

## Supplemental Information

### **The matrix method for calculating the optical properties of an $N+2$ layer stack**

Optical modeling is performed using scattering-matrix formalism to describe transmission and reflection at interfaces as well as propagation through absorbing media (see reference 11). The forward- and reverse-propagating waves on both sides of each interface are related by the continuity equations for the tangential electric and magnetic fields. For  $N$  layers (of thickness  $d_1, d_2, \dots, d_N$ ) arranged between a semi-infinite ambient and substrate (designated layers 0 and  $N+1$ , respectively), the transmission  $T$  and reflection  $R$  coefficients at each wavelength  $\lambda$  are related by:

$$\begin{bmatrix} T \\ 0 \end{bmatrix} = \bar{\beta} \cdot \bar{V}_N \bar{V}_{N-1} \cdots \bar{V}_2 \bar{V}_1 \begin{bmatrix} 1 \\ R \end{bmatrix} \quad (\text{SI.1})$$

We omit the explicit dependence of each term on incident wavelength  $\lambda$  for clarity; this dependence enters through the wavelength-dependent complex index of refraction  $\tilde{n}$  (introduced below). The 2x2 matrix  $\bar{V}_i$  describes the reflection/transmission at interface  $i$  (which separates layers  $i$  and  $i-1$ ) as well as the propagation through layer  $i$  to interface  $i+1$ . The matrices  $\bar{V}_i$  are given by:

$$\bar{V}_i = \frac{1}{2}(1 + p_i) \begin{bmatrix} \exp(-jk_z^i d_i) & 0 \\ 0 & \exp(jk_z^i d_i) \end{bmatrix} \cdot \begin{bmatrix} 1 & \Gamma_i \\ \Gamma_i & 1 \end{bmatrix} \quad (\text{SI.2})$$

where  $p_i = Z_i / Z_{i-1}$  is the ratio of the wave impedances  $Z$  in layers  $i$  and  $i-1$ , and  $\Gamma_i = (1 - p_i)/(1 + p_i)$  is the reflection coefficient at the interface between the layers  $i$  and  $i-1$ . In this form, we have explicitly separated the terms describing propagation through the distance  $d_i$  of layer  $i$  (the

first matrix) from reflection and transmission at the interface (the second matrix). For non-magnetic materials, the wave impedance of a medium differs depending on the polarization of the incident light:

$$Z_i^{TE} = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{\tilde{n}_i \cos(\theta_i)} \quad \text{SI.3}$$

$$Z_i^{TM} = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\cos(\theta_i)}{\tilde{n}_i} \quad \text{SI.4}$$

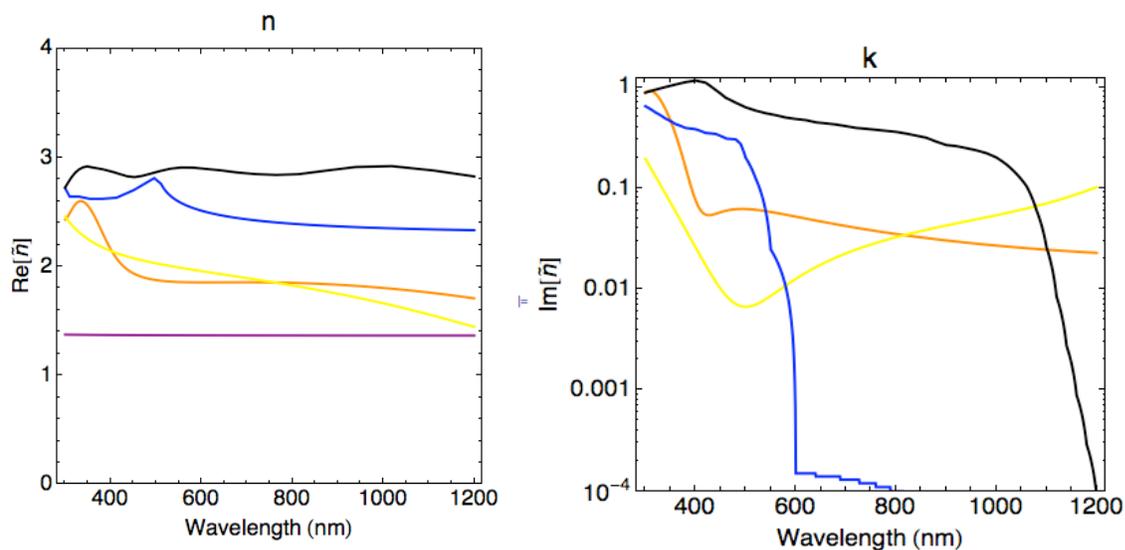
Where  $\mu_0$  and  $\epsilon_0$  are the permeability and permittivity of vacuum,  $\tilde{n}_i$  is the complex index of refraction of layer  $i$  — which is generally wavelength dependent — and  $\theta_i$  is the angle of propagation through layer  $i$ , which can be found by iteratively applying Snell's law ( $\tilde{n}_i \sin(\theta_i) = \tilde{n}_{i-1} \sin(\theta_{i-1})$ ), beginning with the initial angle of incidence  $\theta_0$  at the interface between the ambient and layer 1. The final interface between layer  $N$  and the substrate is described by the 2x2 matrix  $\bar{\beta}$ , which is equivalent to  $\overline{V_{N+1}}$  under the condition  $d_{N+1} = 0$ .

The transmittance  $t$  and reflectance  $r$  are related to  $T$  and  $R$  by:

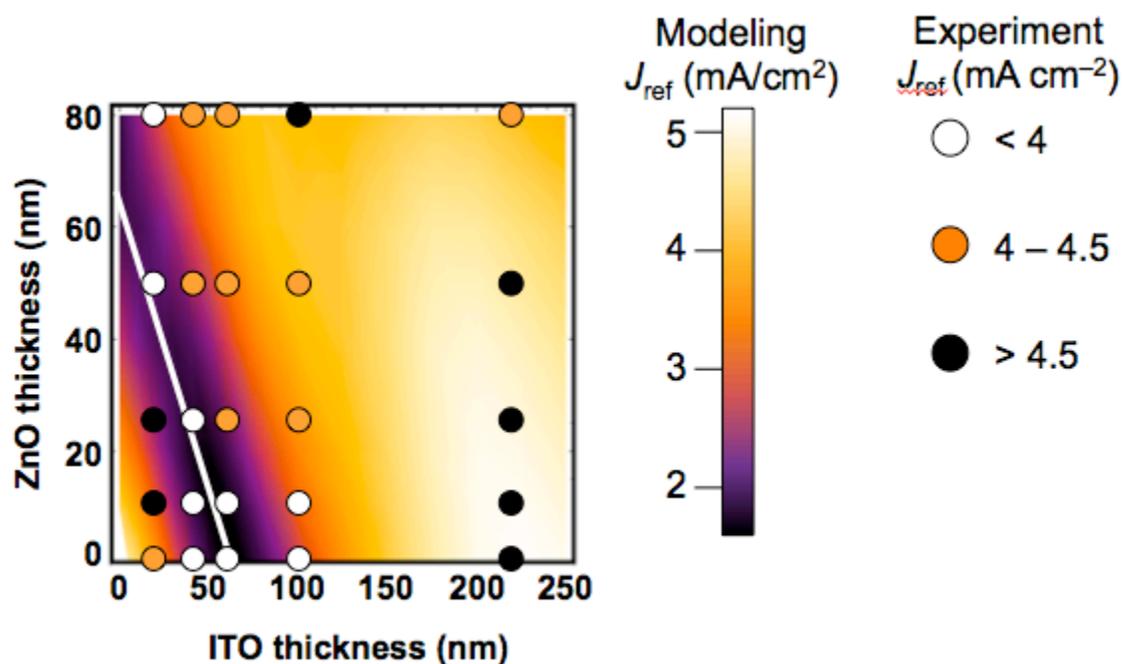
$$r = |R|^2 \quad \text{SI.5}$$

$$t = |T|^2 \frac{Z_0}{\text{Re}[Z_{N+1}^*]} \quad \text{SI.6}$$

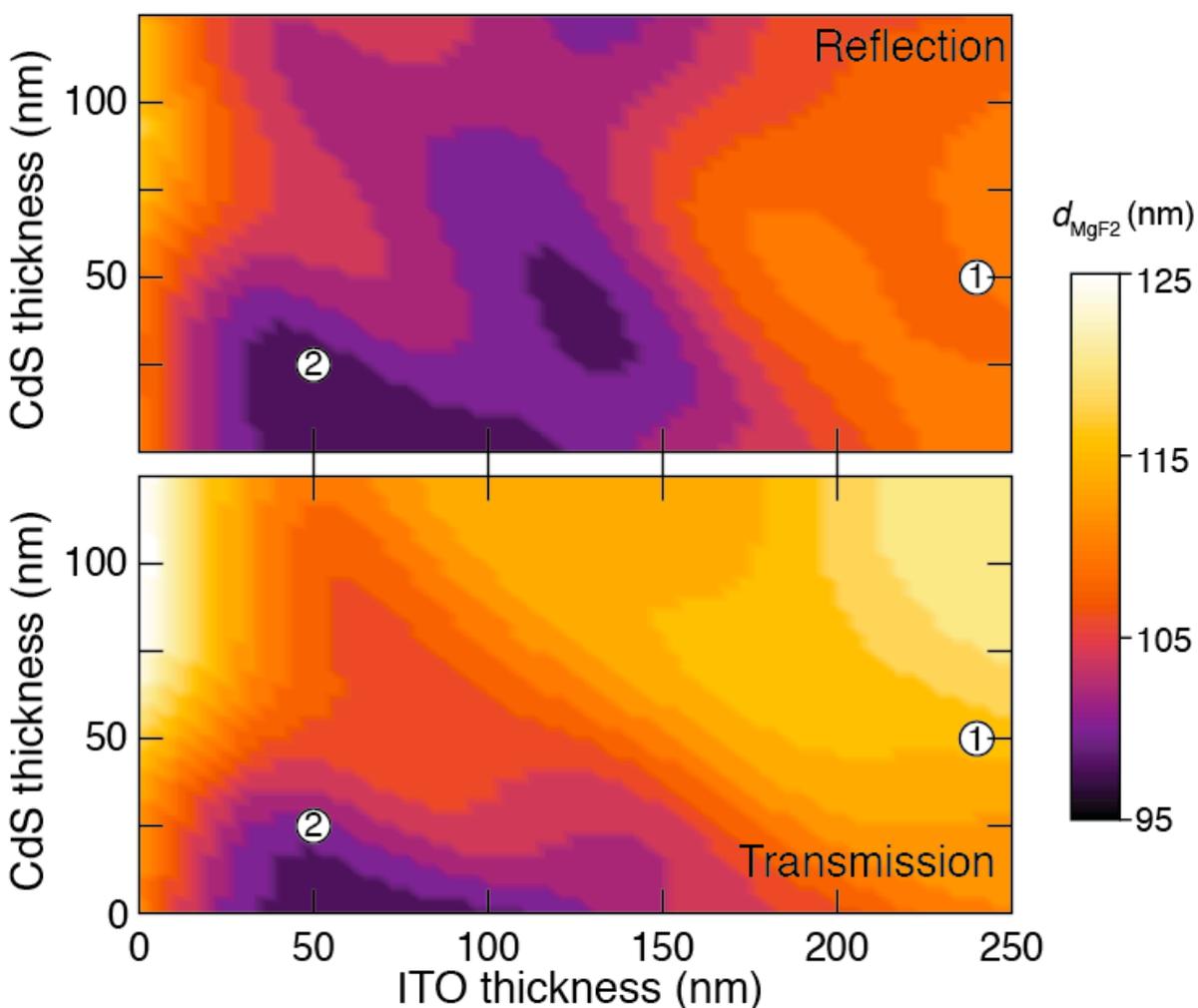
For all modeling performed in the text, the spectrally-dependent complex index of refraction  $\tilde{n}(\lambda)$  is used in equations SI.1 – SI.4 (see Figure SI.1). To model a mirror-like back interface underneath the CZTSSe, we used a purely imaginary index ( $\tilde{n} = i$ ) for all wavelengths.



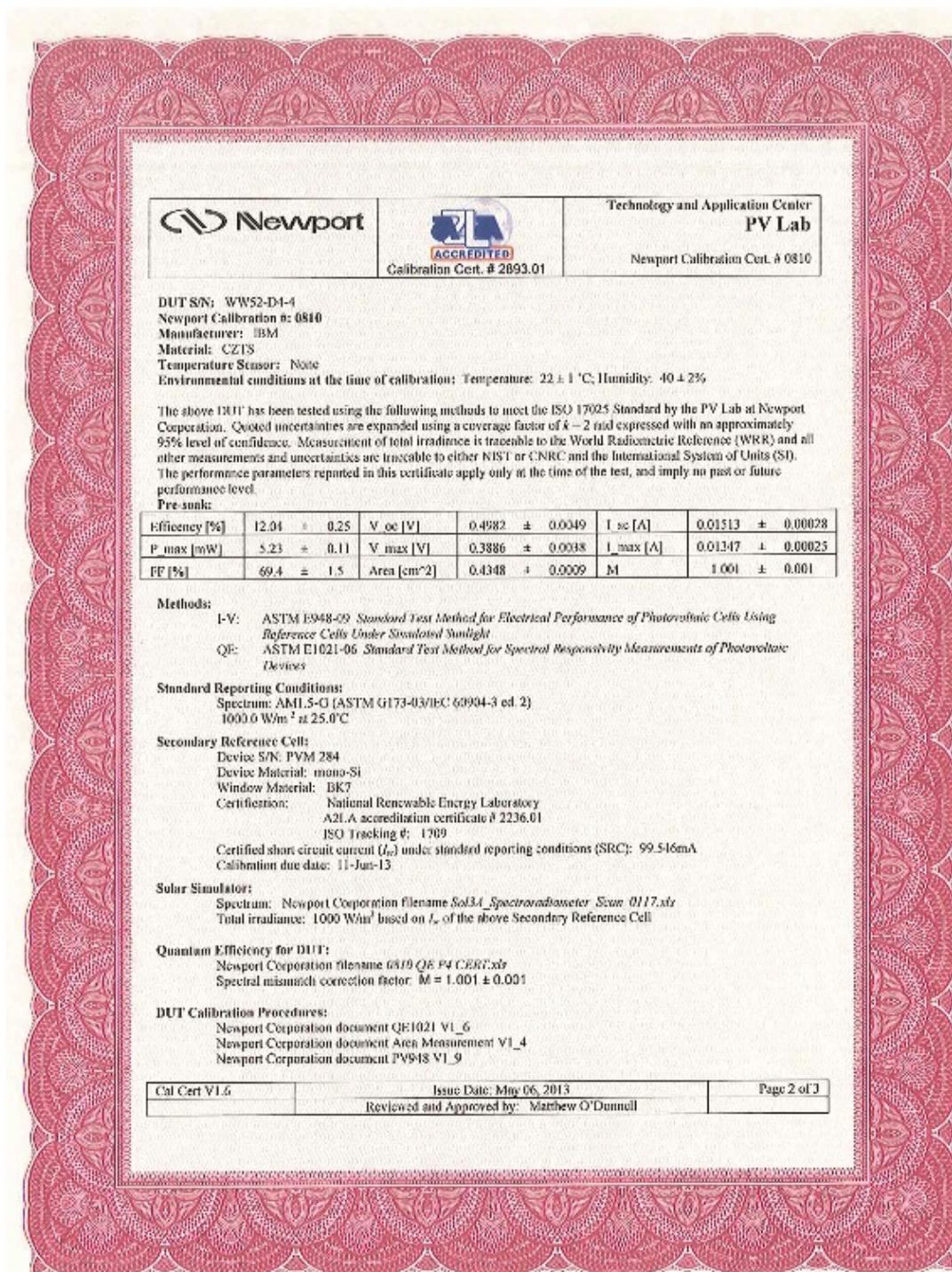
**Figure SI.1 – Index of refraction for optical modeling.** The  $n$ - and  $k$ -values used for the analytical optical modeling of CZTS (black), CdS (blue), ZnO (orange), ITO (yellow), and MgF<sub>2</sub> (purple). For ITO and ZnO, these values were measured by spectroscopic ellipsometry of thin films on Si wafers; for MgF<sub>2</sub>, fits to broadband reflectivity were used in the same geometry. Surface roughness prevented measurements of CdS and CZTSSe; values from literature were therefore used.



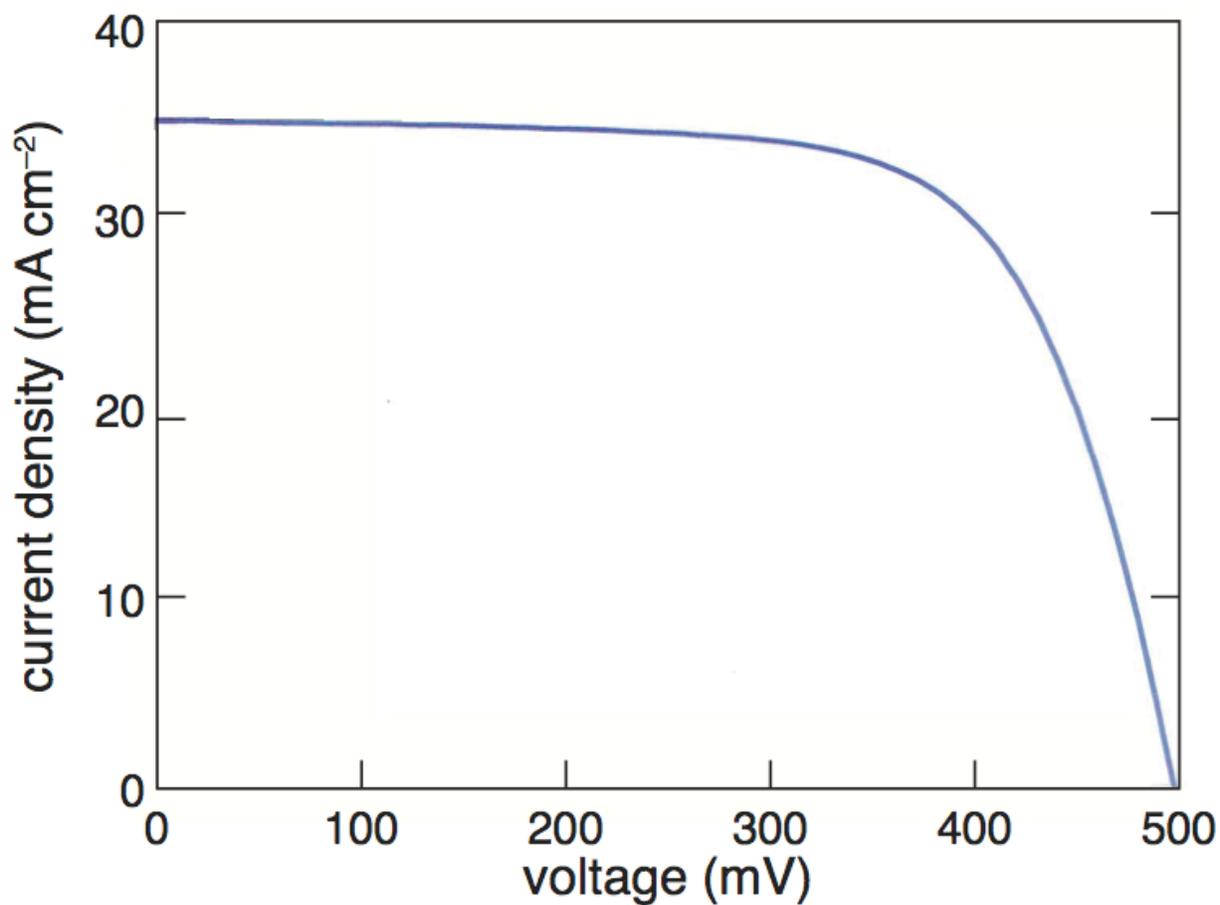
**Figure SI.2 – Reflectivity for varying TCO combinations.** The total reflected photon current is shown for both analytical modeling (density plot, see text) and experiment (overlaid points); reflected-photon current values are obtained by integrating the spectrally-dependent reflectivity against the AM1.5G spectrum between 300 — 1200 nm. Experiment and modeling confirm that, to a good approximation, reflectivity is a function of the total ITO and ZnO thickness due to their similar indices of refraction. The white line is a line of constant thickness ( $d_{\text{TCO}} = d_{\text{ITO}} + d_{\text{ZnO}} = 60 \text{ nm}$ ).



**Figure SI.3 – Optimal thickness of  $\text{MgF}_2$  for the architectures studied in the paper.** Figure 2 in the main text showed total reflected ( $J_R$ ) and transmitted ( $J_T$ ) currents for a wide range of TCO and CdS-layer thicknesses. At each point in Figure 2,  $J_R$  and  $J_T$  were chosen for the optimal value of  $\text{MgF}_2$  thickness. This figure shows that the optimal  $\text{MgF}_2$  thickness ranges from 95 – 125 nm when optimizing (*top*) reflection and (*bottom*) transmission.



**Figure SI.4 – Newport calibration certificate.** Certificate 0810, documenting the efficiency (12.0%) of the champion CZTS solar cell described in the main text.



**Figure SI.5 – Calibrated J-V** Certificate 0810, documenting the efficiency (12.0%) of the champion CZTS solar cell described in the main text.