

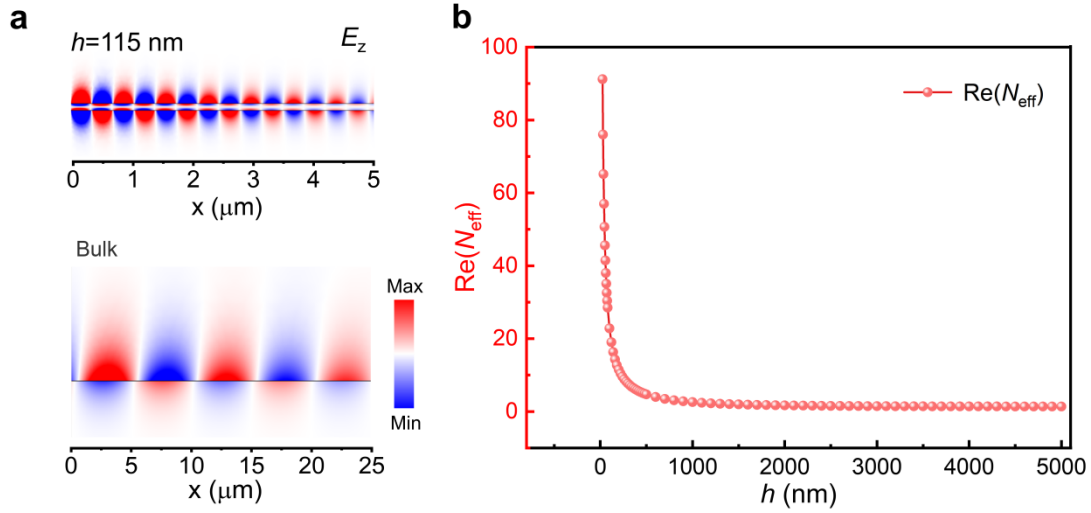
Materials	Interlayer binding energy (meV·Å <sup>-2</sup> )	Interaction strength	Structure	References	
Graphene	22.5	weak vdW interaction		1	
<i>h</i> BN	25.0			1	
WSe <sub>2</sub>	25.6			1	
WS <sub>2</sub>	26.1			1	
MoS <sub>2</sub>	26.3			1	
CrSBr	27.17			2	
V <sub>2</sub> O <sub>5</sub>	28.8			2	
CaSO <sub>4</sub> ·2H <sub>2</sub> O	28.4	weak non-vdW interaction		2	
β-Ga <sub>2</sub> O <sub>3</sub> -(100)B	30.58			3	
P (black)	33.4	strong vdW interaction		1	
SnS	36.5	strong vdW interaction		1	

**Table S2. Classification of non-vdW layered cleavable materials and typical representatives**

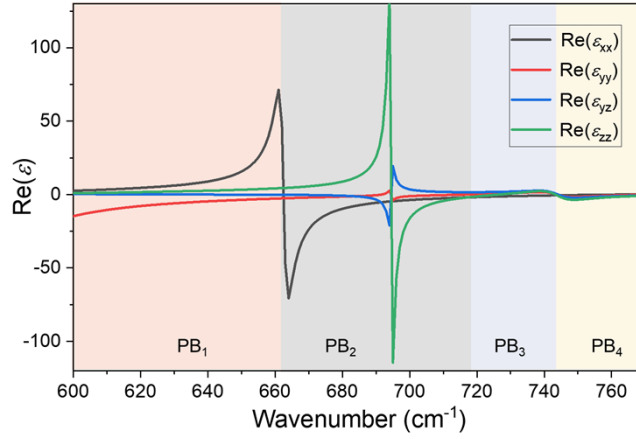
Major Category	Subclass Name	Structural Feature	Dominant Interlayer Interaction	Typical Representatives	References
<b>Ionic Bond-Dominated Type (Most abundant in nature)</b>	Charge-Compensating Cation Group	Negatively charged covalent-ionic framework layers + interlayer large-radius cations	Ionic bonds between cations and framework oxygen atoms	Mica, KBBF ( $\text{KBe}_2\text{BO}_3\text{F}_2$ )	4 5
	Charge-Compensating Anion Group (LDHs Family)	Positively charged metal hydroxide layers + interlayer exchangeable anions	Long-range electrostatic attraction between charged layers and counterions	Hydrotalcite	4
<b>Hydrogen Bond-Dominated Type</b>	Crystalline WaterMediated Hydrogen Bond Group	Strong ionic bond framework layers + interlayer crystalline water molecule layers	Hydrogen bonds between water molecules and framework	Gypsum ( $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ )	6
	Hydroxyl-Mediated Hydrogen Bond Group	Framework surface hydroxyl groups directly bond to oxygen atoms of adjacent layers	Hydroxyl-oxygen hydrogen bonds	Lepidocrocite ( $\gamma\text{-FeO(OH)}$ ), Atacamite ( $\text{Cu}_2\text{Cl(OH)}_3$ )	4
<b>Weak Metallic Bond-Dominated Type (Artificial Synthesize)</b>	MAX Phases (Ternary Carbides / Nitrides)	General formula $\text{M}_{n+1}\text{AX}_n$ , strong M-X covalent bond layers + weak M-A metallic bond layers	Weak M-A metallic bonds	$\text{Ti}_3\text{AlC}_2$ , $\text{Ti}_2\text{AlC}$ , $\text{Nb}_4\text{AlC}_3$	7
	MAB Phases (Ternary Borides)	General formula $\text{M}_{n+1}\text{AB}_{2n}$ , strong M-B covalent bond layers + weak M-A metallic bond layers	Weak M-A metallic bonds	$\text{Ti}_2\text{InB}_2$ , $\text{Zr}_2\text{SB}_2$ , $\text{Hf}_2\text{PB}_2$	8
<b>Weak Covalent Bond-Dominated Type</b>	Semiconductor / Transition Metal Oxides	Stacking of independent metal oxygen polyhedron 2D layers	Weak covalent bonds + partial ionic bonds	$\gamma\text{-WO}_3$ , $\text{SnO}$ , $\beta\text{-Ga}_2\text{O}_3$	9 10 3
	Other Emerging Layered Materials	Diverse layered framework structures	Weak covalent bonds / weak metallic bonds	$\text{Bi}_2\text{O}_2\text{Se}$ , $\text{MgB}_2$ , $\text{KV}_3\text{Sb}_5$	10

**Table S3. Comparison of key polariton performance metrics for  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> and other representative polaritonic platforms**

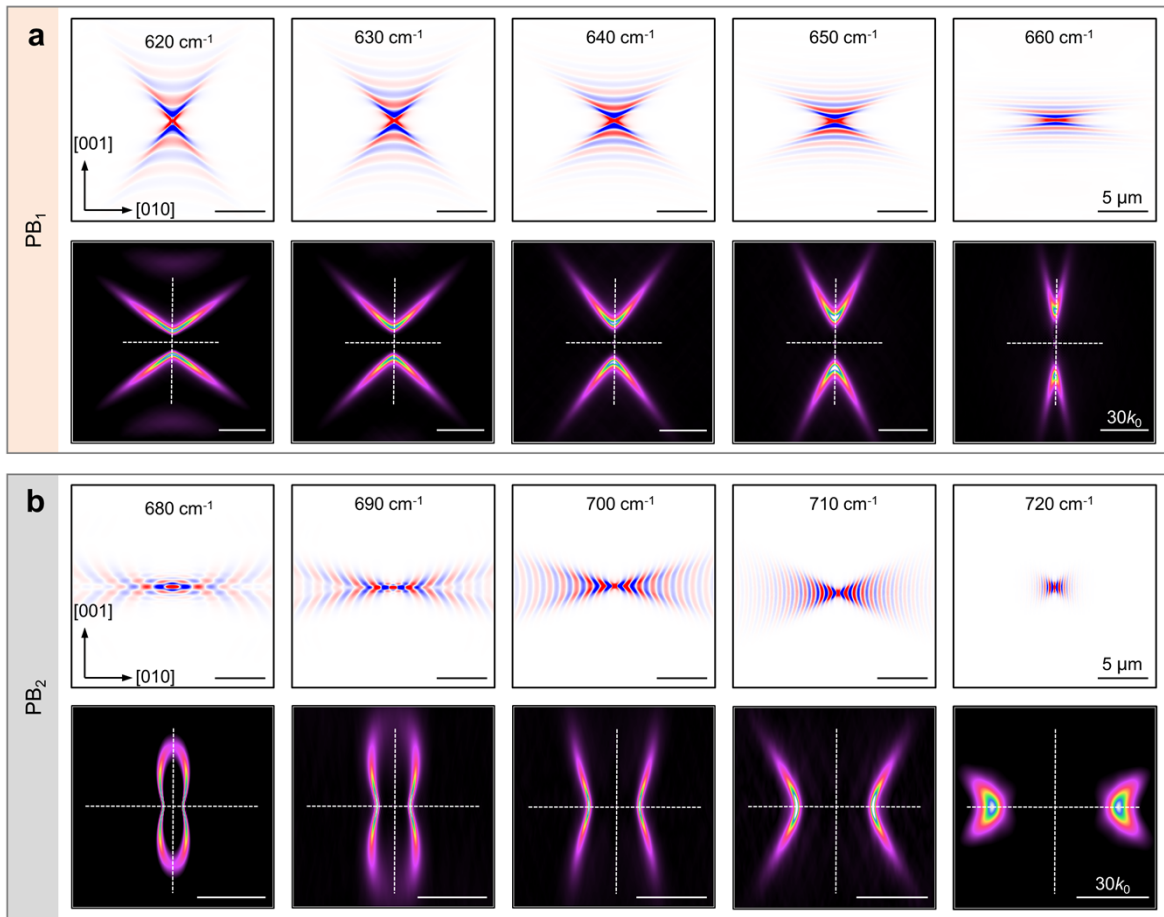
Material	Crystal orientation	Polaritons type	Frequency range (cm <sup>-1</sup> )	Thickness (nm)	Re( $k_p/k_0$ )	$Q$	Ref.
$\beta$ -Ga <sub>2</sub> O <sub>3</sub> (this work)	(100)	elliptic / hyperbolic	600-780	115	22.7 [010]	~3-6	This work
$\alpha$ -MoO <sub>3</sub>	(010)	elliptic / hyperbolic	820-1010	250	11.7 [100] 9.3 [001]	>6	11
hBN	(001)	isotropic	1360-1610	20	13.5	>6	12
$\alpha$ -V <sub>2</sub> O <sub>5</sub>	(010)	elliptic / hyperbolic	750-1050	105	10.7 [100] 12.2 [001]	3.5	13
Gypsum (CaSO <sub>4</sub> ·2H <sub>2</sub> O)	(010)	elliptic / hyperbolic shear	1110-1209	75	25 [100] 10 [001]	–	6



**Fig. S1 Dimensional reduction induces extreme field confinement of polaritons.** (a) Cross-sectional mode profile of phonon polaritons, the out-of-plane component of the optical field  $E_z$  is shown. Up: volume-confined mode within nanoflake. Down: surface mode within bulk crystal. (b) Field compression factor of the polariton mode at 715 cm<sup>-1</sup> against increasing thickness of  $\beta$ -Ga<sub>2</sub>O<sub>3</sub>.

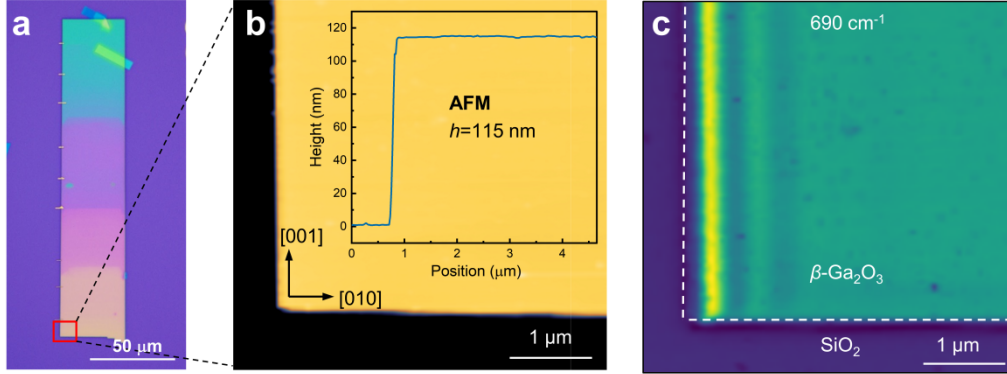


**Fig. S2 Dielectric properties of (100)-oriented  $\beta$ -Ga<sub>2</sub>O<sub>3</sub>.** Frequency-dependent real parts of the dielectric tensor components ( $\text{Re}(\epsilon_{xx})$ ,  $\text{Re}(\epsilon_{yy})$ ,  $\text{Re}(\epsilon_{yz})$ , and  $\text{Re}(\epsilon_{zz})$  for (100) plane as  $xy$ -plane.



**Fig. S3 Frequency-dependent hyperbolic polaritons in a suspended (100)-oriented  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> nanoflake across two different polariton bands.** Near-field patterns (top) and corresponding Fourier-transformed isofrequency contours (IFCs, bottom) across two Reststrahlen bands show frequency-switchable hyperbolic polaritons: (a) In PB<sub>1</sub> (620–

663  $\text{cm}^{-1}$ ), polaritons propagate along the [001] axis. (b) In  $\text{PB}_2$  (680–716  $\text{cm}^{-1}$ ), polaritons propagate along the [010] axis.

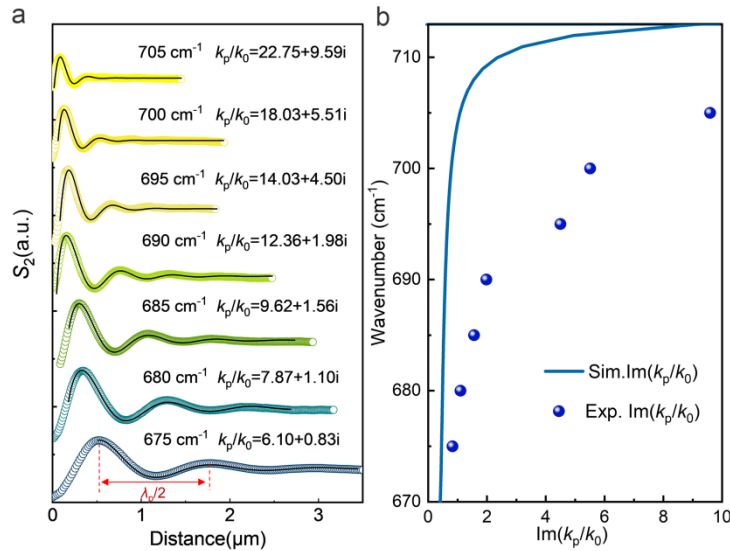


**Fig. S4 CVD-grown (100)-oriented  $\beta\text{-Ga}_2\text{O}_3$  nanoflakes with anisotropic polaritons.** (a) Optical image of a rectangular nanoflake. Orthogonal edges align with crystal axes: short edge  $\parallel$  [010], long edge  $\parallel$  [001]. (b) AFM topography of the region boxed in (a), with height profile (insert) confirming uniform thickness ( $h = 115$  nm). (c) Near-field optical image at 680  $\text{cm}^{-1}$  showing polaritonic fringes parallel to [010], signature of hyperbolic confinement in  $\text{PB}_2$ .

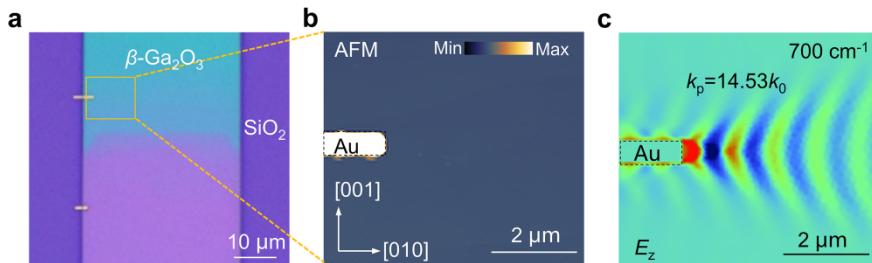
The propagation wavevector (real part  $k'$ ) and attenuation coefficient (imaginary part  $k''$ ) of the polaritons are obtained by analyzing the oscillatory decay of the near-field amplitude ( $S_2$ ) as a function of the tip-to-source distance  $x$  (Fig. 5a). The spatial profile is fitted using a custom MATLAB script based on a one-dimensional damped oscillation model with a power-law prefactor:

$$S_2(x) = A \cdot e^{-k''x} \cdot \frac{\cos(k'x + \varphi)}{x^\beta} + C$$

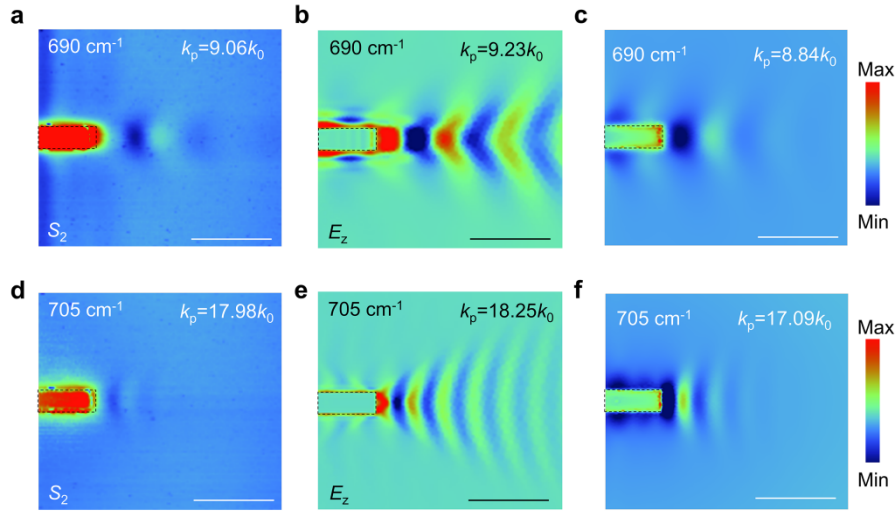
where  $A$  is the amplitude,  $\varphi$  is a phase offset,  $C$  is a constant background, and  $\beta$  is a power-law exponent that accounts for additional distance-dependent decay mechanisms. The propagation wavevector  $k'$  corresponds directly to the oscillation frequency, while the attenuation coefficient  $k''$  quantifies the exponential decay rate of the polariton energy.



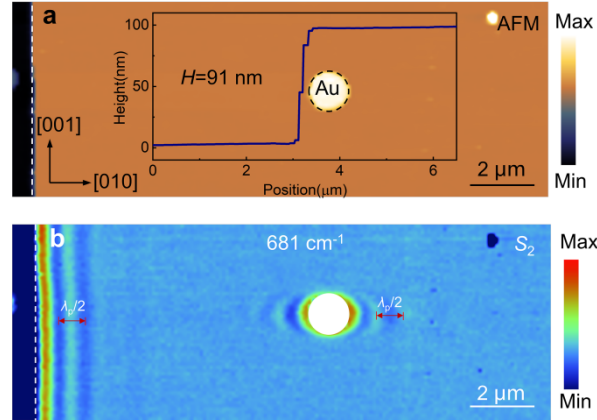
**Fig. S5** (a) Line profiles of polaritonic interference fringes propagating along the [010] direction at different frequencies, as a function of the distance from the flake edge.  $\lambda_p/2$ : the polaritons excited by the tip propagate away and are back-reflected at the flake edges, giving rise to interference fringes. (b) Imaginary part of the polariton wavevector extracted from near-field fringe fitting (blue symbols) compared with the theoretical calculation (blue line). The dielectric function used in the calculation follows Refs. [14] and [15].



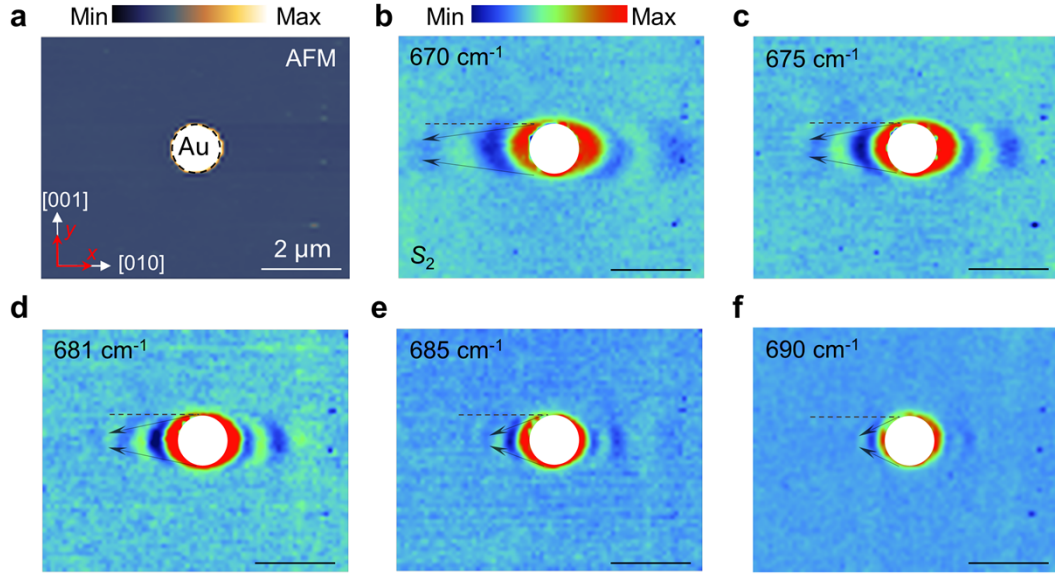
**Fig. S6** (a) Optical microscope image of the  $\beta\text{-Ga}_2\text{O}_3$  nanoflake shown in Fig. 3a. (b) Atomic force microscope image of the region marked by the orange box in (a). The flake thickness is 155 nm. Antenna dimensions: length, 1.5  $\mu\text{m}$ ; width, 500 nm; height, 100 nm. (c) Simulated polaritons launched by a gold antenna at 700  $\text{cm}^{-1}$ , displaying divergent wavefronts. The dielectric function used in the calculation follows Refs. [14] and [15].



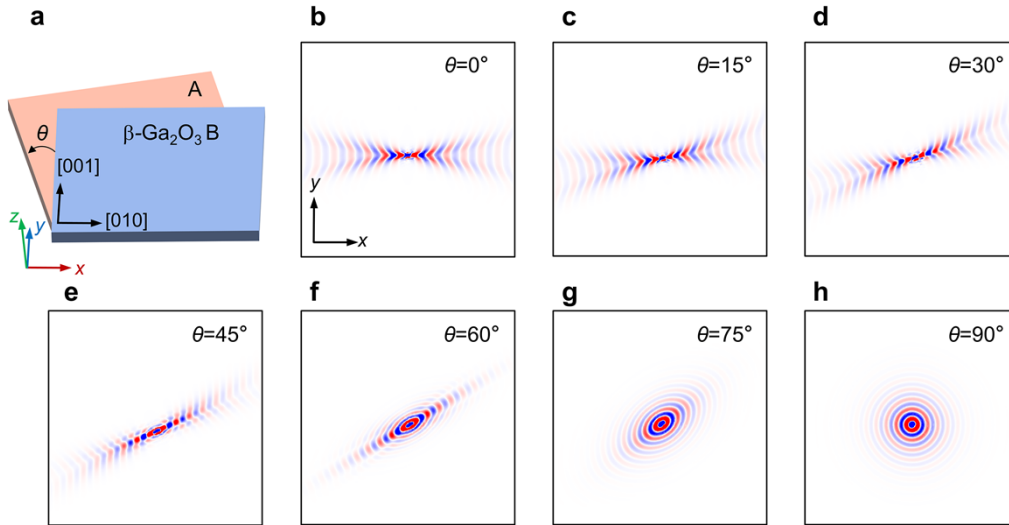
**Fig. S7 Frequency-dependent hyperbolic polaritons excited by gold nanorod on a (100)  $\beta$ - $\text{Ga}_2\text{O}_3$  nanoflake.** (a, d) Experimental near-field images (690 and 705  $\text{cm}^{-1}$ ) of gold-antenna-excited polaritons showing divergent wavefronts. (b, e) Simulated polariton patterns, using the standard dielectric function.<sup>14, 15</sup> (c, f) Simulated polariton patterns with the imaginary parts of the in-plane dielectric components ( $\epsilon_{xx}$  and  $\epsilon_{yy}$ ) increased by a factor of six. Scale bar: 2  $\mu\text{m}$ .



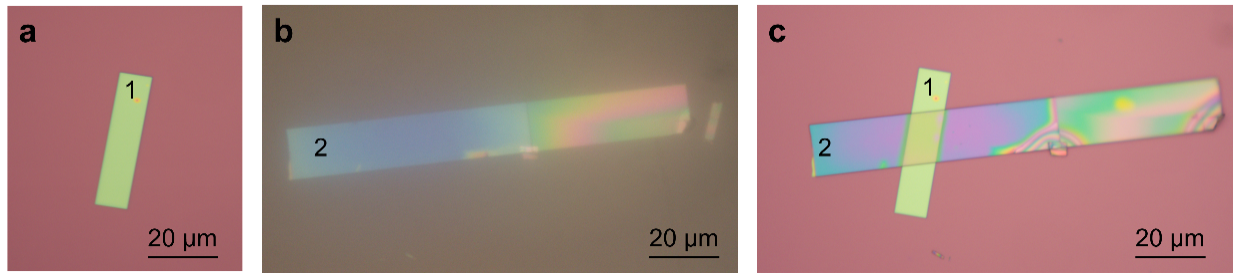
**Fig. S8 Real-space focusing of polaritons by a gold disk on a 91 nm-thick  $\beta$ - $\text{Ga}_2\text{O}_3$  nanoflake.** (a) AFM topography of the gold disk (radius: 650 nm; height: 60 nm) on the flake. (b) Experimental near-field image at 681  $\text{cm}^{-1}$ , showing the focusing of polaritons by the disk.



**Fig. S9 Frequency-dependent focusing of phonon polaritons by a gold disk on a 91-nm-thick  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> nanoflake.** (a) AFM topography of the gold disk. (b–f) Experimental near-field images showing polariton focusing by the gold disk at frequencies from 670 to 690  $\text{cm}^{-1}$ .



**Fig. S10 Rotation-induced topological transition of phonon polaritons.** (a) Schematic of twisted bilayer  $\beta$ -Ga<sub>2</sub>O<sub>3</sub>. Both the top layer ( $\beta$ -Ga<sub>2</sub>O<sub>3</sub> B) and bottom layer ( $\beta$ -Ga<sub>2</sub>O<sub>3</sub> A) are 60 nm thick. The  $x$  and  $y$  axes are aligned with the [010] and [001] crystallographic directions of the top layer ( $\beta$ -Ga<sub>2</sub>O<sub>3</sub> B). The twist angle ( $\theta$ ) is defined as the counterclockwise rotation of the bottom layer ( $\beta$ -Ga<sub>2</sub>O<sub>3</sub> A) relative to the top layer ( $\beta$ -Ga<sub>2</sub>O<sub>3</sub> B). (b–h) Numerical simulations of the out-of-plane electric field component ( $E_z$ ) distributions at 680  $\text{cm}^{-1}$ , illustrating the phonon polariton topology evolution with twist angle. The bilayer's dispersion relation transitions from hyperbolic ( $\theta = 15^\circ$  to  $45^\circ$ ) to elliptical ( $\theta = 75^\circ$ ) and finally to isotropic ( $\theta = 90^\circ$ ).



**Fig. S11 Deterministic assembly of  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> bilayers via dry transfer method.** (a) Bottom-layer  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> nanoflake on SiO<sub>2</sub>/Si substrate. (b) Top-layer  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> nanoflake on PDMS substrate. (c) Stacked bilayer with controlled twist angle.

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