Supplemental Material for
Superlubricity in granular shear flows under external vibrations

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I. FLUCTUATIONS IN THE DISTANCE BETWEEN THE PLANES

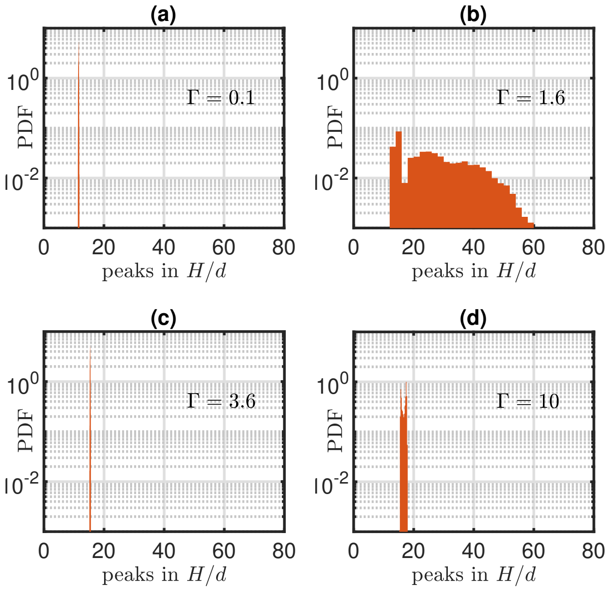

FIG. 1. PDF of the peaks in the distance between the planes for $p/\rho_p V^2 = 1$, $A/d = 1$ and: (a) $\Gamma = 0.1$, (b) $\Gamma = 1.6$, (c) $\Gamma = 3.6$, and (d) $\Gamma = 10$.

As mentioned in the main text, we observed fluctuations in the distance between the shearing and vibrating planes, which induce fluctuations in the macroscopic friction. The Probability Density Function (PDF) of the peaks in H/d, once a steady state is achieved, for different values of Γ, when $p/\rho_p V^2 = 1$ and $A/d = 1$, is shown in Figure 1. The PDF is narrow when Γ is less than 0.1 and larger than, say, 2. In between, the PDF is wide and rather uniform, as in the chaotic regime of a bouncing elastic body [1]. Similarly to the latter, once Γ is greater than 10, the PDF of H/d has two distinct peaks (Figure 1d), hinting at a dynamical bifurcation.

II. INFLUENCE OF POLY-DISPERSITY AND GEOMETRY OF THE PLANES

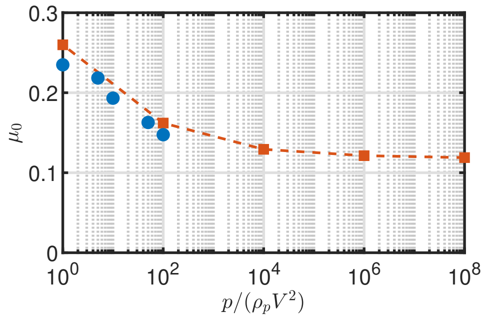

FIG. 2. Macroscopic friction as a function of the imposed pressure in the absence of vibrations measured in the case of: mono-dispersed spheres sheared between planes of regular bumpiness (present simulations, blue circles); and poly-dispersed spheres sheared between planes of irregular bumpiness (after [2], orange squares).

Figure 2 shows the comparisons between the macroscopic friction in the absence of vibrations, μ_0, measured in our simulations –with identical, frictionless spheres sheared between planes of regular geometry, which permit slip at the boundary [3]– and that in discrete simulations with poly-dispersed spheres and planes of irregular geometry [2], over a range of imposed pressure. Both poly-dispersity and the geometry of the boundaries play a minor role in the resistance to shear (the difference is within 10%).

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III. INFLUENCE OF SLIDING FRICTION

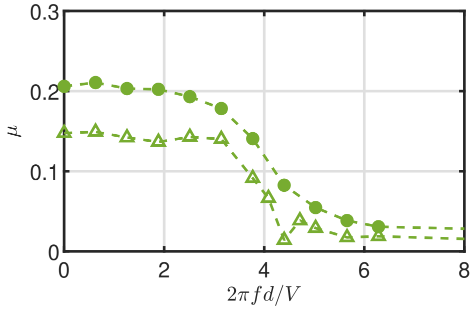

FIG. 3. Measured macroscopic friction as a function of the velocity amplitude when $p/\left(\rho_p V^2\right) = 100$ for: frictionless spheres (open triangles); and frictional spheres ($\mu_p = 0.2$, filled circles).

We have performed DEM simulations using monodispersed frictional spheres (with sliding friction $\mu_p = 0.2$, a typical value measured in particle-particle collision experiments [4]) with imposed pressure $p/\left(\rho_p V^2\right) = 100$, $A/d = 1$ and various velocity amplitudes. As shown in Figure 3, macroscopic friction is about 30% larger than in the frictionless case, but the qualitative behavior is remarkably similar: μ is independent of the velocity amplitude for small $2\pi fd/V$ and reduces dramatically as vibrations intensify.

[1] M. Hubert, F. Ludewig, S. Dorbolo, and N. Vandewalle, Bouncing dynamics of a spring, Physica D **272**, 1 (2014).

[2] A. H. Clark, E. E. Brodsky, H. J. Nasrin, and S. E. Taylor, Frictional weakening of vibrated granular flows, Phys. Rev. Lett. **130**, 118201 (2023).

[3] D. Vescovi, A. S. de Wijn, G. L. Cross, and D. Berzi, Extended kinetic theory applied to pressure-controlled shear flows of frictionless spheres between rigid, bumpy planes, Soft Matter **20**, 8702 (2024).

[4] S. F. Foerster, M. Y. Louge, H. Chang, and K. Allia, Measurements of the collision properties of small spheres, Phys. Fluids **6**, 1108 (1994).