Supplementary information for

Landauer-consistent interpretation of carrier mobility in quasi-ballistic field-effect transistors: Overcoming the limitations of the Y-function method

Dokyoung Lee[1,2], Minseo Cha[1], Hyeonsik Ahn[3], Jusung Kim[1,2], and Sungho Kim[1,2,*]

[1]Division of Electronic and Semiconductor Engineering, Ewha Womans University, Seoul 03760, Republic of Korea

[2]Institute for Multiscale Matter and Systems (IMMS), Ewha Womans University, Seoul 03760, Republic of Korea

[3]Department of Electronic Engineering, Hanbat National University, Daejeon 34158, Republic of Korea

Corresponding author: Sungho Kim (sunghok@ewha.ac.kr)

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Figure S1. Photograph of the low-temperature electrical measurement setup. The device was mounted inside a cryostat and cooled using a Gifford–McMahon (GM) cryocooler, enabling stable temperature control over the range investigated in this study. Gate and drain voltages were applied, and the drain current was measured using a source-measure unit.

Supplementary Note 1. Principle of the basic Y-function method

1.1 Theoretical basis

The Y-function method is a mobility extraction technique based on the transfer characteristic (I_D–V_G) measured in the linear operation regime of a field-effect transistor. In this regime, the drain current can be expressed within the drift–diffusion framework as

$$I_D = \frac{W}{L}\mu_{app}C_{ox}\left(V_G - V_T - \frac{V_D}{2}\right)V_D = \frac{W}{L}\mu_{app}C_{ox}V_{Gt}V_D, \tag{S1.1}$$

where $V_{Gt} = V_G - V_T - V_D/2$ denotes the effective gate drive voltage, W and L are the channel width and length, respectively, and C_{ox} is the gate capacitance per unit area.

The apparent mobility μ_{app} differs from the bulk low-field mobility due to its dependence on the transverse electric field in the channel. This dependence arises from multiple scattering mechanisms, including enhanced phonon scattering due to carrier confinement and surface roughness scattering at high transverse fields. To phenomenologically capture these effects, the apparent mobility is commonly modeled as

$$\mu_{app} = \frac{\mu_0}{1 + \theta_1 V_{Gt} + \theta_2\left(V_{Gt} - \Delta V_T\right)^2}, \tag{S1.2}$$

where μ_0 is the low-field mobility. The parameter θ_1 represents the first-order mobility attenuation, typically associated with phonon scattering and other field-dependent effects that scale approximately linearly with the transverse electric field. The second-order term θ_2 accounts for surface roughness scattering, which becomes significant only above a certain critical field. The voltage shift ΔV_T reflects the delayed onset of this scattering mechanism.

Substituting this mobility model into the linear-regime current expression yields

$$I_D = \frac{G_m V_{Gt}}{1 + \theta_1 V_{Gt} + \theta_2\left(V_{Gt} - \Delta V_T\right)^2}, \tag{S1.3}$$

where $G_m = (W/L)\mu_0 C_{ox} V_D$ is the static transconductance prefactor.

1.2 Basic Y-function method

The basic Y-function method is derived from this drain current expression by considering the low transverse electric field regime. In this regime, typically close to the threshold voltage or under weak-to-moderate inversion, surface roughness scattering is negligible. Consequently, the second-order attenuation term can be ignored, leading to the simplified mobility model

$$\mu_{app} \approx \frac{\mu_0}{1+\theta_1 V_{Gt}}, \tag{S1.4}$$

and the corresponding drain current expression

$$I_D \approx \frac{G_m V_{Gt}}{1+\theta_1 V_{Gt}}. \tag{S1.5}$$

Differentiating I_D with respect to V_G gives the transconductance

$$g_m = \frac{\partial I_D}{\partial V_G} = \frac{G_m}{\left(1+\theta_1 V_{Gt}\right)^2}. \tag{S1.6}$$

The key idea of the Y-function method is to construct a quantity that eliminates the influence of the mobility attenuation factor θ_1. This is achieved by defining the Y-function as

$$Y = \frac{I_D}{\sqrt{g_m}} = \sqrt{G_m} V_{Gt}. \tag{S1.7}$$

The Y-function is strictly linear in V_{Gt} and independent of θ_1. Therefore, despite the presence of field-dependent mobility degradation, the Y-function restores a linear relationship between the measured electrical quantity and the effective gate overdrive. Consequently, from the linear extrapolation of Y as a function of V_G, the threshold voltage V_T can be obtained from the x-axis intercept, while the slope directly yields the low-field mobility μ_0, provided that the device geometry and gate capacitance are known. Because the Y-function is constructed from the ratio of the drain current to the transconductance, both of which are similarly affected by series resistance, the impact of series resistance is also significantly reduced.

Supplementary Note 2. Polynomial Y-function method

Supplementary Note 1 established the Y-function framework based on the drift–diffusion description of the linear-regime drain current and clarified how the basic Y-function method enables extraction of the threshold voltage and low-field mobility by suppressing the first-order mobility attenuation term. This formulation is valid as long as the second-order mobility attenuation associated with surface roughness scattering remains negligible, such that the Y-function preserves a linear dependence on the effective gate drive voltage. However, as the transverse electric field increases, the contribution of the second-order attenuation term becomes non-negligible, and the drain current can no longer be accurately described by a first-order mobility degradation model. Under these conditions, the Y-function inherently deviates from linearity, and parameter extraction based on linear regression becomes sensitive to the selected gate-voltage range. This limitation arises not from the Y-function concept itself, but from the linearization assumption implicit in the basic method.

To address this issue, Supplementary Note 2 summarizes the polynomial Y-function method that has already been reported in the literature and serves here as a concise theoretical overview rather than a re-derivation of a new technique. By retaining the full mobility attenuation model and reformulating the Y-function into a polynomial representation, this approach enables self-consistent and bias-range-independent extraction of the threshold voltage and mobility degradation parameters, while preserving the physical foundations established in Supplementary Note 1.

The polynomial method begins with Eq. (S1.3) which can be further simplified to involve only powers of V_{Gt}. Dividing both the numerator and denominator of Eq. (S1.3) by $(1+ \theta_2 \Delta V_T^2)$ derives a new form of I_D as

$$I_D = \frac{\dfrac{G_m}{1+\theta_2 \Delta V_T{}^2} V_{Gt}}{1+\Theta_1 V_{Gt} + \Theta_2 V_{Gt}{}^2}, \tag{S2.1}$$

where the effective coefficients Θ_1 and Θ_2 are defined as

$$\Theta_1 = \frac{\theta_1 - 2\theta_2 \Delta V_T}{1+\theta_2 \Delta V_T{}^2},\ \Theta_2 = \frac{\theta_2}{1+\theta_2 \Delta V_T{}^2}. \tag{S2.2}$$

For notational simplicity, the scaled prefactor $G_m / \left(1+\theta_2 \Delta V_T{}^2\right)$ is hereafter denoted again by G_m, leading to

$$I_D = \frac{G_m V_{Gt}}{1+\Theta_1 V_{Gt} + \Theta_2 V_{Gt}{}^2}. \tag{S2.3}$$

It should be emphasized that this step corresponds to a redefinition of the effective current prefactor and does not alter the physical meaning of the extracted low-field mobility, which is recovered explicitly in the final step of the extraction procedure.

Using Eq. (S2.3), the Y-function is defined as,

$$Y = \sqrt{\frac{G_m}{1-\Theta_2 V_{Gt}{}^2}} V_{Gt}, \tag{S2.4}$$

which is no longer linear in V_{Gt} due to the presence of Θ_2. To overcome this limitation, an auxiliary ξ-function is introduced as

$$\xi \equiv \frac{1}{Y^2} = \frac{1}{G_m}\left\{\frac{1}{V_{Gt}{}^2} - \Theta_2\right\}. \tag{S2.5}$$

Here, it is assumed that the exact value of V_T is unknown, and a rough estimator V_T^* is introduced which differs from V_T by an error, $\varepsilon = V_T^* - V_T$. As a result, $V_{Gt}^* = V_{Gt} - \varepsilon$, and ξ-function can be rewritten as

$$\xi = \frac{1}{G_m}\left\{\frac{1}{\left(V_{Gt}{}^{*}+\varepsilon\right)^{2}} - \Theta_2\right\}. \qquad \text{(S2.6)}$$

If V_T^* is close enough to V_T (i.e., $\varepsilon/V_{Gt}^* \rightarrow 0$), $1/(1+\varepsilon/V_{Gt}^*)^2$ can be approximated as

$$\frac{1}{\left(1+\varepsilon/V_{Gt}{}^{*}\right)^{2}} \approx 1 - \frac{2\varepsilon}{V_{Gt}{}^{*}}. \qquad \text{(S2.7)}$$

Then, Eq. (S2.7) can be expressed as a polynomial function of $1/V_{Gt}^*$,

$$\xi = \frac{1}{G_m}\left\{\frac{1}{V_{Gt}{}^{*2}}\frac{1}{\left(1+\varepsilon/V_{Gt}{}^{*}\right)^{2}} - \Theta_2\right\} \approx \frac{1}{G_m}\left\{\frac{1}{V_{Gt}{}^{*2}} - \frac{2\varepsilon}{V_{Gt}{}^{*3}} - \Theta_2\right\}. \qquad \text{(S2.8)}$$

This polynomial form constitutes the core principle of the polynomial Y-function method. For a given provisional V_T^*, polynomial regression of ξ as a function of $1/V_{Gt}^*$ allows the error term ε to be extracted. The threshold voltage estimate is then iteratively updated according to

$$V_{T,i+1}{}^{*} = V_{T,i}{}^{*} - \varepsilon_i, \qquad \text{(S2.9)}$$

until convergence is achieved. This recursive procedure yields a unique and bias-range-independent determination of V_T and G_m.

Once V_T and G_m are accurately obtained, the effective mobility attenuation function is evaluated using Eq. (S2.10),

$$\Theta_{eff} = \frac{G_m}{I_D} - \frac{1}{V_{Gt}} = \Theta_1 + \Theta_2 V_{Gt}. \qquad \text{(S2.10)}$$

Linear regression of Θ_{eff} yields Θ_1 and Θ_2, from which the physical mobility degradation parameters θ_1, θ_2, ΔV_T are finally recovered using Eq. (S2.2).

Supplementary Note 3. Landauer formalism and mobility definitions in quasi-ballistic transport

3.1 Landauer formulation of low-field transport

In nanoscale FETs, carrier transport cannot always be described solely within the drift–diffusion framework, particularly when the channel length becomes comparable to the carrier mean free path. In this regime, the Landauer formalism provides a more general description by treating current flow as a transmission problem through a finite-length channel.

Under low drain bias (linear regime), the drain current can be expressed as

$$I_D = \left[\frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f}{\partial E} \right) dE \right] \cdot V_D \equiv G \cdot V_D \, , \tag{S3.1}$$

where q is the elementary charge, h is Planck's constant, T(E) is the energy-dependent transmission probability, M(E) is the density of conducting modes, and f(E) is the Fermi–Dirac distribution. For sufficiently small V_D, the integral term can be identified as the channel conductance G.

Within the Landauer framework, the transmission probability can be approximated by a simple expression involving the channel length L and the carrier mean free path λ:

$$T(E) = \frac{\lambda}{\lambda + L} \, . \tag{S3.2}$$

This expression naturally captures the continuous transition between the diffusive limit (L>>λ) and the ballistic limit (L<<λ). As a result, finite channel length alone can impose an intrinsic limitation on the current, even in the absence of strong scattering.

3.2 Drift–diffusion interpretation and definition of apparent mobility

Experimentally, the drain current in the linear regime is commonly interpreted using a drift–diffusion-based description, in which the current is written as the product of channel charge and drift velocity. The inversion charge per unit area is given by $Q_n = qn_s$, where n_s is the sheet carrier density. The lateral electric field in the channel can be approximated as $E_x \approx V_D/L$, and the drift velocity is written as $v_d = \mu E_x$. The drain current then follows as

$$I_D = WQ_n v_d = W(qn_s)\mu\left(\frac{V_D}{L}\right) = \frac{W}{L} qn_s \mu V_D . \tag{S3.3}$$

When this expression is used to interpret measured low-field characteristics, the mobility inferred from the measured current is referred to as the apparent mobility, denoted μ_{app}. Accordingly,

$$I_D = \frac{W}{L} qn_s \mu_{app} V_D . \tag{S3.4}$$

It is emphasized that μ_{app} is not a fundamental transport parameter, but an effective quantity obtained by mapping the measured current onto a drift–diffusion-like expression.

3.3 Relation between apparent mobility and Landauer conductance

By equating the Landauer expression for the drain current [Eq. (S3.1)] with the drift–diffusion form [Eq. (S3.4)], the apparent mobility can be formally defined as

$$\mu_{app} = \frac{G}{qn_s}\frac{L}{W} . \tag{S3.5}$$

Using the Landauer conductance expressed in terms of the transmission probability [Eqs. (S3.1) and (S3.2)], and assuming an energy-independent mean free path λ, the conductance can be written as

$$G = \frac{\lambda}{\lambda + L} G_0, \quad G_0 = \frac{2q^2}{h}\int M(E)\left(-\frac{\partial f}{\partial E}\right)dE , \tag{S3.6}$$

where G_0 denotes the ballistic (injection-limited) conductance corresponding to T = 1.

Using Eq. (S3.5), the apparent mobility becomes

$$\mu_{app} = \frac{\lambda}{\lambda + L} \frac{G_0}{qn_s} \frac{L}{W} . \tag{S3.7}$$

In the ballistic injection limit (T→1, or λ>>L), the apparent mobility obtained by mapping the Landauer current onto the drift–diffusion form approaches the prefactor in Eq. (S3.7). Since this limit corresponds to transport constrained solely by the finite channel length, independent of scattering processes, this prefactor is naturally identified as the ballistic mobility, denoted as μ_{ball}. Accordingly, Eq. (S3.7) can be rewritten as

$$\mu_{app} = \frac{\lambda}{\lambda + L} \mu_{ball} . \tag{S3.8}$$

3.4 Effective mobility and ballistic mobility

To express Eq. (S3.8) in a physically transparent form, it is convenient to define mobility components associated with the two characteristic length scales governing transport: the mean free path λ and the channel length L. The effective (scattering-limited) mobility is defined as

$$\mu_{eff} \equiv \frac{\lambda v_T}{2k_B T / q} . \tag{S3.9}$$

where $v_T = (2k_BT/\pi m^*)^{1/2}$ is the unidirectional thermal velocity, k_B is Boltzmann's constant, and T is the temperature. This quantity reflects intrinsic scattering processes within the channel.

Similarly, the ballistic mobility introduced in Eq. (S3.7) can be evaluated in terms of the same velocity scale as

$$\mu_{ball} \equiv \frac{L v_T}{2k_B T / q} . \tag{S3.10}$$

The use of the same velocity scale v_T in both μ_{eff} and μ_{ball} reflects their common origin in the Landauer conductance prefactor. Importantly, the unidirectional thermal velocity v_T represents the average velocity of forward-moving carriers and is physically equivalent to the injection velocity in the Landauer transport framework under non-degenerate conditions. In this sense, both μ_{eff} and μ_{ball} are ultimately governed by carrier injection statistics rather than by purely drift-based transport. As a consequence, although μball corresponds to the scattering-free transport limit, it is not temperature independent. Since $v_T{\sim}T^{1/2}$, the ballistic mobility follows $\mu_{ball}{\sim}T^{-1/2}$. This intrinsic temperature dependence arises from carrier injection statistics at the source, rather than from scattering processes within the channel.

From the definitions in Eqs. (S3.9) and (S3.10), the ratio of the effective mobility to the ballistic mobility directly reflects the ratio between the mean free path and the channel length.

$$\frac{\mu_{eff}}{\mu_{ball}} \equiv \frac{\lambda}{L}. \qquad \text{(S3.11)}$$

Using Eq. (S3.11), the transmission factor in Eq. (S3.8) can be rewritten purely in terms of mobility components as

$$\frac{\lambda}{\lambda + L} = \frac{\mu_{eff}}{\mu_{eff} + \mu_{ball}}. \qquad \text{(S3.12)}$$

Substituting Eq. (S3.12) into Eq. (S3.8) yields

$$\mu_{app} = \frac{\mu_{eff}\,\mu_{ball}}{\mu_{eff} + \mu_{ball}}. \qquad \text{(S3.13)}$$

This expression is equivalently written in the Matthiessen-like form

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_{eff}} + \frac{1}{\mu_{ball}}. \qquad \text{(S3.14)}$$

Equations (S3.8)–(S3.14) demonstrate that the apparent mobility extracted from low-field electrical measurements reflects two independent limitations on carrier transport: scattering within the channel and finite channel length. In the diffusive limit ($L \gg \lambda$), the apparent mobility approaches the effective mobility ($\mu_{app} \approx \mu_{eff}$). In contrast, in the quasi-ballistic and ballistic regimes, the apparent mobility progressively saturates toward the ballistic mobility ($\mu_{app} \approx \mu_{ball}$), and further improvements in scattering-limited mobility no longer translate into proportional increases in the measured mobility. This Landauer-based formulation provides the conceptual foundation for interpreting mobility parameters extracted using drift–diffusion-based techniques, such as the Y-function method, when applied to nanoscale transistors.

Supplementary Note 4. Landauer-consistent mobility model for Y-function analysis

4.1 Connection between the Y-function mobility model and Landauer transport

Supplementary Notes 1 and 2 formulated the Y-function method by assuming that the linear-regime drain current can be interpreted within a drift–diffusion framework. In this picture, the measured drain current is written as in Equation (S1.1). However, Supplementary Note 3 showed that when the channel length L becomes comparable to the carrier mean free path λ, the mobility inferred from Equation (S1.1) does not represent a purely scattering-limited mobility. Instead, within the Landauer formalism, the apparent mobility is related to the effective (scattering-limited) mobility μ_{eff} and the ballistic mobility μ_{ball} through the Matthiessen-like relation, given by Equation (S3.14). This relation provides a natural starting point for extending the Y-function mobility model to include quasi-ballistic transport effects.

4.2 Reinterpretation of the Y-function mobility attenuation model

In Supplementary Notes 1 and 2, the field dependence of the mobility was modeled phenomenologically as,

$$\mu_{app}(V_{Gt}) = \frac{\mu_0}{1+\theta_1 V_{Gt} + \theta_2 \left(V_{Gt} - \Delta V_T\right)^2}. \tag{S1.2}$$

In long-channel devices operating in the diffusive regime, this expression can be directly interpreted as the apparent mobility. In the quasi-ballistic regime, however, Eq. (S3.14) implies that the quantity described by the attenuation model should instead be identified with the *scattering-limited* mobility. Accordingly, we reinterpret Eq. (S1.2) as a model for the effective mobility,

$$\mu_{eff}(V_{Gt}) = \frac{\mu_0}{F(V_{Gt})}, \tag{S4.1}$$

where the attenuation function $F(V_{Gt})$ is defined using the same parameters as in Supplementary Note 1,

$$F(V_{Gt}) \equiv 1 + \theta_1 V_{Gt} + \theta_2 (V_{Gt} - \Delta V_T)^2. \tag{S4.2}$$

Substituting Equation (S4.1) into the Landauer relation, Equation (S3.14), yields

$$\frac{1}{\mu_{app}} = \frac{F(V_{Gt})}{\mu_0} + \frac{1}{\mu_{ball}}. \tag{S4.3}$$

Rearranging Equation (S4.3), the apparent mobility measured in electrical experiments becomes

$$\mu_{app}(V_{Gt}) = \frac{\mu_0 \mu_{ball}}{\mu_0 + \mu_{ball} F(V_{Gt})}. \tag{S4.4}$$

For notational convenience, we introduce a dimensionless parameter that quantifies the relative importance of ballistic transport,

$$\eta \equiv \frac{\mu_0}{\mu_{ball}}, \tag{S4.5}$$

which depends on temperature and channel length through μ_{ball}. Using Eq. (S4.5), Eq. (S4.4) can be rewritten in a compact form,

$$\mu_{app}(V_{Gt}) = \frac{\mu_0}{F(V_{Gt}) + \eta}. \tag{S4.6}$$

In the diffusive limit ($\mu_{ball} \to \infty$, $\eta \to 0$), Eq. (S4.6) reduces exactly to the conventional Y-function mobility model of Eq. (S1.2). In the quasi-ballistic limit ($\mu_{eff} >> \mu_{ball}$), the apparent mobility saturates toward μ_{ball}, independent of further increases in μ_{eff}.

4.3 Landauer-consistent drain current model

Substituting Equation (S4.6) into the linear-regime drain current expression in Equation (S1.1) yields

$$I_D = \frac{W}{L} C_{ox} V_D \frac{\mu_0 V_{Gt}}{F(V_{Gt}) + \eta} = \frac{G_m V_{Gt}}{1 + \theta_1 V_{Gt} + \theta_2 (V_{Gt} - \Delta V_T)^2 + \eta}. \quad \text{(S4.7)}$$

Equation (S4.7) has the same mathematical structure as the polynomial Y-function model introduced in Supplementary Note 2, with the sole modification that an additional gate-voltage-independent term η appears in the denominator.

4.4 Breakdown of the conventional polynomial Y-function fitting scheme

The polynomial Y-function method summarized in Supplementary Note 2 relies on the drain-current model given by Equation (S2.3), which is mathematically equivalent to Equation (S1.3) and assumes that the attenuation of the apparent mobility can be fully described by a gate-voltage-dependent function of the form $F(V_{Gt})$. Under this assumption, all transport limitations are implicitly absorbed into the parameters μ_0, θ_1, θ_2, and ΔV_T, and the polynomial reformulation of the Y-function enables robust extraction of these parameters through the ξ-function and iterative threshold-voltage correction through Equations (S2.4)–(S2.9).

However, Supplementary Note 3 established that in the quasi-ballistic regime the apparent mobility inferred from electrical measurements obeys the Landauer relation of Equation (S3.14), rather than representing a purely scattering-limited quantity. As shown in Supplementary Note 4, this leads to the modified drain-current expression of Equation (S4.7), in which an additional gate-voltage-independent term η appears in the denominator. This additional term fundamentally alters the structure of the attenuation function assumed in the polynomial Y-function method. In particular, the total attenuation can no longer be written solely as a polynomial function of V_{Gt}, but instead takes the form

$$F_{corr}(V_{Gt}) = F(V_{Gt}) + \eta \, . \tag{S4.8}$$

As a result, the core assumption underlying the polynomial Y-function fitting—namely that all deviations from ideal linear behavior in the Y-function originate from gate-field-dependent mobility degradation—is no longer valid. When the conventional polynomial Y-function method is applied to data governed by Equation (S4.7), the fitting algorithm attempts to compensate for the missing constant offset η by distorting the coefficients θ_1 and θ_2. This manifests as unphysical trends, such as negative θ_1 values or anomalous temperature dependences of θ_2, as observed experimentally in the main text. Therefore, in the presence of quasi-ballistic transport, the systematic breakdown of the polynomial Y-function method should not be interpreted as a numerical instability or lack of fitting robustness, but rather as a direct consequence of an incomplete transport model.

Supplementary Note 5. Revised fitting strategy under the Landauer-consistent model

5.1 Scope and objective

Supplementary Note 4.4 established that the conventional polynomial Y-function fitting scheme becomes inadequate when quasi-ballistic transport contributes significantly to the drain current, due to the presence of a gate-voltage-independent Landauer correction term η. In the Landauer-consistent mobility model, the apparent mobility is expressed as Equation (S4.6),

$$\mu_{app}(V_{Gt}) = \frac{\mu_0}{F(V_{Gt}) + \eta},$$

where the Landauer correction parameter η is defined as Equation (S4.5),

$$\eta \equiv \frac{\mu_0}{\mu_{ball}}.$$

Since the ballistic mobility μ_{ball} can be determined solely by the channel length and temperature through Eq. (S3.10), $\mu_{ball}(T)$ is calculated a priori for each temperature. With $\mu_{ball}(T)$ fixed, the Landauer-consistent drain current model used for fitting is given by Equation (S4.7),

$$I_D = \frac{W}{L} C_{ox} V_D \frac{\mu_0 V_{Gt}}{1 + \theta_1 V_{Gt} + \theta_2 (V_{Gt} - \Delta V_T)^2 + \eta}.$$

and the set of fitting parameters is reduced to $\{\mu_0, \theta_1, \theta_2, \Delta V_T\}$.

5.2 Practical fitting procedure

For each temperature, the fitting is performed as follows.

(1) **Threshold-voltage extraction using the polynomial Y-function method**: The threshold voltage V_T is first extracted independently using the polynomial Y-function method described in Supplementary Note 2. Specifically, V_T is iteratively refined using the ξ-function formalism [Equations (S2.4)–(S2.9)]. The converged V_T is then fixed and used in all subsequent fitting steps. Importantly, V_T is not treated as an adjustable fitting parameter in the Landauer-consistent analysis.

(2) **Selection of fitting window:** Using the fixed threshold voltage, the effective gate overdrive is defined as $V_{Gt} = V_G – V_T – V_D/2$. A common gate-voltage range is selected for all temperatures, and only data points satisfying $V_{Gt} \geq V_{Gt,min}$ are included. Here, $V_{Gt,min}$ is a user-defined lower bound on the effective gate overdrive voltage, introduced to restrict the fitting to the strong-inversion regime where the polynomial mobility attenuation model and the Landauer-consistent transport description remain physically meaningful. This restriction ensures numerical stability and confines the fitting to the regime where the underlying transport model remains valid.

(3) **Simultaneous fitting of I_D and g_m:** With V_T fixed, **t**he drain current I_D and transconductance $g_m = \partial I_D / \partial V_G$ are fitted simultaneously using the Landauer-consistent current model [Equation (S4.7)] and its analytical derivative with respect to V_G. A weighted nonlinear least-squares procedure is employed to minimize the combined residual of I_D and g_m.

(4) **Extraction of the parameters:** The parameters {μ_0, θ_1, θ_2, ΔV_T} are obtained directly from the optimization. The Landauer correction parameter is then evaluated a posteriori as

$$\eta(T) \equiv \frac{\mu_0}{\mu_{ball}(T)},$$

providing a quantitative measure of the degree of quasi-ballistic transport.

5.3 Validity of threshold-voltage extraction using the Y-function method under Landauer-consistent transport

A potential concern is whether the threshold voltage extracted using the conventional Y-function method remains valid when the drain-current model is modified to include Landauer-type quasi-ballistic corrections. This issue arises because the Landauer-consistent model explicitly departs from the drift–diffusion assumption underlying the original Y-function formulation.

Importantly, the Landauer correction enters the drain-current expression as a gate-voltage-independent term η in the mobility denominator. As a result, while it alters the magnitude and gate-voltage dependence of the apparent mobility in the strong-inversion regime, it does not modify the fundamental condition defining the onset of inversion. The threshold voltage remains determined by the electrostatic condition for channel formation and charge control, rather than by the detailed transport mechanism governing carrier transmission once the channel is formed.

Within the Y-function formalism, the extraction of V_T relies primarily on the geometric structure of the Y-function and its extrapolated intercept, rather than on the specific functional form of the mobility attenuation. In Supplementary Note 2, V_T is obtained through a recursive ξ-function procedure that effectively identifies the gate-voltage offset at which the effective gate overdrive vanishes. Because the Landauer correction does not introduce any additional gate-voltage-dependent shift in this condition, its impact on V_T extraction is inherently weak.

This distinction is also evident from the different sensitivities of V_T and mobility-related parameters to the transport model. While the mobility attenuation parameters θ_1 and θ_2 are directly affected by the presence of a correction term η—leading to unphysical values when analyzed within a purely drift–diffusion framework—the extracted V_T remains stable and reproducible across different fitting strategies. In this sense, the breakdown of the polynomial Y-function method under quasi-ballistic transport should be understood as a failure of mobility interpretation, not of threshold-voltage determination.

Accordingly, in this work, the Y-function method is retained exclusively as a robust tool for threshold-voltage extraction, while mobility parameters are obtained using a Landauer-consistent numerical fitting approach. This separation of roles reflects the different physical origins of V_T and apparent mobility and ensures that the advantages of the Y-function method are preserved without extending its applicability beyond its valid physical domain.

5.4 Pseudo-code for the Landauer-consistent I_D + g_m fitting

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Given: measured ID–VG at multiple temperatures, device constants (W, L, Cox, VD),
       and a common VG fitting window [VGmin, VGmax].

For each temperature T:

  1) Threshold-voltage extraction (polynomial Y-function)
     - From measured ID(VG), compute gm(VG) = dID/dVG.
     - Within [VGmin, VGmax], extract VT using the polynomial Y-function
       and ξ-function iteration (Supplementary Note 2, Eqs.(S2.4)–(S2.9)).
     - Fix the extracted VT for subsequent fitting steps.

  2) Compute μball(T)
     - Evaluate μball(T) from the Landauer expression (Eq.(S3.10)).
     - Keep μball(T) fixed during fitting (physically constrained strategy).

  3) Select fitting dataset
     - Define VGt = VG – VT – VD/2 using the fixed VT.
     - Keep data points satisfying VGt ≥ VGt,min.

  4) Simultaneous nonlinear fitting of ID and gm
     - Fit parameters: {μ0, θ1, θ2, ΔVT}.
     - Model: Landauer-consistent current (Eq.(S4.7)),
              with η constrained by η = μ0/μball(T) (Eq.(S4.5)).
     - Compute gm analytically as gm = ∂ID/∂VG from the model.
     - Minimize a weighted least-squares residual of ID and gm
       using MATLAB lsqnonlin.

  5) Post-processing
     - Report fitted parameters {μ0, θ1, θ2, ΔVT}.
     - Evaluate η(T) a posteriori as η(T) = μ0/μball(T).

End for
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