

Table S1. Experimental parameters and refinement details for the structures of **Nd**, **Ho**, **Tm** at 100 and 300K.

Identification code	Nd_100K	Nd_300K	Ho_100K
Deposition number (CCDC)	2528619	2528623	2528620
Empirical formula	C ₂₇ H ₂₉ N ₂ NdO ₆	C ₂₇ H ₂₉ N ₂ NdO ₆	C ₂₇ H ₂₉ HoN ₂ O ₆
Formula weight	621.76	621.76	642.45
Temperature/K	100(2)	300(2)	100(2)
Crystal system	monoclinic	monoclinic	monoclinic
Space group	P2 ₁ /n	P2 ₁ /n	P2 ₁ /n
a/Å	9.4200(9)	9.520(5)	9.4176(7)
b/Å	20.973(2)	21.161(11)	20.7896(10)
c/Å	14.6628(14)	14.926(7)	14.4163(11)
α/°	90	90	90
β/°	98.810(3)	99.216(16)	98.198(3)
γ/°	90	90	90
Volume/Å ³	2862.8(5)	2968(3)	2793.7(3)
Z	4	4	4
ρ _{calc} /g/cm ³	1.443	1.391	1.527
μ/mm ⁻¹	1.853	1.787	2.873
F(000)	1252.0	1252.0	1280.0
Crystal size/mm ³	0.24 × 0.08 × 0.03	0.24 × 0.08 × 0.03	0.24 × 0.08 × 0.03
Radiation	MoKα (λ = 0.71073)	MoKα (λ = 0.71073)	MoKα (λ = 0.71073)
2θ range for data collection/°	3.884 to 50.496	4.74 to 50.498	3.918 to 53.998
Index ranges	-11 ≤ h ≤ 11 0 ≤ k ≤ 25 0 ≤ l ≤ 17	-11 ≤ h ≤ 11 -25 ≤ k ≤ 25 -17 ≤ l ≤ 17	-12 ≤ h ≤ 12 -26 ≤ k ≤ 22 -18 ≤ l ≤ 18
Reflections collected	5190	48426	25790
Independent reflections	5190 [R _{int} = 0.0980, R _{sigma} = 0.0805]	5371 [R _{int} = 0.1147 R _{sigma} = 0.0568]	6086 [R _{int} = 0.0465, R _{sigma} = 0.0426]
Data/restraints/parameters	5190/0/332	5371/7/331	6086/0/331
Goodness-of-fit on F ²	1.083	1.071	1.067
Final R indexes [I ≥ 2σ (I)]	R ₁ = 0.0467 wR ₂ = 0.0822	R ₁ = 0.0401 wR ₂ = 0.0763	R ₁ = 0.0378 wR ₂ = 0.0738
Final R indexes [all data]	R ₁ = 0.0797 wR ₂ = 0.0986	R ₁ = 0.0794 wR ₂ = 0.0970	R ₁ = 0.0464 wR ₂ = 0.0767
Largest diff. peak/hole / e Å ⁻³	1.20/-0.73	0.98/-0.59	1.38/-1.28

Table S1 (continued).

Identification code	Ho_300K	Tm_100K	Tm_300K
Deposition number (CCDC)	2528622	2528621	2528624
Empirical formula	C ₂₇ H ₂₉ HoN ₂ O ₆	C ₂₇ H ₂₉ N ₂ O ₆ Tm	C ₂₇ H ₂₉ N ₂ O ₆ Tm
Formula weight	642.45	646.45	646.45
Temperature/K	300(2)	100(2)	300(2)
Crystal system	monoclinic	monoclinic	monoclinic
Space group	P2 ₁ /n	P2 ₁ /n	P2 ₁ /n
a/Å	9.4837(7)	9.3976(4)	9.482(3)
b/Å	21.0014(14)	20.7896(9)	21.013(6)
c/Å	14.6551(10)	14.3501(6)	14.641(6)
α/°	90	90	90
β/°	98.797(2)	98.241(2)	98.921(18)
γ/°	90	90	90
Volume/Å ³	2884.5(3)	2774.7(2)	2881.8(17)
Z	4	4	4
ρ _{calc} /cm ³	1.479	1.548	1.490
μ/mm ⁻¹	2.782	3.238	3.118
F(000)	1280.0	1288.0	1288.0
Crystal size/mm ³	0.12 × 0.1 × 0.02	0.28 × 0.08 × 0.04	0.28 × 0.08 × 0.04
Radiation	MoKα (λ = 0.71073)	MoKα (λ = 0.71073)	MoKα (λ = 0.71073)
2Θ range for data collection/°	3.878 to 52.76	4.798 to 50.7	4.76 to 55.752
Index ranges	-11 ≤ h ≤ 11 -26 ≤ k ≤ 26 -18 ≤ l ≤ 18	-11 ≤ h ≤ 11 -25 ≤ k ≤ 25 -17 ≤ l ≤ 17	-12 ≤ h ≤ 12 -27 ≤ k ≤ 27 -19 ≤ l ≤ 19
Reflections collected	36092	41183	54804
Independent reflections	5894 [R _{int} = 0.0595 R _{sigma} = 0.0426]	5089 [R _{int} = 0.0768 R _{sigma} = 0.0427]	6883 [R _{int} = 0.0570 R _{sigma} = 0.0340]
Data/restraints/parameters	5894/7/331	5089/0/331	6883/6/331
Goodness-of-fit on F ²	1.197	1.033	1.048
Final R indexes [I ≥ 2σ (I)]	R ₁ = 0.0437 wR ₂ = 0.0797	R ₁ = 0.0314 wR ₂ = 0.0624	R ₁ = 0.0329 wR ₂ = 0.0590
Final R indexes [all data]	R ₁ = 0.0655 wR ₂ = 0.0854	R ₁ = 0.0420 wR ₂ = 0.0657	R ₁ = 0.0478 wR ₂ = 0.0641
Largest diff. peak/hole / e Å ⁻³	0.70/-0.70	1.04/-1.13	1.07/-1.02

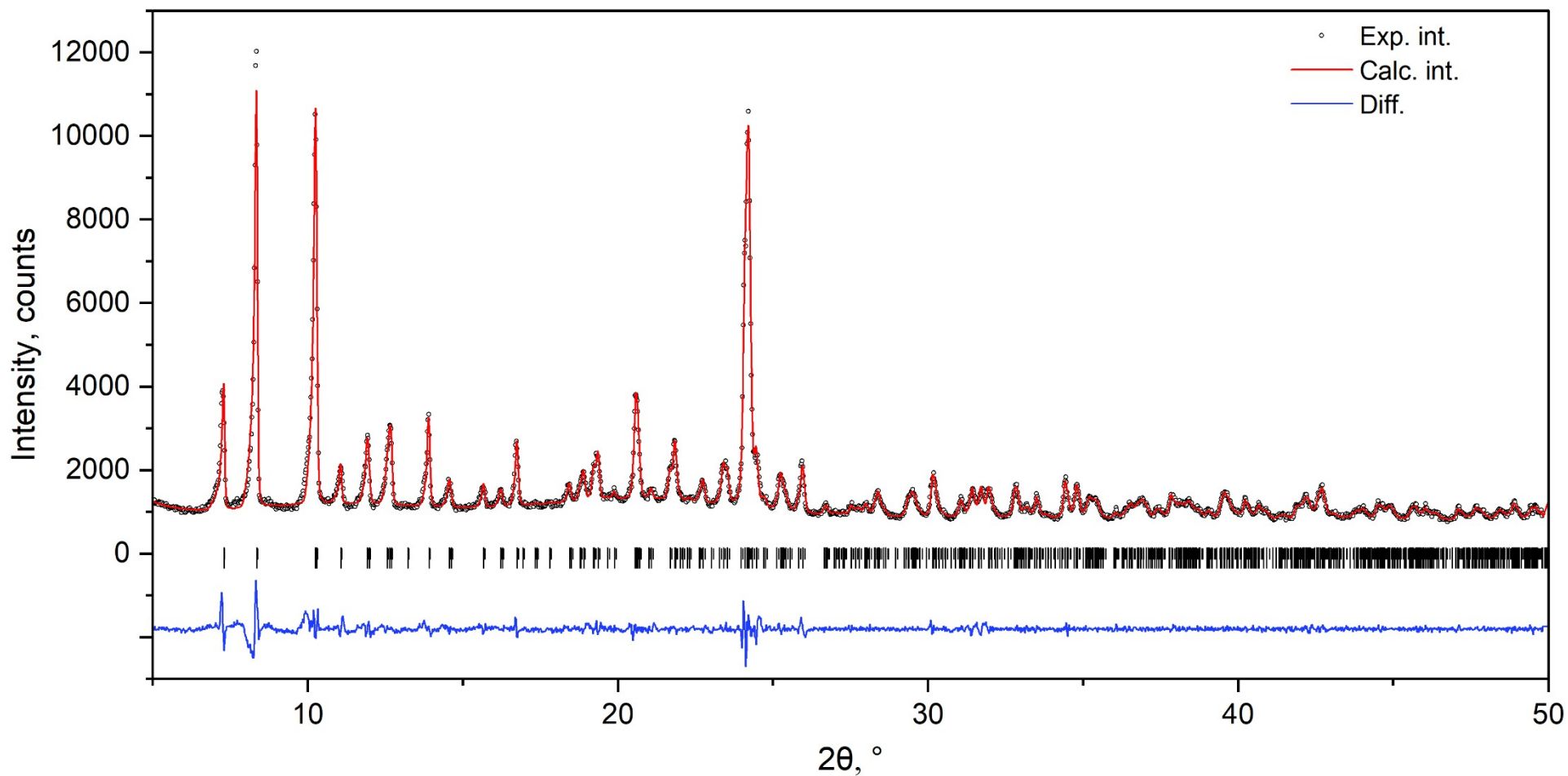


Fig. S1. Le Bail refinement results for powder pattern of Nd. The picture shows experimental (black round marks) and calculated (red line) data with difference curve ($I(\text{exp})-I(\text{calc})$, blue line) and $\text{CuK}_{\alpha 1}$ (major black ticks) and $\text{CuK}_{\alpha 2}$ (minor black ticks) peaks positions.

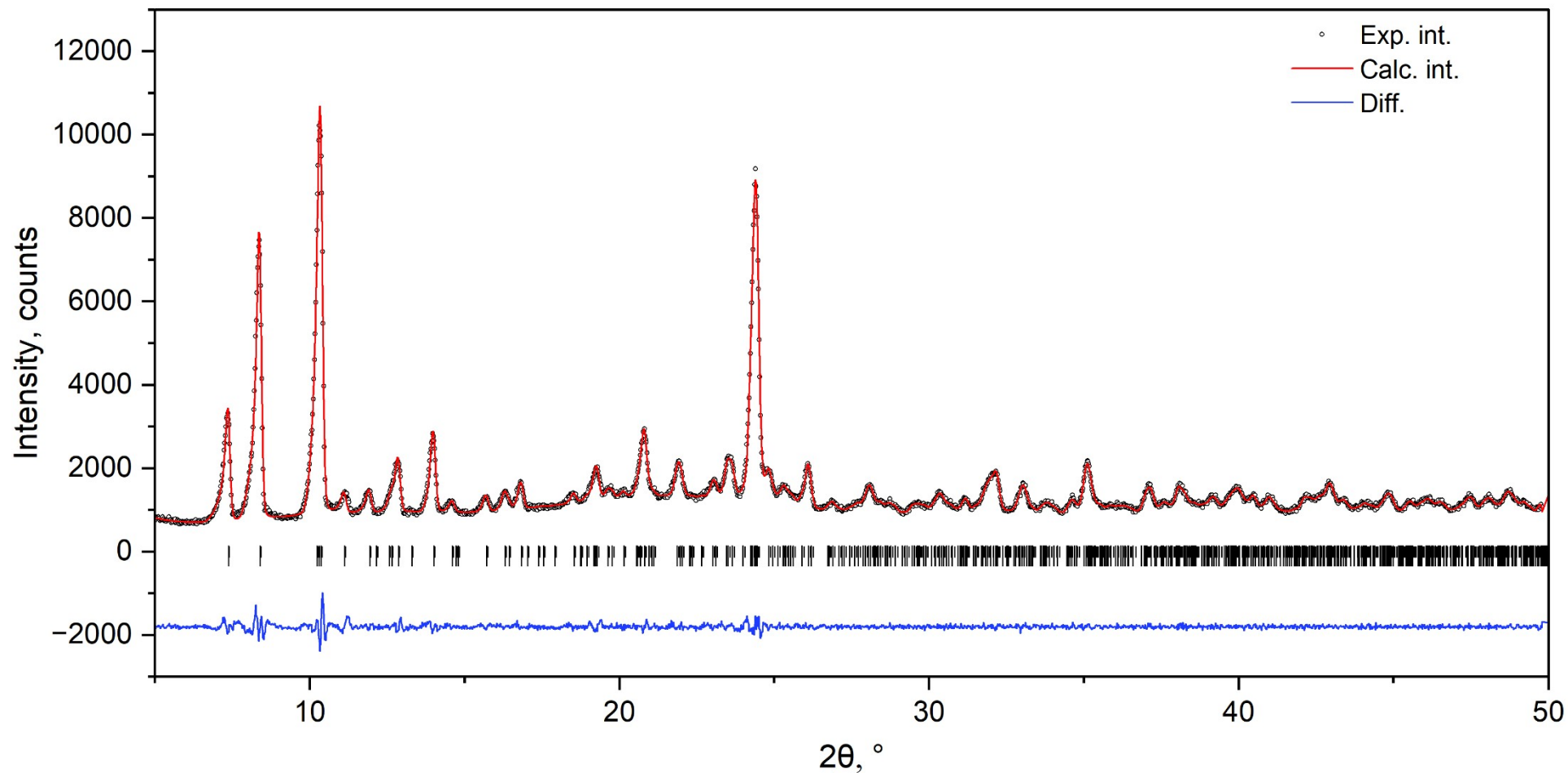


Fig. S2. Le Bail refinement results for powder pattern of **Ho**. The picture shows experimental (black round marks) and calculated (red line) data with difference curve ($I(\text{exp})-I(\text{calc})$, blue line) and $\text{CuK}_{\alpha 1}$ (major black ticks) and $\text{CuK}_{\alpha 2}$ (minor black ticks) peaks positions.

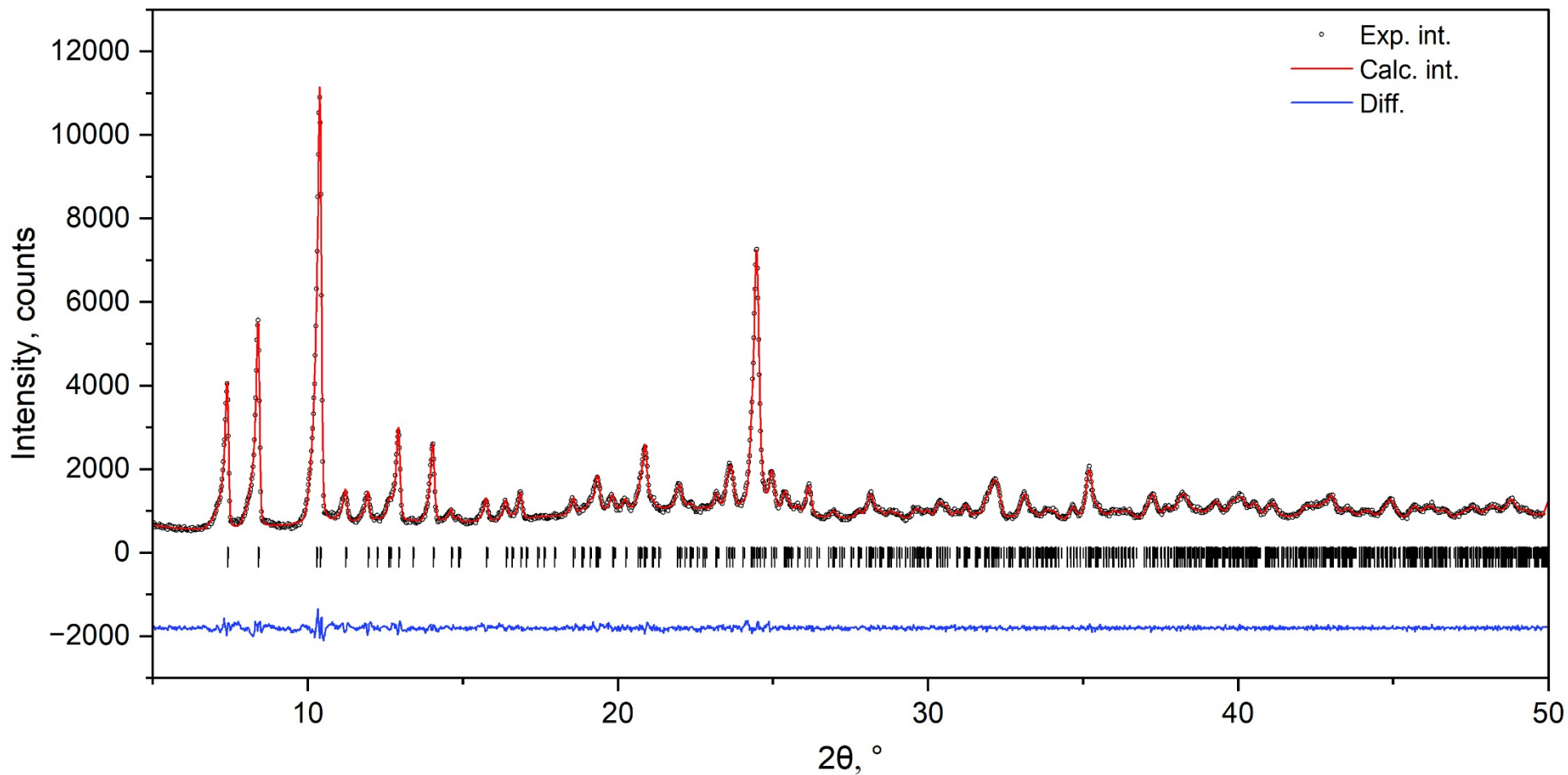


Fig. S3. Le Bail refinement results for powder pattern of **Tm**. The picture shows experimental (black round marks) and calculated (red line) data with difference curve ($I(\text{exp}) - I(\text{calc})$, blue line) and $\text{CuK}_{\alpha 1}$ (major black ticks) and $\text{CuK}_{\alpha 2}$ (minor black ticks) peaks positions.

Table S2. Le Bail refinement indicators and refined parameters for powder patterns of polycrystalline samples of complexes synthesized.

Sample	SG	a , Å	b , Å	c , Å	α , °	β , °	γ , °	R_p	wR_p	GOF
Nd	P2 ₁ /n	9.5783(6)	21.1574(7)	14.9666(6)	90	99.488(6)	90	3.55	5.07	1.87
Ho	P2 ₁ /n	9.5917(7)	21.0569(14)	14.7422(9)	90	99.190(7)	90	2.84	3.80	1.41
Tm	P2 ₁ /n	9.5285(7)	21.0098(11)	14.6330(9)	90	98,605(6)	90	2.84	3.79	1.29

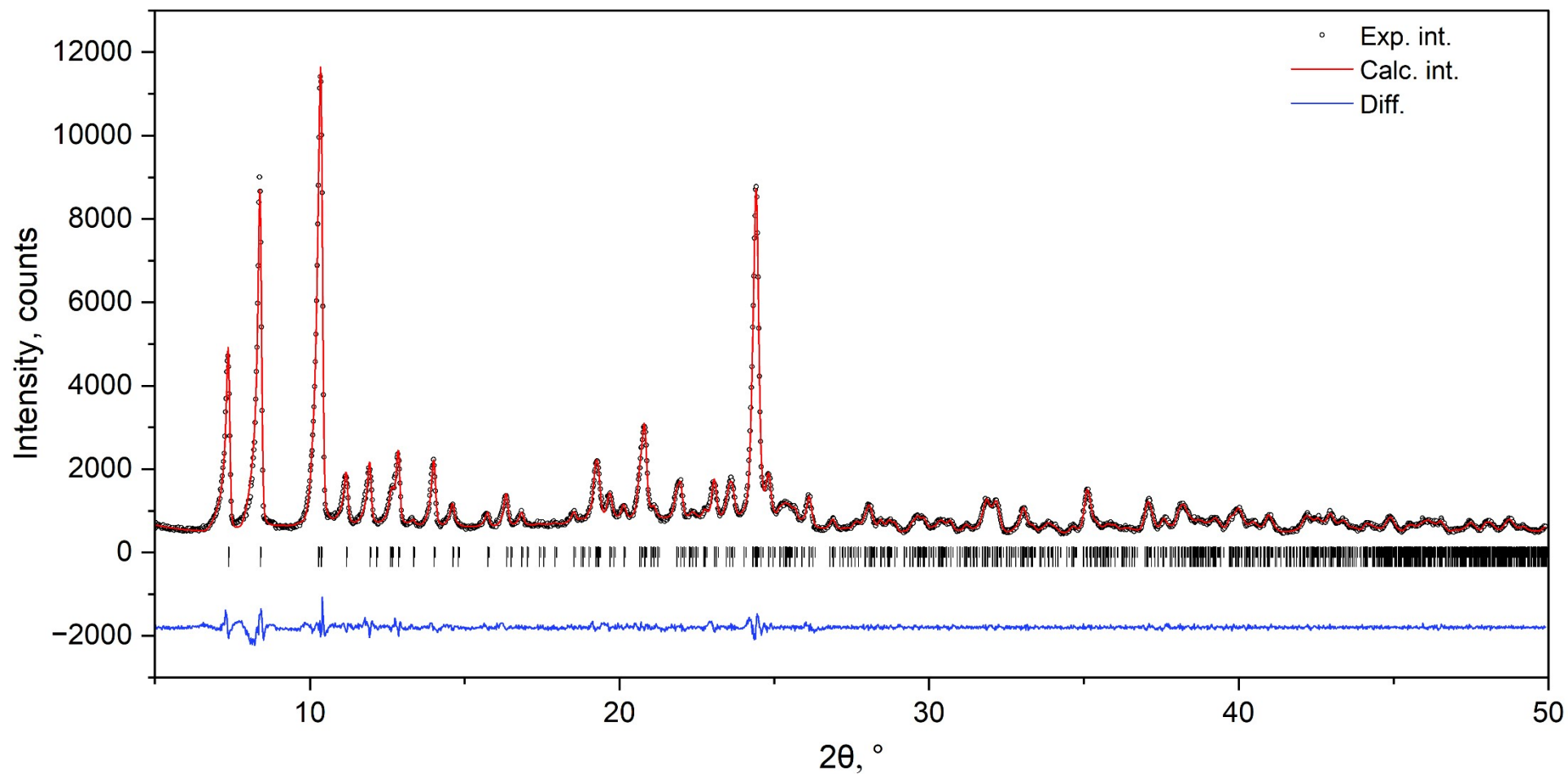


Fig. S4. Le Bail refinement results for powder pattern of Nd@Y. The picture shows experimental (black round marks) and calculated (red line) data with difference curve ($I(exp)-I(calc)$, blue line) and CuK_{a1} (major black ticks) and CuK_{a2} (minor black ticks) peaks positions.

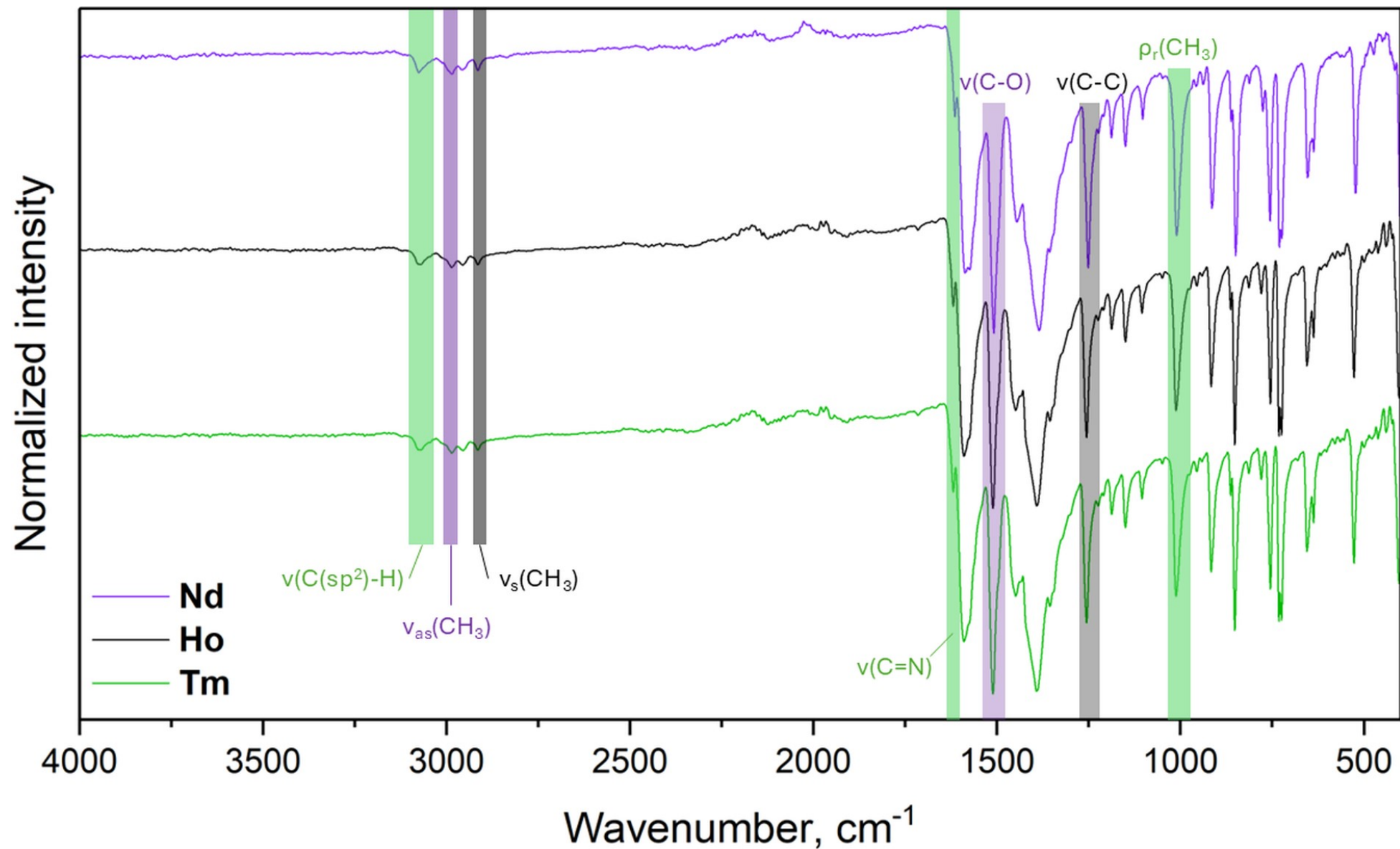


Fig. S5. Normalized ATR-IR spectra of **Nd**, **Ho** and **Tm** with the assignments for the most characteristic bands.

Table S3. SHAPE calculations for structures studied at 100 and 300K.

Structure	OP-8	HPY-8	HBPY-8	CU-8	SAPR-8	TDD-8	JGBF-8	JETBPY-8	JBTPR-8	BTPR-8	JSD-8	TT-8	ETBPY-8
Nd 100K	30.932	22.172	17.063	10.197	0.603	2.697	16.875	28.243	2.920	2.455	5.446	11.038	23.451
Nd 300K	30.667	22.108	16.921	9.838	0.576	2.656	16.830	28.194	3.062	2.532	5.697	10.596	23.211
Ho 100K	29.668	22.581	16.559	9.792	0.375	2.238	17.013	28.107	2.704	2.226	4.951	10.646	24.032
Ho 300K	29.866	22.661	16.596	9.822	0.391	2.529	17.012	28.027	2.912	2.425	5.286	10.671	23.936
Tm 100K	29.411	22.795	16.539	9.840	0.353	2.187	16.835	27.924	2.688	2.229	4.836	10.673	24.206
Tm 300K	29.808	22.745	16.582	9.870	0.396	2.458	16.925	28.000	2.901	2.449	5.191	10.718	24.100

Reference shapes

Coordination number = 8

OP-8 (D_{8h}) – Octagon

HPY-8 (C_{7v}) – Heptagonal pyramid

HBPY-8 (D_{6h}) – Hexagonal bipyramid

CU-8 (O_h) – Cube

SAPR-8 (D_{4d}) – Square antiprism

TDD-8 (D_{2d}) – Triangular dodecahedron

JGBF-8 (D_{2d}) – Johnson – Gyrobifastigium J26

JETBPY-8 (D_{3h}) - Johnson – Elongated triangular bipyramid J14

JBTP-8 (C_{2v}) - Johnson – Biaugmented trigonal prism J50

BTPR-8 (C_{2v}) – Biaugmented trigonal prism

JSD-8 (D_{2d}) – Snub Disphenoid J84

TT-8 (T_d) – Triakis tetrahedron

ETBPY-8 (D_{3h}) – Elongated trigonal bipyramid

Table S4. Geometric parameters of the square antiprismatic polyhedra in structures of **Nd**, **Ho** and **Tm** at 100 and 300 K.

Structure	Nd_100K	Nd_300K	Ho_100K	Ho_300K	Tm_100K	Tm_300K
Average rms deviation for O ₁ O ₂ O ₃ O ₄ , Å	0.041	0.025	0.033	0.015	0.039	0.024
Average rms deviation for O ₅ O ₆ N ₁ N ₂ , Å	0.000	0.005	0.018	0.006	0.016	0.002
Angle between antiprism bases, °	3.6	3.1	2.1	2.3	1.7	2.0
Range of the flatness angle, °	53.7-58.8	53.8-59.0	55.0-59.1	54.7-59.2	55.2-59.2	54.9-59.0
Average Ln-O distances, Å	2.403(4)	2.399(4)	2.312(3)	2.313(4)	2.291(3)	2.292(3)
Average Ln-N distances, Å	2.647(5)	2.650(5)	2.537(4)	2.549(5)	2.515(3)	2.538(3)

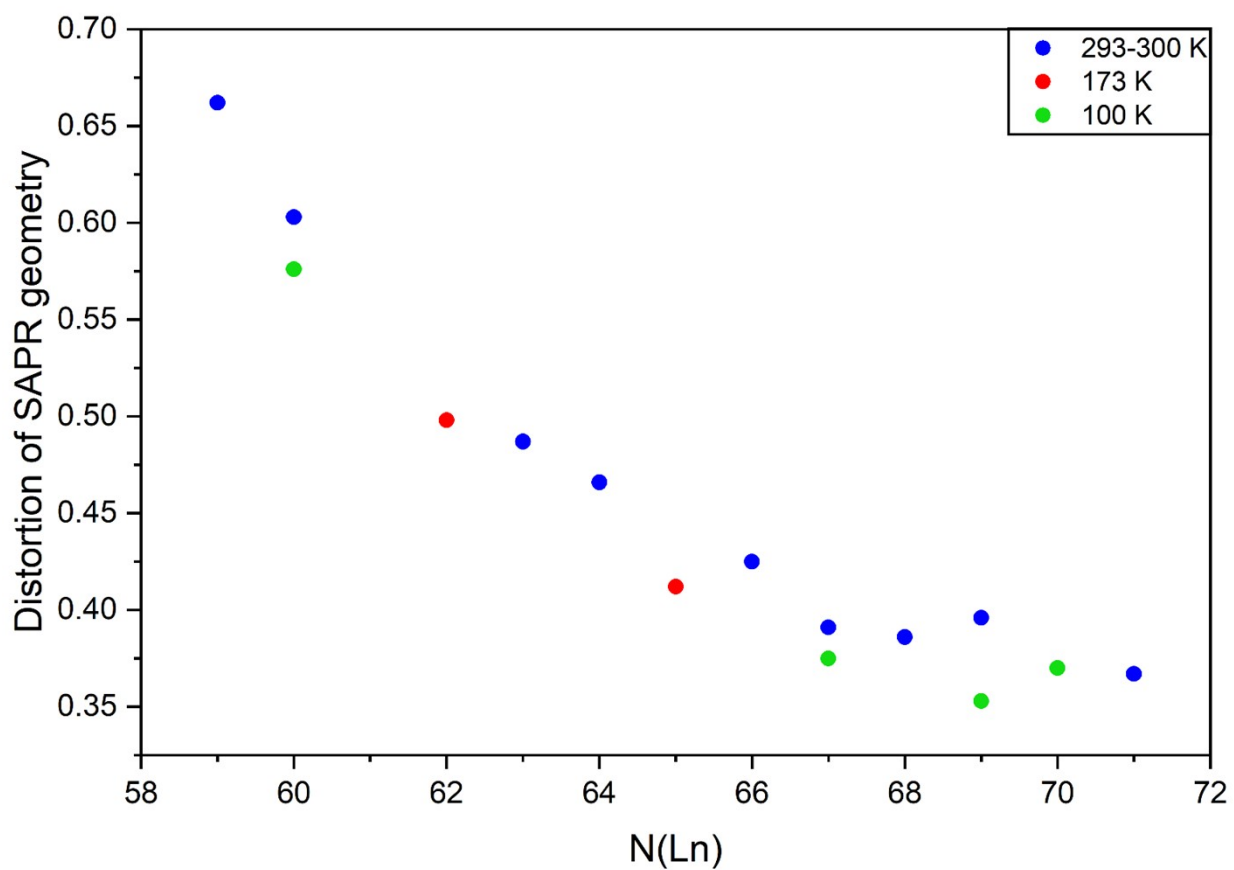


Fig. S6. The distortion of the coordination polyhedron in $[\text{Ln}(\text{acac})_3(\text{phen})]$ structures as function of the lanthanide number.

Table S5. Selected bond lengths (Ln environment) for the studied structures.

Nd_100K			Nd_300K			Ho_100K		
Atom	Atom	Length/Å	Atom	Atom	Length/Å	Atom	Atom	Length/Å
Nd1	O6	2.368(4)	Nd1	O6	2.379(4)	Ho1	O6	2.288(3)
Nd1	O3	2.389(4)	Nd1	O3	2.388(4)	Ho1	O3	2.311(3)
Nd1	O4	2.419(4)	Nd1	O4	2.409(4)	Ho1	O4	2.321(3)
Nd1	O5	2.414(4)	Nd1	O5	2.401(4)	Ho1	O5	2.309(3)
Nd1	O2	2.399(4)	Nd1	O2	2.403(4)	Ho1	O2	2.311(3)
Nd1	N2	2.678(5)	Nd1	N2	2.667(5)	Ho1	N2	2.560(4)
Nd1	O1	2.429(4)	Nd1	O1	2.415(4)	Ho1	O1	2.334(3)
Nd1	N1	2.616(5)	Nd1	N1	2.633(4)	Ho1	N1	2.514(4)
Ho_300K			Tm_100K			Tm_300K		
Atom	Atom	Length/Å	Atom	Atom	Length/Å	Atom	Atom	Length/Å
Ho1	O6	2.292(4)	Tm1	O6	2.265(3)	Tm1	O6	2.269(3)
Ho1	O3	2.306(4)	Tm1	O3	2.284(3)	Tm1	O3	2.284(3)
Ho1	O4	2.328(4)	Tm1	O4	2.311(3)	Tm1	O4	2.311(3)
Ho1	O5	2.312(4)	Tm1	O5	2.290(3)	Tm1	O5	2.289(3)
Ho1	O2	2.304(4)	Tm1	O2	2.287(3)	Tm1	O2	2.288(3)
Ho1	N2	2.571(5)	Tm1	N2	2.539(3)	Tm1	N2	2.561(3)
Ho1	O1	2.335(4)	Tm1	O1	2.309(3)	Tm1	O1	2.309(3)
Ho1	N1	2.527(5)	Tm1	N1	2.490(3)	Tm1	N1	2.515(3)

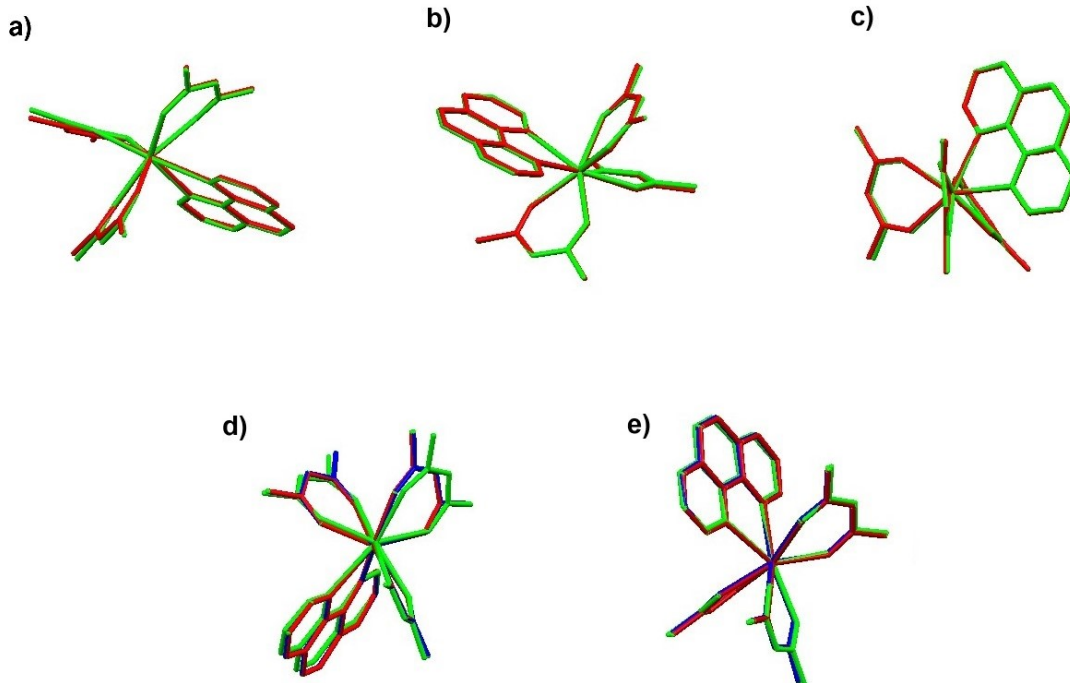
**Fig. S7.** Overlaying of the $[\text{Ln}(\text{acac})_3(\text{phen})]$ fragments: a) Nd_100K (green) and Nd_300K (red); b) Ho_100K (green) and Ho_300K; c) Tm_100K (green) and Tm_300K (red); d) Nd_100K (green), Ho_100K (blue) and Tm_100K (red); e) Nd_300K (green), Ho_300K (blue) and Tm_300K (red).

Table S6. The shortest Ln...Ln distances in the studied structures.

Structure	Ln...Ln shortest distances		
Nd_100K	8.537(1)	9.064(1)	9.420(1)
Nd_300K	8.448(4)	9.122(5)	9.520(5)
Ho_100K	7.9708(8)	8.9852(8)	9.4176(8)
Ho_300K	8.2154(8)	9.0612(8)	9.4837(8)
Tm_100K	7.9460(6)	8.9614(6)	9.3976(5)
Tm_300K	8.188(3)	9.056(4)	9.482(4)

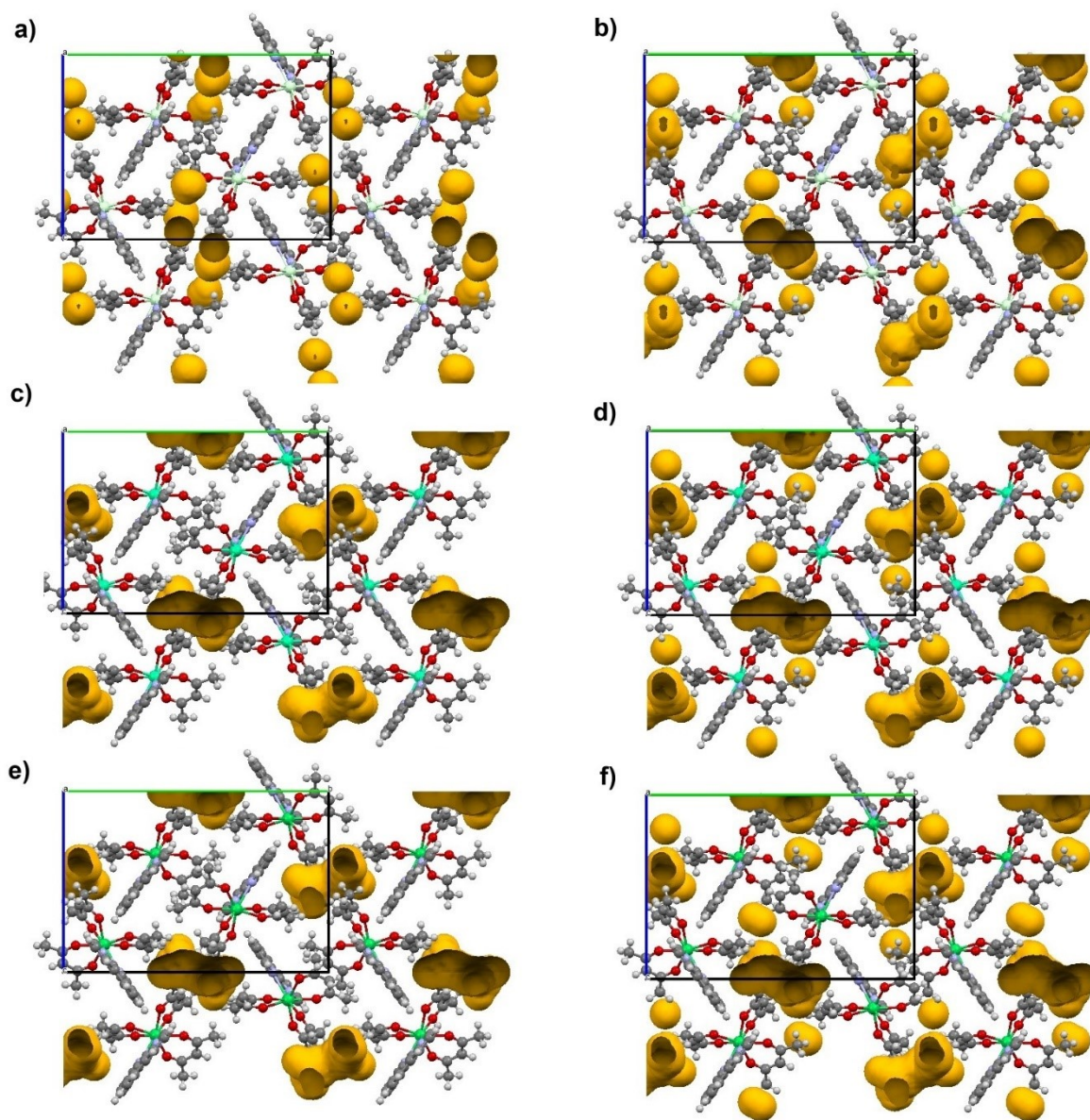


Fig. S8. Visualization of the voids (probe radius 1.2 Å) in structures **Nd_100K** (a), **Nd_300K** (b), **Ho_100K** (c), **Ho_300K** (d), **Tm_100K** (e) and **Tm_300K** (f).

Table S7. Voids (probe radius 1.2 Å) parameters in the structures of **Nd**, **Ho**, **Tm** at 100 and 300K.

Compound	Temperature, K	Unit cell volume, Å ³	Unit cell voids volume, Å ³	Voids fraction, %
Nd	100	2862.8(5)	100	3.5
Nd	300	2968(3)	155	5.2
Gd	293	2916.99(17)	198	6.8
Ho	100	2793.7(3)	197	7.0
Ho	300	2884.5(3)	215	7.5
Er	295	2881.0(6)	198	7.6
Tm	100	2774.7(2)	191	6.9
Tm	300	2881.8(17)	232	8.0

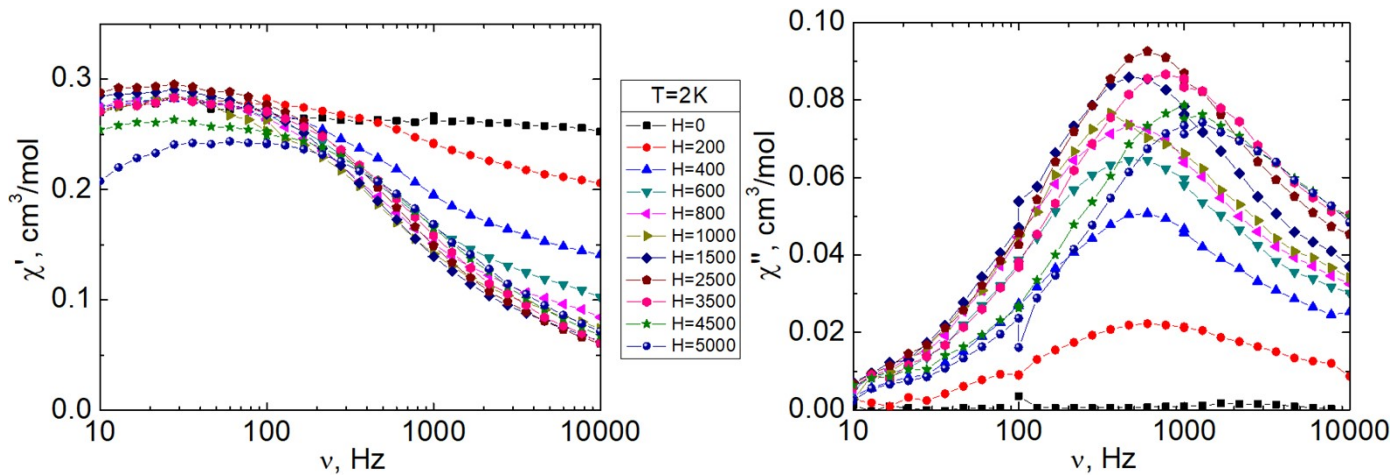
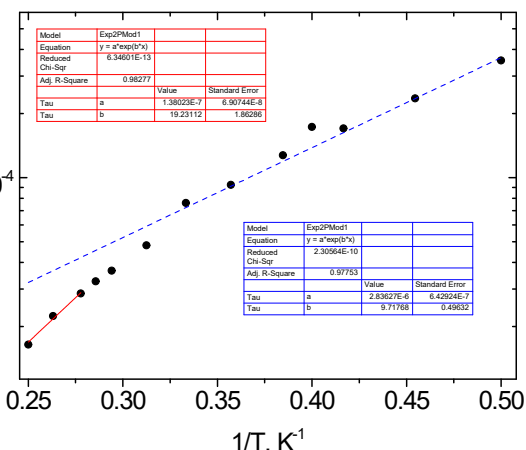
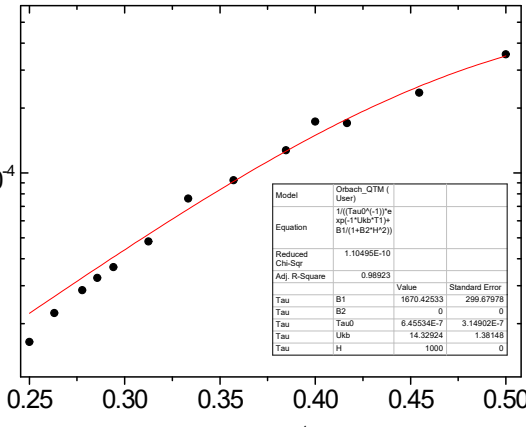
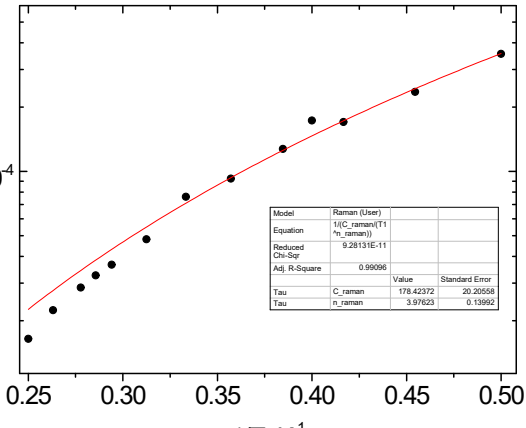
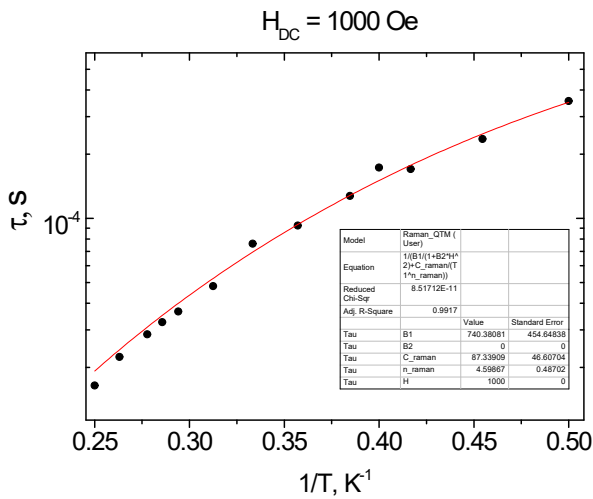


Fig. S9. Frequency dependencies of the in-phase (χ' , left) and out-of-phase (χ'' , right) components of the dynamic magnetic susceptibility of complex **Nd** measured at $T=2\text{K}$ under various DC magnetic fields. Solid lines are visual guides.

Table S8. Various approximations of τ vs. $1/T$ dependency for complex Nd under a DC field of 1000 Oe.

Dependence of the relaxation time τ on the reciprocal temperature for Tb complex 1 ($H = 1$ kOe, $T = 2-4$ K).	Fit function, temperature range, and the best-fit parameters with uncertainties.																																																
<p style="text-align: center;">$H_{DC} = 1000$ Oe</p>  <table border="1" data-bbox="231 369 470 470"> <thead> <tr> <th>Model</th> <th>Exp2PMod1</th> <th></th> <th></th> </tr> </thead> <tbody> <tr> <td>Equation</td> <td>$y = a \cdot \exp(b/x)$</td> <td></td> <td></td> </tr> <tr> <td>Reduced Chi-Sqr</td> <td>6.34601E-13</td> <td></td> <td></td> </tr> <tr> <td>Adj. R-Square</td> <td>0.98277</td> <td></td> <td></td> </tr> <tr> <td>Tau</td> <td>a</td> <td>1.38022E-7</td> <td>6.90144E-8</td> </tr> <tr> <td>Tau</td> <td>b</td> <td>19.23112</td> <td>1.86296</td> </tr> </tbody> </table> <table border="1" data-bbox="446 560 678 660"> <thead> <tr> <th>Model</th> <th>Exp2PMod1</th> <th></th> <th></th> </tr> </thead> <tbody> <tr> <td>Equation</td> <td>$y = a \cdot \exp(b/x)$</td> <td></td> <td></td> </tr> <tr> <td>Reduced Chi-Sqr</td> <td>2.30564E-10</td> <td></td> <td></td> </tr> <tr> <td>Adj. R-Square</td> <td>0.97753</td> <td></td> <td></td> </tr> <tr> <td>Tau</td> <td>a</td> <td>2.83627E-6</td> <td>6.42624E-7</td> </tr> <tr> <td>Tau</td> <td>b</td> <td>9.71168</td> <td>6.48692</td> </tr> </tbody> </table>	Model	Exp2PMod1			Equation	$y = a \cdot \exp(b/x)$			Reduced Chi-Sqr	6.34601E-13			Adj. R-Square	0.98277			Tau	a	1.38022E-7	6.90144E-8	Tau	b	19.23112	1.86296	Model	Exp2PMod1			Equation	$y = a \cdot \exp(b/x)$			Reduced Chi-Sqr	2.30564E-10			Adj. R-Square	0.97753			Tau	a	2.83627E-6	6.42624E-7	Tau	b	9.71168	6.48692	<p>Orbach</p> $\tau = \tau_0 \cdot \exp\{\Delta E/kT\}$ <p>$T = 3.6-4$ K $\Delta E/k = 19 \pm 2$ K $\tau_0 = 1.4 \cdot 10^{-7} \pm 7 \cdot 10^{-8}$ s $R^2 = 0.98277$ (red line)</p> <p style="text-align: center;">Meaningless fit!!!</p> <p>$T = 2-4$ K $\Delta E/k = 9.7 \pm 0.5$ K $\tau_0 = 2.8 \cdot 10^{-6} \pm 6 \cdot 10^{-7}$ s $R^2 = 0.97753$ (blue line)</p> <p style="text-align: center;">Meaningless fit!!!</p>
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<p style="text-align: center;">$H_{DC} = 1000$ Oe</p>  <table border="1" data-bbox="470 1086 718 1265"> <thead> <tr> <th>Model</th> <th>Orbach_QTM (User)</th> <th></th> <th></th> </tr> </thead> <tbody> <tr> <td>Equation</td> <td>$1/(\tau_0 \cdot \exp(-\Delta E/kT) + B)$</td> <td></td> <td></td> </tr> <tr> <td>Reduced Chi-Sqr</td> <td>1.10485E-10</td> <td></td> <td></td> </tr> <tr> <td>Adj. R-Square</td> <td>0.98923</td> <td></td> <td></td> </tr> <tr> <td>Tau</td> <td>B1</td> <td>1670.42533</td> <td>296.67976</td> </tr> <tr> <td>Tau</td> <td>B2</td> <td>0</td> <td>0</td> </tr> <tr> <td>Tau</td> <td>tau0</td> <td>6.45534E-7</td> <td>3.14902E-7</td> </tr> <tr> <td>Tau</td> <td>Ukb</td> <td>14.32624</td> <td>1.38148</td> </tr> <tr> <td>Tau</td> <td>H</td> <td>1000</td> <td>0</td> </tr> </tbody> </table>	Model	Orbach_QTM (User)			Equation	$1/(\tau_0 \cdot \exp(-\Delta E/kT) + B)$			Reduced Chi-Sqr	1.10485E-10			Adj. R-Square	0.98923			Tau	B1	1670.42533	296.67976	Tau	B2	0	0	Tau	tau0	6.45534E-7	3.14902E-7	Tau	Ukb	14.32624	1.38148	Tau	H	1000	0	<p>Orbach+QTM</p> $\tau^{-1} = \tau_0^{-1} \cdot \exp\{-\Delta E/kT\} + B$ <p>$T = 2-4$ K $\Delta E/k = 14 \pm 1$ K $\tau_0 = 6 \cdot 10^{-7} \pm 3 \cdot 10^{-7}$ s $B = 1670 \pm 300$ s⁻¹ $R^2 = 0.98923$</p> <p style="text-align: center;">Meaningless fit!!!</p>												
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Raman+ QTM

$$\tau^{-1} = C_{\text{Raman}} T^{n_{\text{Raman}}} + B$$

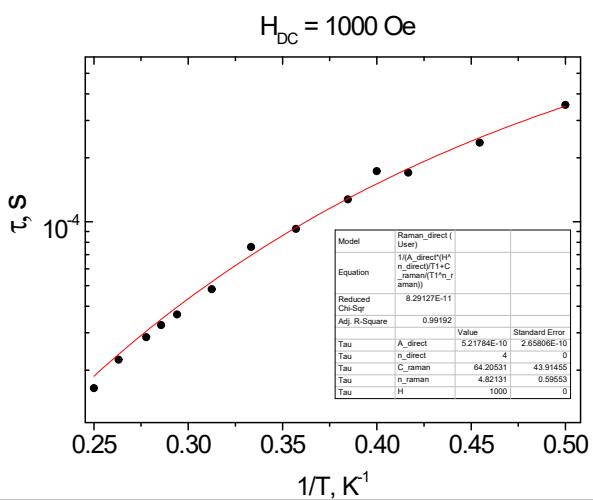
$$T = 2-4 \text{ K}$$

$$C_{\text{Raman}} = 87 \pm 47 \text{ s}^{-1} \text{K}^{-n_{\text{Raman}}}$$

$$n_{\text{Raman}} = 4.6 \pm 0.5$$

$$B = 740 \pm 455 \text{ s}^{-1}$$

$$R^2 = 0.9917$$



Raman+Direct

$$\tau^{-1} = C_{\text{Raman}} T^{n_{\text{Raman}}} + A_{\text{direct}} T^4$$

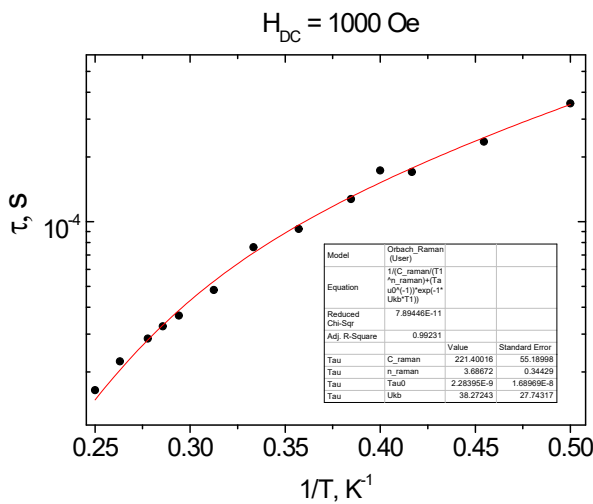
$$T = 2-4 \text{ K}$$

$$C_{\text{Raman}} = 64 \pm 44 \text{ s}^{-1} \text{K}^{-n_{\text{Raman}}}$$

$$n_{\text{Raman}} = 4.8 \pm 0.6$$

$$A_{\text{direct}} = 5 \cdot 10^{-10} \pm 3 \cdot 10^{-10} \text{ K}^{-1} \text{Oe}^{-4} \text{ s}^{-1}$$

$$R^2 = 0.99192$$



Orbach+Raman

$$\tau^{-1} = \tau_0^{-1} \cdot \exp\left\{-\frac{\Delta E}{kT}\right\} + C_{\text{Raman}} T^{n_{\text{Raman}}}$$

$$T = 2-4 \text{ K}$$

$$\Delta E/k = 38 \pm 28 \text{ K}$$

$$\tau_0 = 2 \cdot 10^{-9} \pm 1.7 \cdot 10^{-8} \text{ s}$$

$$C_{\text{Raman}} = 221 \pm 55 \text{ s}^{-1} \text{K}^{-n_{\text{Raman}}}$$

$$n_{\text{Raman}} = 3.7 \pm 0.3 \text{ (if } n_{\text{Raman}} \text{ is not fixed, then it tends to be 0)}$$

$$R^2 = 0.99231$$

Over-parametrization!!!

$$\tau^{-1} = \tau_0^{-1} \cdot \exp\left\{-\frac{\Delta E}{kT}\right\} + C_{\text{Raman}} T^{n_{\text{Raman}}}$$

$$T = 2-4 \text{ K}$$

$$\Delta E/k = 31 \text{ K (fixed)}$$

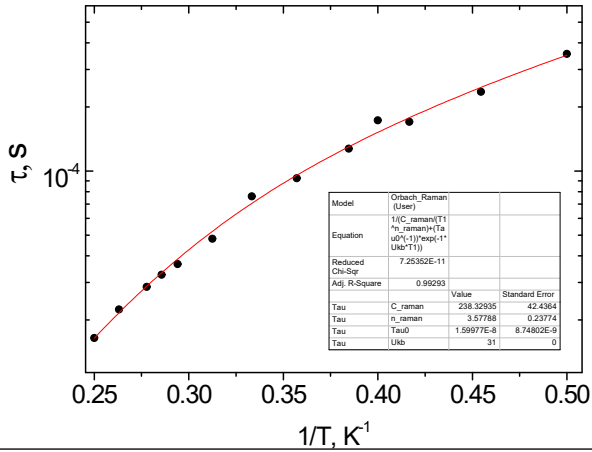
$$\tau_0 = 1.6 \cdot 10^{-8} \pm 9 \cdot 10^{-9} \text{ s}$$

$$C_{\text{Raman}} = 238 \pm 42 \text{ s}^{-1} \text{K}^{-n_{\text{Raman}}}$$

$$n_{\text{Raman}} = 3.6 \pm 0.2$$

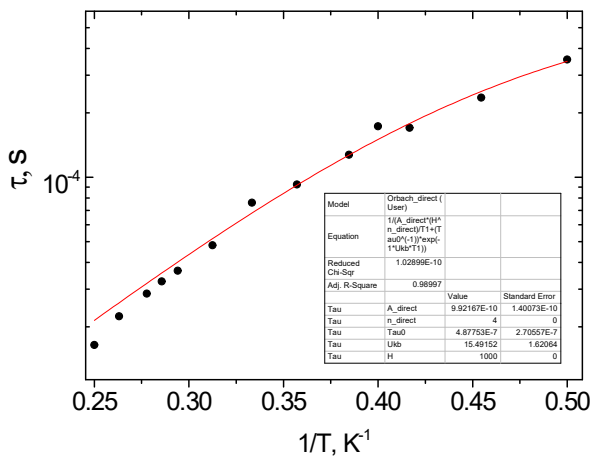
$$R^2 = 0.99293$$

$H_{DC} = 1000 \text{ Oe}$



Meaningless fit!!!

$H_{DC} = 1000 \text{ Oe}$



Orbach+Direct

$$\tau^{-1} = \tau_0^{-1} \cdot \exp\{-\Delta E/kT\} + A_{direct} T H^4$$

$$T = 2-4 \text{ K}$$

$$\Delta E/k = 15 \pm 2 \text{ K}$$

$$\tau_0 = 5 \cdot 10^{-7} \pm 3 \cdot 10^{-7} \text{ s}$$

$$A_{direct} = 1 \cdot 10^{-9} \pm 1 \cdot 10^{-10} \text{ K}^{-1} \text{Oe}^{-4} \text{s}^{-1}$$

$$R^2 = 0.98997$$

Meaningless fit!!!

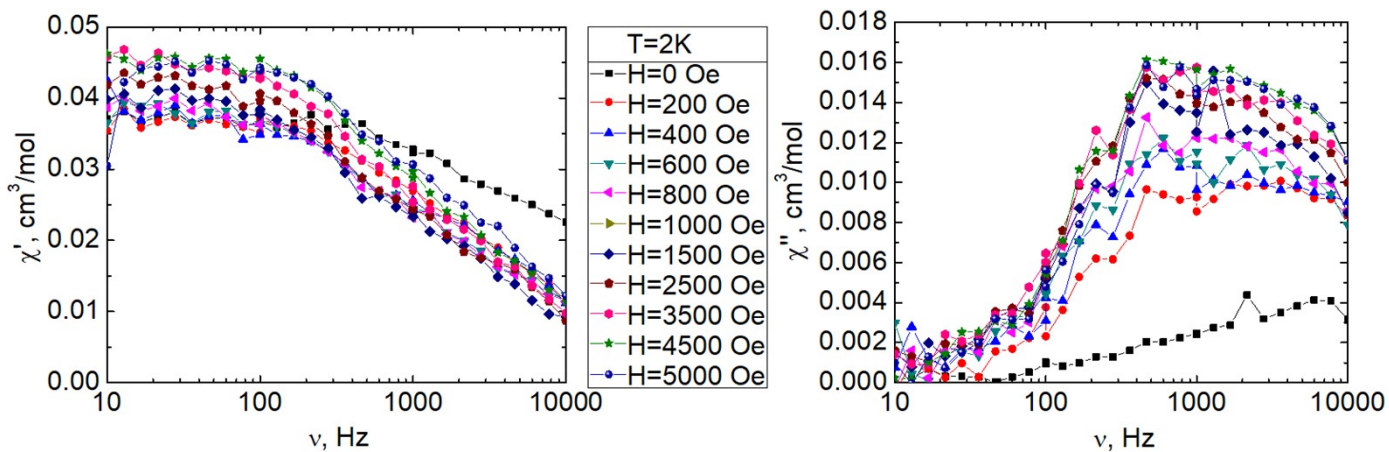


Fig. S10. Frequency dependencies of real (left) and imaginary (right) components of dynamic magnetic susceptibility of **Nd@Y** sample at 2 K in various DC magnetic fields. Solid lines are visual guides.

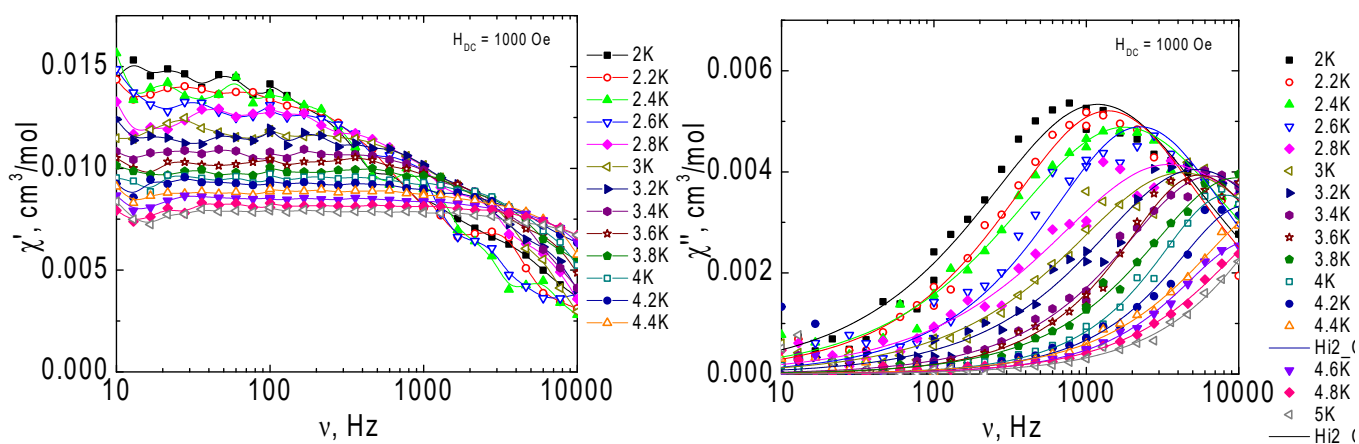
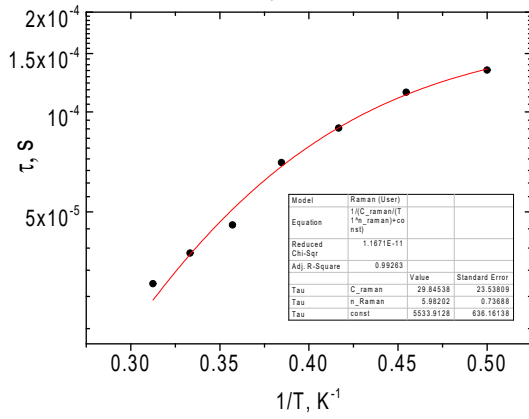


Fig. S11. Frequency dependencies of real (left) and imaginary (right) components of magnetic susceptibility for **Nd@Y** sample in 1000 Oe DC magnetic field. Solid lines on the left section are visual guides. Solid lines on the right section represent best fits by the generalized Debye model.

Table S9. Various approximations of τ vs. $1/T$ dependency for Nd@Y sample under a DC field of 1000 Oe.

Dependence of the relaxation time τ on the reciprocal temperature for Tb complex 1 ($H = 1$ kOe, $T = 2-3.2$ K).	Fit function, temperature range, and the best-fit parameters																								
<p style="text-align: center;">$H_{DC} = 1000$ Oe</p> <p>Model: Orbach (User) Equation: $\tau = \tau_0 \cdot \exp\{\Delta E/kT\}$ Reduced Chi-Sqr: 4.5533E-13 Adj. R-Square: 0.99221</p> <table border="1"> <thead> <tr> <th>Parameter</th> <th>Value</th> <th>Standard Error</th> </tr> </thead> <tbody> <tr> <td>Tau</td> <td>tau0</td> <td>1.8552E-6</td> </tr> <tr> <td>Tau</td> <td>DE_div_k</td> <td>8.9378</td> </tr> <tr> <td>Tau</td> <td>const</td> <td>0</td> </tr> </tbody> </table> <p>Model: Orbach (User) Equation: $\tau = \tau_0 \cdot \exp\{\Delta E/kT\} + B$ Reduced Chi-Sqr: 9.93641E-11 Adj. R-Square: 0.93713</p> <table border="1"> <thead> <tr> <th>Parameter</th> <th>Value</th> <th>Standard Error</th> </tr> </thead> <tbody> <tr> <td>Tau</td> <td>tau0</td> <td>4.32979E-6</td> </tr> <tr> <td>Tau</td> <td>DE_div_k</td> <td>8.99249</td> </tr> <tr> <td>Tau</td> <td>const</td> <td>0.79131</td> </tr> </tbody> </table>	Parameter	Value	Standard Error	Tau	tau0	1.8552E-6	Tau	DE_div_k	8.9378	Tau	const	0	Parameter	Value	Standard Error	Tau	tau0	4.32979E-6	Tau	DE_div_k	8.99249	Tau	const	0.79131	<p>Orbach $\tau = \tau_0 \cdot \exp\{\Delta E/kT\}$ $T = 2.8-3.2$ K $\Delta E/k = 8.98 \pm 0.56$ K $\tau_0 = 1.9 \cdot 10^{-6} \pm 4 \cdot 10^{-7}$ s $R^2 = 0.9922$ (red line)</p> <p style="text-align: center;">Meaningless fit!!!</p> <p>$T = 2-3.2$ K $\Delta E/k = 7.0 \pm 0.8$ K $\tau_0 = 4.3 \cdot 10^{-6} \pm 1.6 \cdot 10^{-6}$ s $R^2 = 0.93713$ (blue line)</p> <p style="text-align: center;">Meaningless fit!!!</p>
Parameter	Value	Standard Error																							
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<p style="text-align: center;">$H_{DC} = 1000$ Oe</p> <p>Model: Orbach (User) Equation: $\tau^{-1} = \tau_0^{-1} \cdot \exp\{-\Delta E/kT\} + B$ Reduced Chi-Sqr: 8.9594E-12 Adj. R-Square: 0.99437</p> <table border="1"> <thead> <tr> <th>Parameter</th> <th>Value</th> <th>Standard Error</th> </tr> </thead> <tbody> <tr> <td>Tau</td> <td>tau0</td> <td>1.4439E-7</td> </tr> <tr> <td>Tau</td> <td>DE_div_k</td> <td>17.5572</td> </tr> <tr> <td>Tau</td> <td>const</td> <td>6326.83697</td> </tr> </tbody> </table>	Parameter	Value	Standard Error	Tau	tau0	1.4439E-7	Tau	DE_div_k	17.5572	Tau	const	6326.83697	<p>Orbach+QTM $\tau^{-1} = \tau_0^{-1} \cdot \exp\{-\Delta E/kT\} + B$ $T = 2-3.2$ K $\Delta E/k = 17 \pm 2$ K $\tau_0 = 1.4 \cdot 10^{-7} \pm 8 \cdot 10^{-8}$ s $B = 6326 \pm 369$ s⁻¹ $R^2 = 0.9944$</p> <p style="text-align: center;">Meaningless fit!!!</p>												
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<p style="text-align: center;">$H_{DC} = 1000$ Oe</p> <p>Model: Raman (User) Equation: $\tau^{-1} = C_{Raman} T^{n_{Raman}}$ Reduced Chi-Sqr: 4.83742E-11 Adj. R-Square: 0.96314</p> <table border="1"> <thead> <tr> <th>Parameter</th> <th>Value</th> <th>Standard Error</th> </tr> </thead> <tbody> <tr> <td>Tau</td> <td>C_Raman</td> <td>909.36571</td> </tr> <tr> <td>Tau</td> <td>n_Raman</td> <td>2.96055</td> </tr> <tr> <td>Tau</td> <td>const</td> <td>0</td> </tr> </tbody> </table>	Parameter	Value	Standard Error	Tau	C_Raman	909.36571	Tau	n_Raman	2.96055	Tau	const	0	<p>Raman $\tau^{-1} = C_{Raman} T^{n_{Raman}}$ $T = 2-3.2$ K $C_{Raman} = 909 \pm 200$ s⁻¹ K^{-nRaman} $n_{Raman} = 2.96 \pm 0.26$ $R^2 = 0.96311$</p> <p>Moderate R^2</p>												
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Tau	const	0																							

$H_{DC} = 1000 \text{ Oe}$



Raman+ QTM

$$\tau^{-1} = C_{\text{Raman}} T^{n_{\text{Raman}}} + B$$

$$T = 2-3.2\text{K}$$

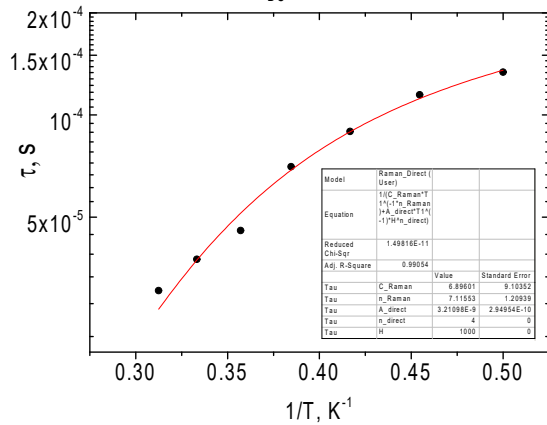
$$C_{\text{Raman}} = 30 \pm 24 \text{ s}^{-1} \text{K}^{-n_{\text{Raman}}}$$

$$n_{\text{Raman}} = 6.0 \pm 0.7$$

$$B = 5533 \pm 636 \text{ s}^{-1}$$

$$R^2 = 0.99263$$

$H_{DC} = 1000 \text{ Oe}$



Raman+Direct

$$\tau^{-1} = C_{\text{Raman}} T^{n_{\text{Raman}}} + A_{\text{direct}} T^H$$

$$T = 2-3.2\text{K}$$

$$C_{\text{Raman}} = 6.8 \pm 9 \text{ s}^{-1} \text{K}^{-n_{\text{Raman}}}$$

$$n_{\text{Raman}} = 7.1 \pm 1.2$$

$$A_{\text{direct}} = 3 \cdot 10^{-9} \pm 3 \cdot 10^{-10} \text{ K}^{-1} \text{Oe}^{-4} \text{ s}^{-1}$$

$$R^2 = 0.99054$$

Meaningless fit!!!

Table S10. CASSCF computed energy levels (the composition of the effective g' - tensor (assuming a pseudospin $S = \frac{1}{2}$) and major (>10%) components of the wave function for each mJ state of the lowest atomic multiplet $J=9/2$ of Nd^{3+} . KDs involved in magnetic relaxation are specified.

KD	E,K	g_x	g_y	g_z	Wavefunction decomposition, %
1	0.0	0.366	1.941	4.164	$52 \pm 9/2\rangle 26 \pm 5/2\rangle 12 \pm 1/2\rangle$
2	141.7	3.089	1.734	0.345	$15 \pm 9/2\rangle 29 \pm 7/2\rangle 10 \pm 5/2\rangle 27 \pm 3/2\rangle 20 \pm 1/2\rangle$
3	261.9	0.447	1.195	4.794	$18 \pm 9/2\rangle 24 \pm 7/2\rangle 14 \pm 5/2\rangle 10 \pm 3/2\rangle 35 \pm 1/2\rangle$
4	397.5	3.152	2.526	0.513	$28 \pm 7/2\rangle 41 \pm 3/2\rangle 24 \pm 1/2\rangle$
5	490.1	0.584	2.011	3.651	$13 \pm 9/2\rangle 15 \pm 7/2\rangle 46 \pm 5/2\rangle 16 \pm 3/2\rangle 10 \pm 1/2\rangle$

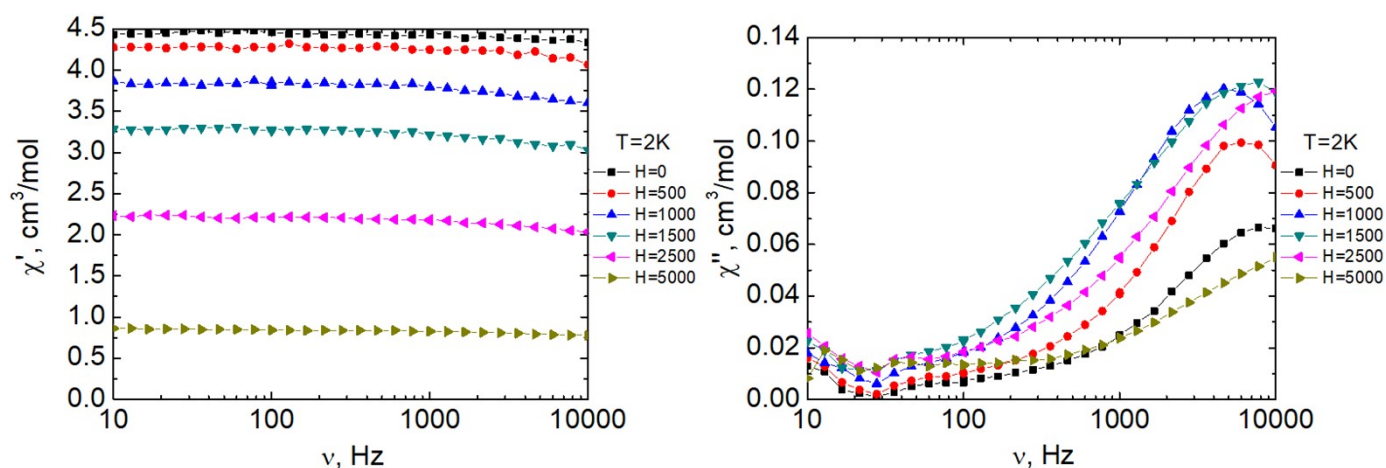


Fig. S12. Frequency dependencies of the in-phase (χ' , left) and out-of-phase (χ'' , right) components of the dynamic magnetic susceptibility of complex **Ho** measured at $T=2\text{K}$ under various DC magnetic fields. Solid lines are visual guides.

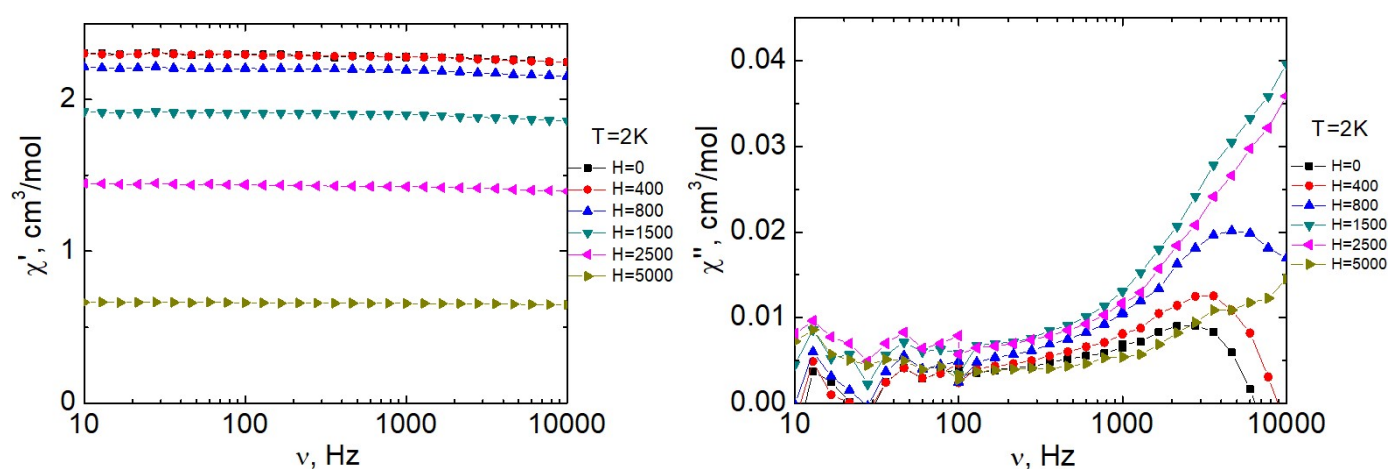


Fig. S13. Frequency dependencies of the in-phase (χ' , left) and out-of-phase (χ'' , right) components of the dynamic magnetic susceptibility of complex **Tm** measured at $T=2\text{K}$ under various DC magnetic fields. Solid lines are visual guides.