Electronic Supplementary Material (ESI) for Energy & Environmental Science

Supplementary Information

Theoretical Study of Contact-Mode Triboelectric Nanogenerators as an Effective Power Source

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1. Derivation of Equation 17-19 for uniform velocity motion

For uniform velocity motion, we have:

$$x(t) = vt \qquad \text{(S1)}$$

Substitute Equation S1 into Equation 14, we can obtain:

$$Q(t) = \sigma S - \sigma S\, exp[-\frac{1}{RS\varepsilon_0}(d_0 t + \frac{1}{2}vt^2)] - \frac{\sigma d_0}{R\varepsilon_0} exp[-\frac{1}{RS\varepsilon_0}(d_0 t + \frac{1}{2}vt^2)] \int_0^t exp\left[\frac{1}{RS\varepsilon_0}\left(d_0 z + \frac{1}{2}vz^2\right)\right] dz \quad \text{(S2)}$$

Equation S2 is very complicated. To simplify it, we can define the following constant:

$$A = \frac{d_0}{RS\varepsilon_0} \qquad \text{(S3.1)}$$

$$B = \frac{1}{2RS\varepsilon_0} v \quad \text{(S3.2)}$$

And Equation S2 can be simplified as:

$$Q(t) = \sigma S - \sigma S exp(-At - Bt^2) - \sigma S A exp(-At - Bt^2) \int_0^t exp(Az + Bz^2)\, dz \quad \text{(S4)}$$

Now we are going to prove that:

$$\int_0^t exp(Az + Bz^2)\, dz = \frac{1}{\sqrt{B}} exp(At + Bt^2) Dawson\left(\frac{A}{2\sqrt{B}} + \sqrt{B}t\right) - \frac{1}{\sqrt{B}} Dawson\left(\frac{A}{2\sqrt{B}}\right) \quad \text{(S5)}$$

Proof:

$$\int_0^t exp(Az + Bz^2)\, dz = \int_0^t exp\left((\sqrt{B}z + \frac{A}{2\sqrt{B}})^2 - \frac{A^2}{4B}\right) dz$$

$$= exp\left(-\frac{A^2}{4B}\right)\int_0^t exp\left((\sqrt{B}z + \frac{A}{2\sqrt{B}})^2\right) dz$$

$$= \frac{1}{\sqrt{B}} exp\left(-\frac{A^2}{4B}\right)\int_{\frac{A}{2\sqrt{B}}}^{\sqrt{B}t+\frac{A}{2\sqrt{B}}} exp(z^2)\, dz$$

$$= \frac{1}{\sqrt{B}} exp\left(-\frac{A^2}{4B}\right)\int_0^{\sqrt{B}t+\frac{A}{2\sqrt{B}}} exp(z^2)\, dz - \frac{1}{\sqrt{B}} exp\left(-\frac{A^2}{4B}\right)\int_0^{\frac{A}{2\sqrt{B}}} exp(z^2)\, dz$$

$$= \frac{1}{\sqrt{B}} exp(At + Bt^2) exp[-\left(\sqrt{B}t + \frac{A}{2\sqrt{B}}\right)^2]\int_0^{\sqrt{B}t+\frac{A}{2\sqrt{B}}} exp(z^2)\, dz$$
$$- \frac{1}{\sqrt{B}} exp\left(-\frac{A^2}{4B}\right)\int_0^{\frac{A}{2\sqrt{B}}} exp(z^2)\, dz$$

$$= \frac{1}{\sqrt{B}} exp(At + Bt^2) Dawson\left(\frac{A}{2\sqrt{B}} + \sqrt{B}t\right) - \frac{1}{\sqrt{B}} Dawson\left(\frac{A}{2\sqrt{B}}\right)$$

Therefore,

$$Q(t) = \sigma S[1 - exp(-At - Bt^2) + \sqrt{2}F exp(-At - Bt^2) \times Dawson\left(\frac{F}{\sqrt{2}}\right) - \sqrt{2}F \times Dawson(\frac{F}{\sqrt{2}} + \sqrt{B}t)] \qquad (S6)$$

This is exactly the same equation as Equation 17. Similarly, Equation 18 and 19 can be obtained.

2. Basic property of Dawson's integral

For a Dawson's integral, it has the following basic property:

Its Maclaurin series can be given by:

$$Dawson(x) = x - \frac{2}{3}x^3 + O(x^5) \quad \text{(S7)}$$

Its asymptotic series can be given by:

$$Dawson(x) = \frac{1}{2x} + \frac{1}{4x^3} + O(\frac{1}{x^5}) \quad \text{(S8)}$$

Its derivative can be given by:

$$\frac{dDawson(x)}{dx} = 1 - 2xDawson(x) \quad \text{(S9)}$$

3. Analytical Equation derivation when t exceeds x_{max}/v

When t exceeds x_{max}/v, x equals to x_{max}. Therefore, Equation 12 can be simplified as

$$R\frac{dQ}{dt} = -\frac{Q}{S\varepsilon_0}(d_0 + x_{max}) + \frac{\sigma x_{max}}{\varepsilon_0} \qquad \text{(S10)}$$

From Equation S10, given the boundary condition that Q ($t = x_{\mathrm{max}}/v$) = Q_0 , Q is calculated as:

$$Q = \frac{\sigma S x_{max}}{d_0 + x_{max}} - \left(\frac{\sigma S x_{max}}{d_0 + x_{max}} - Q_0\right)\exp\left[-\frac{d_0 + x_{max}}{RS\varepsilon_0}\left(t - \frac{x_{max}}{v}\right)\right] \qquad \text{(S11)}$$

In Equation S11, Q_0 can be calculated by assigning t equals to x_{max}/v into Equation 14.

And when t exceeds x_{max}/v, the current I can be given by:

$$I = \left(\frac{\sigma S x_{max}}{d_0 + x_{max}} - Q_0\right)\frac{d_0 + x_{max}}{RS\varepsilon_0}\exp\left[-\frac{d_0 + x_{max}}{RS\varepsilon_0}\left(t - \frac{x_{max}}{v}\right)\right] \qquad \text{(S12)}$$

The current decays exponentially with time. The decay time constant τ is given by:

$$\tau = \frac{RS\varepsilon_0}{d_0 + x_{max}} \qquad \text{(S13)}$$

When R is small, the current decays at a fast speed. While when R is sufficiently large, τ is sufficiently large and the current will decay at a fairly low speed.

4. Derivation of Equation 23 and Equation 24

When R is small, utilizing the first order of Equation S8, Equation 17 can be simplified as:

$$Q(t) = \frac{\sigma v}{R\varepsilon_0}\frac{t}{A + 2Bt} \qquad (S14)$$

Therefore,

$$I = \frac{dQ}{dt} = \frac{\sigma v}{R\varepsilon_0}\frac{A}{(A + 2Bt)^2} \qquad (S15)$$

When R is large, utilizing the first order of Equation S7, Equation 17 can be simplified as:

$$Q(t) = \frac{\sigma v}{R\varepsilon_0 B}[\frac{1}{2} - \frac{1}{2}exp(-At - Bt^2) - \frac{At}{2}] \qquad (S16)$$

And this time when R is sufficiently large, A and B are close to 0. Therefore,

$$exp(-At - Bt^2) \approx 1 - At - Bt^2 \qquad (S17)$$

Therefore,

$$Q(t) = \frac{\sigma v t^2}{2R\varepsilon_0} \qquad (S18)$$

$$I = \frac{dQ}{dt} = \frac{\sigma v}{R\varepsilon_0}t \qquad (S19)$$

Thus,

$$V = IR = \frac{\sigma v t}{\varepsilon_0} \qquad (S20)$$

5. Derivation of the optimum resistance relationship with TENG parameters

Proof of $t_0 = \frac{x_{max}}{v} f(F, y) \qquad (S21)$

From $(\frac{dI}{dt})_{t=t_0} = 0$, we can get:

$$\sum_{n=1}^{\infty} \frac{n\alpha_n v^n}{x_{max}^{n-1} d_0} {t_0}^{n-1} \times \exp\left[-\frac{d_0^2}{RS\varepsilon_0 v}\left(\frac{vt_0}{d_0} + \sum_{n=1}^{\infty} \frac{\alpha_n v^{n+1}}{(n+1)x_{max}^{n-1} d_0^2} {t_0}^{n+1}\right)\right] + (1$$
$$+ \sum_{n=1}^{\infty} \frac{\alpha_n v^n}{x_{max}^{n-1} d_0} {t_0}^n) \times \exp\left[-\frac{d_0^2}{RS\varepsilon_0 v}\left(\frac{vt_0}{d_0} + \sum_{n=1}^{\infty} \frac{\alpha_n v^{n+1}}{(n+1)x_{max}^{n-1} d_0^2} {t_0}^{n+1}\right)\right]$$
$$\times \left(-\frac{d_0^2}{RS\varepsilon_0 v}\right) \times (\frac{v}{d_0} + \frac{v}{d_0} \sum_{n=1}^{\infty} \frac{\alpha_n v^n}{x_{max}^{n-1} d_0} {t_0}^n) + \frac{d_0^2}{RS\varepsilon_0 v} \sum_{n=1}^{\infty} \frac{n\alpha_n v^{n+1}}{x_{max}^{n-1} d_0^2} {t_0}^{n-1}$$
$$\times \exp\left[-\frac{d_0^2}{RS\varepsilon_0 v}\left(\frac{vt_0}{d_0} + \sum_{n=1}^{\infty} \frac{\alpha_n v^{n+1}}{(n+1)x_{max}^{n-1} d_0^2} {t_0}^{n+1}\right)\right]$$
$$\times \int_0^{t_0} \exp\left[\frac{d_0^2}{RS\varepsilon_0 v}\left(\frac{vz}{d_0} + \sum_{n=1}^{\infty} \frac{\alpha_n v^{n+1}}{(n+1)x_{max}^{n-1} d_0^2} \mathrm{z}^{n+1}\right)\right] dz - \left(\frac{d_0^2}{RS\varepsilon_0 v}\right)^2 (\frac{v}{d_0}$$
$$+ \sum_{n=1}^{\infty} \frac{\alpha_n v^{n+1}}{x_{max}^{n-1} d_0^2} {t_0}^n) \exp\left[-\frac{d_0^2}{RS\varepsilon_0 v}\left(\frac{vt_0}{d_0} + \sum_{n=1}^{\infty} \frac{\alpha_n v^{n+1}}{(n+1)x_{max}^{n-1} d_0^2} {t_0}^{n+1}\right)\right]$$
$$\times (\frac{v}{d_0} + \frac{v}{d_0} \sum_{n=1}^{\infty} \frac{\alpha_n v^n}{x_{max}^{n-1} d_0} {t_0}^n)$$
$$\times \int_0^{t_0} \exp\left[\frac{d_0^2}{RS\varepsilon_0 v}\left(\frac{vz}{d_0} + \sum_{n=1}^{\infty} \frac{\alpha_n v^{n+1}}{(n+1)x_{max}^{n-1} d_0^2} \mathrm{z}^{n+1}\right)\right] dz + \frac{d_0^2}{RS\varepsilon_0 v} (\frac{v}{d_0}$$
$$+ \frac{v}{d_0} \sum_{n=1}^{\infty} \frac{\alpha_n v^n}{x_{max}^{n-1} d_0} {t_0}^n) = 0 \qquad (S22)$$

The parameter *w* was defined as: $w = t_0 \frac{v}{x_{max}}$,

Noting that

$$\int_0^{t_0} exp\left[\frac{d_0^2}{RS\varepsilon_0 v}\left(\frac{vz}{d_0}+\sum_{n=1}^{\infty}\frac{\alpha_n v^{n+1}}{(n+1)x_{max}^{n-1}d_0^2}z^{n+1}\right)\right]dz$$
$$=\int_0^{w} exp\left[\frac{d_0^2}{RS\varepsilon_0 v}\left(\frac{x_{max}}{d_0}z+\frac{x_{max}^2}{d_0^2}\sum_{n=1}^{\infty}\frac{\alpha_n z^{n+1}}{n+1}\right)\right]\frac{x_{max}}{v}dz$$
$$=\frac{x_{max}}{v}\int_0^{w}\exp\left[F^2\left(yz+y^2\sum_{n=1}^{\infty}\frac{\alpha_n z^{n+1}}{n+1}\right)\right]dz \quad (S23)$$

The above equation can be simplified as

$$\frac{1}{F^2}\sum_{n=1}^{\infty}n\alpha_n w^{n-1}\, exp[-F^2(yw+y^2\sum_{n=1}^{\infty}\frac{\alpha_n}{n+1}w^{n+1})]$$
$$-\, exp[-F^2(yw+y^2\sum_{n=1}^{\infty}\frac{\alpha_n}{n+1}w^{n+1})]\left(1+y\sum_{n=1}^{\infty}\alpha_n w^n\right)^2$$
$$+\, y\sum_{n=1}^{\infty}n\alpha_n w^{n-1}\, exp[-F^2(yw+y^2\sum_{n=1}^{\infty}\frac{\alpha_n}{n+1}w^{n+1})]$$
$$\times\int_0^{w} exp\left[F^2\left(yz+y^2\sum_{n=1}^{\infty}\frac{\alpha_n z^{n+1}}{n+1}\right)\right]dz$$
$$-\, F^2 y\left(1+y\sum_{n=1}^{\infty}\alpha_n w^n\right)^2 exp[-F^2(yw$$
$$+\, y^2\sum_{n=1}^{\infty}\frac{\alpha_n}{n+1}w^{n+1})]\int_0^{w} exp\left[F^2\left(yz+y^2\sum_{n=1}^{\infty}\frac{\alpha_n z^{n+1}}{n+1}\right)\right]dz+1$$
$$+\, y\sum_{n=1}^{\infty}\alpha_n w^n=0 \quad (S24)$$

Therefore, we know that *w* is only a function (*f*) of *F* and *y*

$$w=f(\mathrm{F},\mathrm{y}) \qquad (S25)$$

Therefore, $t_0=\frac{x_{max}}{v}w=\frac{x_{max}}{v}f(F,y) \qquad (S26)$

So the peak value of current will happen at

$t_{max}=\min\left(t_0,\frac{x_{max}}{v}\right)=\frac{x_{max}}{v}\min(f(F,y),1)=\frac{x_{max}}{v}M(\mathrm{F},\mathrm{y}) \quad (S27)$

Substitute this back to the general equation for current (Equation 27)

Noting that

$$1+\frac{1}{d_0}\sum_{n=1}^{\infty}\frac{\alpha_n v^n}{x_{max}^{n-1}}t_{max}^n = 1+y\sum_{n=1}^{\infty}\alpha_n M^n(F,y) \qquad (S28.1)$$

$$-\frac{1}{RS\varepsilon_0}\left(d_0 t_{max}+\sum_{n=1}^{\infty}\frac{\alpha_n v^n}{(n+1)x_{max}^{n-1}}t_{max}^{n+1}\right) = -F^2yM(F,y)(1+y\sum_{n=1}^{\infty}\frac{\alpha_n M^n(F,y)}{n+1}) \qquad (S28.2)$$

$$\int_0^{t_{max}}\exp\left[\frac{1}{RS\varepsilon_0}\left(d_0 z+\sum_{n=1}^{\infty}\frac{\alpha_n v^n}{(n+1)x_{max}^{n-1}}z^{n+1}\right)\right]dz = \frac{x_{max}}{v}\int_0^{\mathrm{M(F,y)}}\exp\left[F^2y\left(z+y\sum_{n=1}^{\infty}\frac{\alpha_n z^{n+1}}{n+1}\right)\right]dz \qquad (S28.3)$$

Therefore,

$$\begin{aligned}I_{max} = \frac{\sigma d_0}{R\varepsilon_0}\Bigg\{&-1+\left(1+y\sum_{n=1}^{\infty}\alpha_n M^n(F,y)\right)\\ &\times\exp\left[-F^2yM(F,y)(1+y\sum_{n=1}^{\infty}\frac{\alpha_n M^n(F,y)}{n+1})\right]\\ &+F^2y\left(1+y\sum_{n=1}^{\infty}\alpha_n M^n(F,y)\right)\\ &\times\exp\left[-F^2yM(F,y)(1+y\sum_{n=1}^{\infty}\frac{\alpha_n M^n(F,y)}{n+1})\right]\\ &\times\int_0^{\mathrm{M(F,y)}}\exp\left[F^2y\left(z+y\sum_{n=1}^{\infty}\frac{\alpha_n z^{n+1}}{n+1}\right)\right]dz\Bigg\}\end{aligned} \qquad (S29)$$

Therefore, the peak value of current I_{max} can be written as

$$I_{max} = \frac{\sigma d_0}{R\varepsilon_0}G(F,y) \qquad (S30)$$

The function $G(F,y)$ is given by:

$$G(F,y) = -1 + \left(1 + y\sum_{n=1}^{\infty}\alpha_n M^n(F,y)\right)$$
$$\times \exp\left[-F^2 yM(F,y)(1 + y\sum_{n=1}^{\infty}\frac{\alpha_n M^n(F,y)}{n+1})\right]$$
$$+ F^2 y\left(1 + y\sum_{n=1}^{\infty}\alpha_n M^n(F,y)\right)$$
$$\times \exp\left[-F^2 yM(F,y)(1 + y\sum_{n=1}^{\infty}\frac{\alpha_n M^n(F,y)}{n+1})\right]$$
$$\times \int_0^{\mathrm{M(F,y)}} \exp\left[F^2 y\left(z + y\sum_{n=1}^{\infty}\frac{\alpha_n z^{n+1}}{n+1}\right)\right]dz \qquad (S31)$$

During this time period, maximum transient power output can be calculated as:

$$P_{max} = I_{max}^2 R = \frac{(\sigma d_0)^2}{\varepsilon_0^2} \times \frac{1}{R} G^2(F,y) = \frac{\sigma^2 Sv}{\varepsilon_0} F^2 G^2(F,y) \qquad (S32)$$

The optimized load resistance satisfied the following equation,

$$\frac{\partial P_{max}}{\partial R} = 0 \qquad (S33)$$

Considering that only F contains R, Equation S28 can be simplified as

$$\frac{\partial P_{max}}{\partial F}\frac{\partial F}{\partial R} = 0 \qquad (S34)$$

$$\left[2FG^2(F,y) + 2G(F,y)\frac{\partial G}{\partial F}\right]\frac{\partial F}{\partial R} = 0 \qquad (S35)$$

Noting that at R_{opt}, G is not 0 and

$$\frac{\partial F}{\partial R} = -\frac{F}{2R} \qquad (S36)$$

Equation S35 can be simplified as the following equation

$$G(F_{opt}, y) + F_{opt}\left(\frac{\partial G}{\partial \mathrm{F}}\right)_{F=F_{opt}} = 0 \qquad (S37)$$

Equation S37 is exactly the same as Equation 36.

6. Derivation of Equation 39

At a specific case when the moving mode is in uniform velocity motion, we have the following equation.

$$\alpha_n = 0\ (n \neq 1)\ \ \alpha_1 = 1 \quad M(F,y) = 1 \qquad (S38)$$

Put this value back into Equation S31, we can have:

$$G(F,y) = -1 + (1+y)\,exp\left[-\frac{1}{2}F^2y(2+y)\right] + F^2y(1+y)\,exp\left[-\frac{1}{2}F^2y(2+y)\right]\int_0^1 exp\left[F^2y\left(z+\frac{yz^2}{2}\right)\right]dz \quad (S39)$$

From Equation S5, we can obtain:

$$Fy\int_0^1 exp\left[F^2y\left(z+\frac{yz^2}{2}\right)\right]dz = -\sqrt{2}Dawson\left(\frac{F}{\sqrt{2}}\right) + \sqrt{2}\,exp\left[\frac{1}{2}F^2y(2+y)\right]Dawson\left[\frac{F}{\sqrt{2}}(1+y)\right] \ (S40)$$

Therefore,

$$G(F,y) = -1 + (1+y)\,exp\left[-\frac{1}{2}F^2y(2+y)\right]\left[1-\sqrt{2}FDawson\left(\frac{F}{\sqrt{2}}\right) + \sqrt{2}F(1+y)Dawson\left[\frac{F}{\sqrt{2}}(1+y)\right]\right] \qquad (S41)$$

Substitute Equation S41 into Equation S37.

Noting that:

$$\frac{\partial G}{\partial F} = -Fy(2+y)(1+y)\,exp\left[-\frac{1}{2}F^2y(2+y)\right]\left[1-\sqrt{2}FDawson\left(\frac{F}{\sqrt{2}}\right)\right]$$
$$-\sqrt{2}(1+y)\,exp\left[-\frac{1}{2}F^2y(2+y)\right]\left[Dawson\left(\frac{F}{\sqrt{2}}\right)+\frac{F}{\sqrt{2}}\right.$$
$$\left.-F^2Dawson\left(\frac{F}{\sqrt{2}}\right)\right]$$
$$+\sqrt{2}(1+y)\left\{Dawson\left[\frac{F}{\sqrt{2}}(1+y)\right]+\frac{F}{\sqrt{2}}(1+y)\right.$$
$$\left.-F^2(1+y)^2Dawson\left[\frac{F}{\sqrt{2}}(1+y)\right]\right\} \quad (S42)$$

We can get

$$[F^2(1+y)^2-1]\times\left\{1-(1+y)\,exp\left[-\frac{1}{2}F^2y(2+y)\right]\right\}+[F^2(1+y)^2-2]$$
$$\times\left\{\sqrt{2}(1+y)F\,exp\left[-\frac{1}{2}F^2y(2+y)\right]\times Dawson\left(\frac{\sqrt{2}}{2}F\right)-\sqrt{2}F(1+y)\right.$$
$$\left.\times Dawson\left[\frac{\sqrt{2}}{2}F(1+y)\right]\right\}=0 \quad (S43)$$

It can be simplified as

$$[F^2(1+y)^2-1]\times(-G)$$
$$=\sqrt{2}(1+y)F\,exp\left[-\frac{1}{2}F^2y(2+y)\right]\times Dawson\left(\frac{\sqrt{2}}{2}F\right)-\sqrt{2}F(1+y)$$
$$\times Dawson\left[\frac{\sqrt{2}}{2}F(1+y)\right] \qquad (S44)$$

When y is larger than 10, which means the gap is sufficiently larger than the thickness of dielectrics. The right half of Equation S43 is close to 0. Therefore, it can be solved approximately to the following solution.

$$F_{opt} = H(y) \approx \frac{1}{1+y} \qquad (S45)$$

The comparison of the exact solution of Equation S44 (numerically calculated by Matlab) and the approximate solution of Equation S45 is given by the following figure.

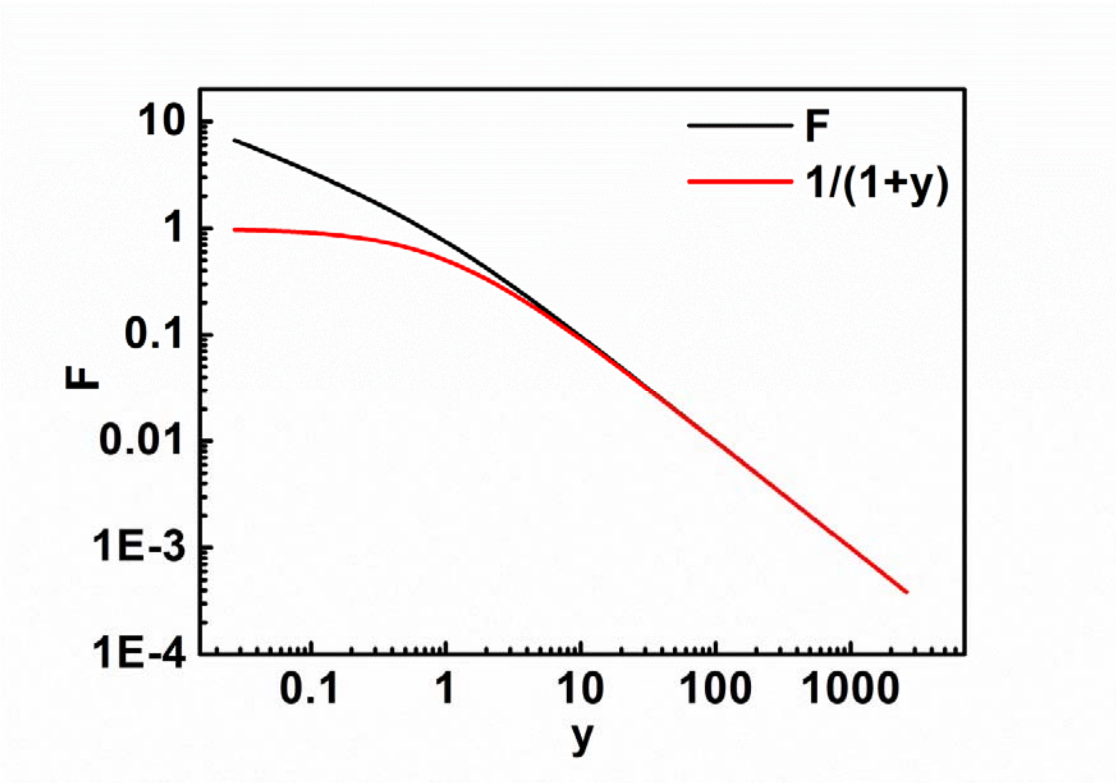

Fig. S1 Comparison of the exact solution from Equation S44 and the approximate solution from Equation S45.

7. Optimum resistance relationship with the thickness of the dielectric and the gap

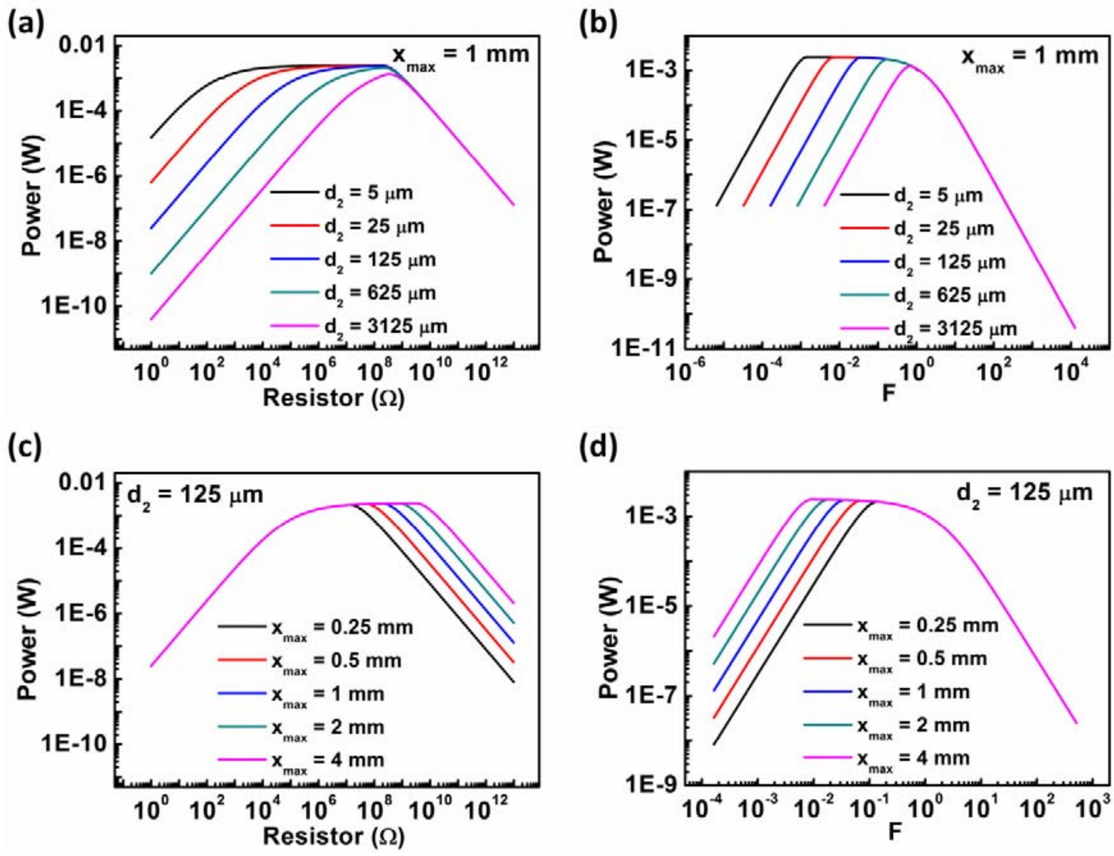

Fig. S2 Influence of thicknesses of the dielectrics and the gap on the optimum resistance. (a-b) Relationship of (a) R_{opt} (b) F_{opt} with x_{max} with maintaining the same dielectric thickness. (c-d) Relationship of (c) R_{opt} (d) F_{opt} with d_2 with maintaining the same gap thickness.