

## Supplementary Information:

### Fill Factor:

The fill factor,  $F$ , is commonly defined as the ratio of the active thermoelectric material to the area of the ceramic plate. This is illustrated in Fig. 1 in the top view of a pair of thermoelectric legs. The cross sectional area of one leg (either the p-type or n-type leg) is  $A_{c,i}=AF/2$ . Given this definition of  $F$ , factors of two are present in Eqn. 8 that yield the factor of four in Eqns. 19 and 25. A different definition of  $F$  may change these pre-factors.

### Heat Exchanger Cost:<sup>17</sup>

The costs of heat exchangers are typically reported in units of dollars per thermal conductance [\$/ (W/K)] which can be expressed in units of [\$/m<sup>2</sup>] by multiplying  $C_{HX}$  by the heat exchanger's  $U$ -value. The resulting areal cost of real heat exchangers<sup>S1</sup> is linear with the  $U$ -value as shown in Fig S1. Approximating each line as passing through the origin, the slope is the  $C_{HX}$  value for that class of heat exchangers and for that specific design demand conductance (*i.e.*,  $Q/(T_H-T_L)$ ); different heat exchanger types and different design requirements will have different  $C_{HX}$  values.

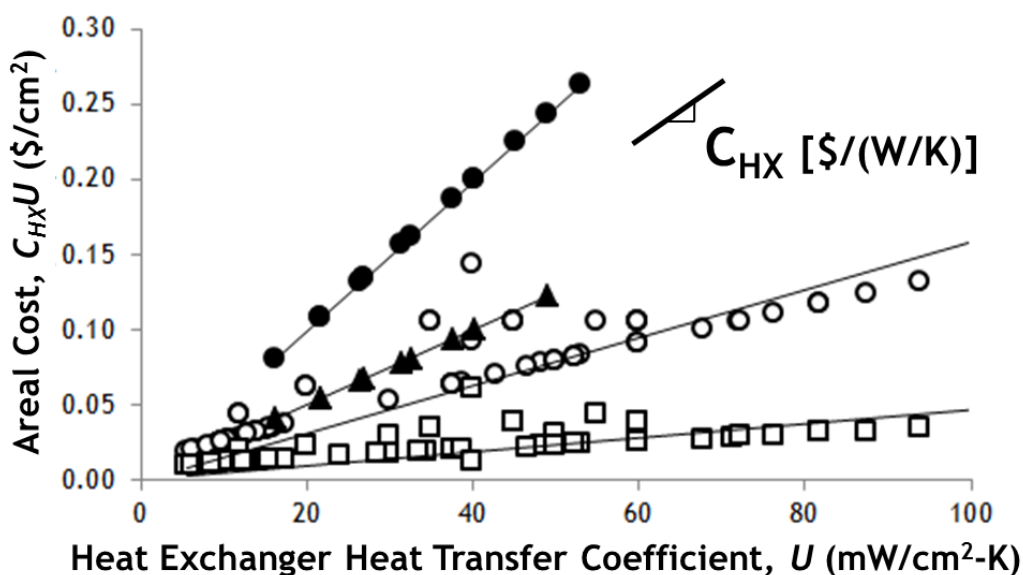


Figure S1: Heat exchanger costs. Typical areal cost as a function of heat transfer coefficient for tube and shell (open points) and plate and fin heat exchangers (solid points). The cost depends on the heat flow  $Q_H$  and temperature difference ( $T_H-T_L$ ). For  $K_H=Q_H/(T_H-T_L)=5$  kW/K (circles), 10 kW/K (triangles), and 30 kW/K (squares). Data extracted from Ref. 18.

### Universal Axes for Fig. 2

The universal axes in Fig. 2 are motivated by recognizing that, with  $m=1$ , Eqn. 25 can be recast as:

$$\frac{G}{G_0 \tilde{L}_{HX}} \approx (y+x)^2 \left( 1 + \frac{a}{x} + \frac{2}{xy} \right) \quad (\text{S1})$$

where

$$x \equiv \frac{\tilde{L}}{\sqrt{\tilde{L}_{HX}}} \text{ and } y \equiv \frac{F}{\frac{1}{2} \sqrt{\tilde{L}_{HX}}} \quad (\text{S2})$$

and

$$a \equiv \frac{\tilde{L}_C}{\sqrt{\tilde{L}_{HX}}} = \frac{C''}{\sqrt{kC'''C_{HX}}} \quad (\text{S3})$$

For most realistic materials and applications,  $a \ll 1$  or is at most on the order of 1.<sup>1</sup> Plotting Eqn. S1 for different  $a$  shows that the differences in Fig. 2 between the cases  $a=0$  and  $a=1$  are only at the level of tens of percent in the vicinity of the characteristic point of diminishing returns. Such errors may be considered negligible for the present analysis, considering the large overall variations in  $G$  by factors of 10 or more. Setting  $a$  to 0 in Eqn. S1 shows that  $\left( \frac{G}{G_0 \tilde{L}_{HX}} \right)$  becomes purely a function of the dimensionless quantities  $x$  and  $y$  defined in Eqn. S2, thus justifying the universal axes used in Fig. 2 of the main text. In other words, as long as a given material and application correspond to  $a$  of around 1 or smaller, the  $G$  surface in Fig. 2 can be applied directly and quantitatively by using the universal axes. For any such scenario with negligible  $a$ , the characteristic point of diminishing returns is  $(x,y)=(1,1)$ , and Eqn. S1 can be simplified to

$$\frac{G}{G_0 \tilde{L}_{HX}} \approx (y+x)^2 \left( 1 + \frac{2}{xy} \right) \quad (\text{S4})$$

This convenient form can be used to quantify the behavior near the point of diminishing returns. For example, along the line  $x=y$  (equivalent to  $F=\tilde{L}/2$  and corresponding to the trough of Fig. 2), at  $x=y=1$ , Eqn. S4 shows that  $G$  will be within 50% of its ultimate best case. Selecting smaller values of  $x$  and  $y$  will bring  $G$  closer to this ultimate limit; for example, at  $x=y=1/2$ ,  $G$  is only 12.5% greater than the best case.

### Cost-Dominant Regime Map for Fig. 5

The cost-dominant regime map in Fig. 5 was created by considering the additive cost terms in Eqn. 17. Comparing the areal cost to the heat exchanger cost produces the conditional statement that

$$\text{if } C''AF \leq C_{HX}UA, \text{ then } \frac{C''F}{C_{HX}U} \leq 1. \quad (\text{S5})$$

Similarly, comparing the volumetric costs to the areal costs produces the conditional statement that

$$\text{if } C'''LAF \leq C''AF, \text{ then } \frac{L}{L_c} \leq 1. \quad (\text{S6})$$

Finally comparing the volumetric costs to the heat exchanger costs produces the conditional statement that

$$\text{if } C'''LAF \leq C_{HX}UA, \text{ then } \frac{C''F}{C_{HX}U} \leq \left( \frac{L}{L_c} \right)^{-1} \quad (\text{S7})$$

which is a diagonal line on the log-log plot.

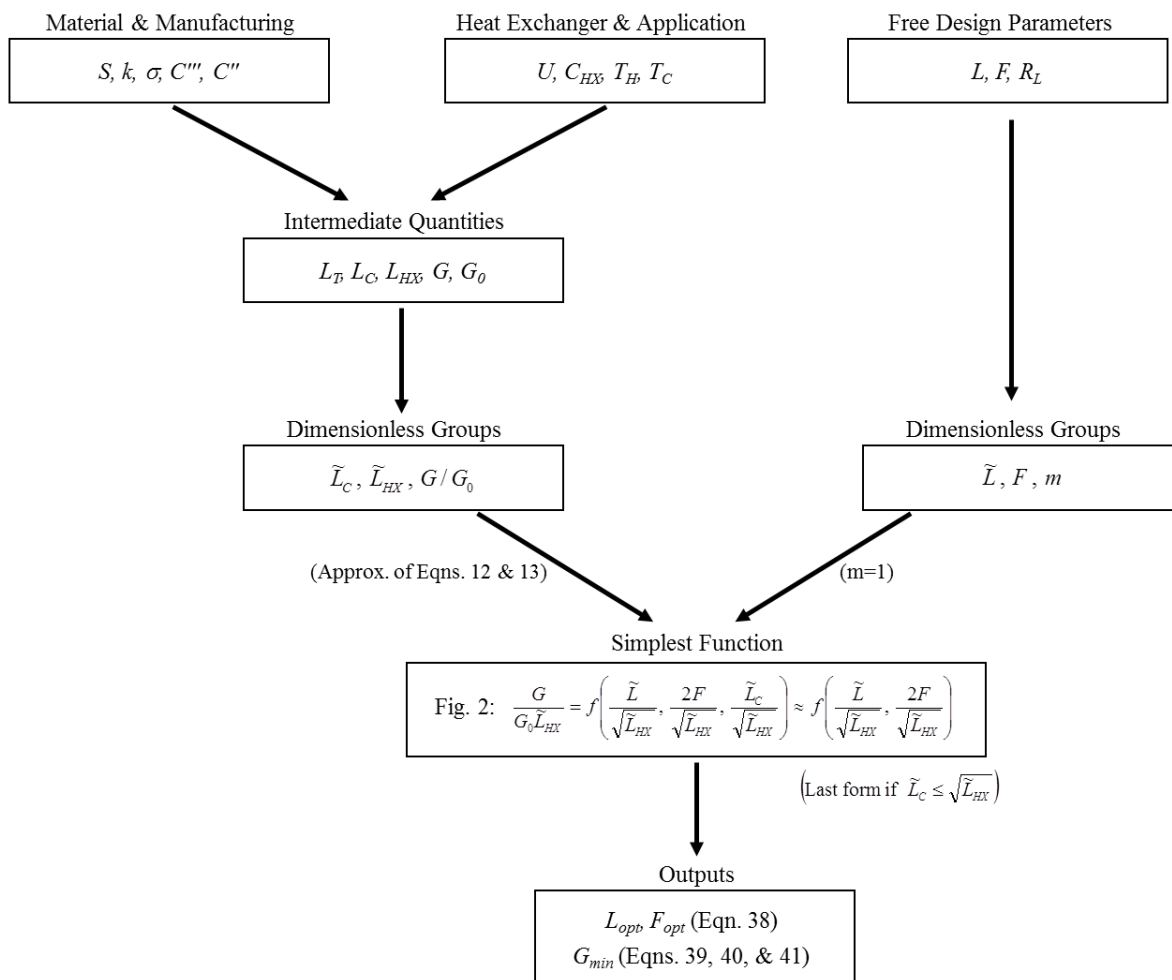


Figure S2: Schematic flow chart of the analysis.